Aspects of mathematical creativity in children aged 11 - 12.

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ASPECTS OF MATHEMATICAL CREATIVITY IN CHILDREN AGED 11 - 12

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ABSTRACT

This study considers the relevance of ideas associated with creativity to children's experiences of mathematics. In the first emphasis of the study criteria for mathematical creativity tests are clarified, using two main constructs: (a) the ability to break from mental sets, by overcoming fixations in mathematics, on either content universe or algorithms; (b) divergent production in mathematical situations, involving problem solving, problem posing or redefinition. A battery of mathematical creativity tests was developed, administered to 283 children, aged 11 - 12, and evaluated by analysis of pupils' responses.

In the second emphasis of the study, six hypotheses were investigated concerning the relationships between mathematical creativity and the following personality and attitudinal characteristics: willingness to take risks and nonconformity in mathematical contexts, category width, self-concept in mathematics, anxiety towards mathematics and test anxiety. Tests related to these characteristics were administered to the same 11 - 12 year old pupils. The hypotheses were examined both by consideration of correlations within various bands of mathematics attainment, and by analysis of profiles of individual high and low mathematically creative pupils. Clearest results were obtained within the highest attaining group. A tentative description emerges of a typical high mathematically creative, high attaining pupil: one who is willing to risk reasoned judgments in mathematical situations involving some uncertainty, one who has a high self-concept in mathematics, and low levels of anxiety towards mathematics and probably towards tests in general. The pupil is also likely to show a tendency to think in broad categories, concentrating on similarities rather than differences in the coding

of information.

In considering the implications of this research for mathematics teachers attention is drawn to some conflicts between mathematical creativity and other desirable behaviours in mathematics.

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CHAPTER 1

INTRODUCTION TO THE STUDY

This study is an investigation into mathematical creativity in schoolchildren. There are two main emphases in the investigation. The first is the recognition and assessment of some aspects of creativity in mathematics. The second is the identification of some significant characteristics which might be typical of the pupil showing creative ability in mathematics.

The Need for the Study

Interest in the study of creativity in educational research can be traced back to the presidential address to the American Psychological Association given by J. P. Guilford (1950). In this address he indicated that less than 0.2 percent of the literature in psychology was devoted to the subject of creativity. One of Guilford's concerns was that conventional assessment of students' potential was based mainly upon the notion of intelligence, which emphasised convergent thinking. Guilford argued that divergent thinking, which played an important part in creativity, was equally important. Since then research into this area of educational psychology has been gathering momentum. This is shown by the fact that Guilford (1970) could report that by 1969 investigation into creativity had increased to 1.4 percent of the literature. In 1983 the Educational Resources Information Center database on educational materials was found by the present author to hold no less than 4,358 articles and reports referring to the subject of creativity.

However, much of the interest in creativity has excluded consideration of this concept within the area of mathematics education. Hudson (1966) suggested that divergent thinking was more favoured by artists, and likewise convergent thinking by scientists and mathematicians. This dichotomy may have contributed to many educationalists' perception of creativity as having most relevance in the arts rather than in the sciences and mathematics, although Hudson himself states clearly that creativity should not be equated with divergent thinking. It must be acknowledged that the very nature of mathematics as it exists within the conventional school curriculum does make it seem more appropriate to associate it with convergent rather than divergent thinking. But this does not preclude the possibility that opportunities for divergent thinking in mathematics might exist and that the ability for divergent production in mathematics may be evidently valid and potentially useful for students of the subject. One of the concerns of the present study is to explore the relevance and validity of notions associated with creativity in general, such as divergent thinking, to school mathematics.

Interest in creativity has not excluded mathematics however. Barron (1969) made a study of creative persons which extended beyond artists, architects and writers, to include mathematical and scientific researchers who were perceived by professionals in these fields as being highly creative. He sought to show how the personal characteristics of these creative adults bear upon the processes of mathematical or scientific creativity at that level. One of the concerns of the present study is to investigate relationships between personality characteristics and creative ability in mathematics as demonstrated by schoolchildren.

Creativity, inventiveness and associated ideas feature in a number of attempts to specify goals and objectives for teaching mathematics (e.g. Hollands, 1972; Wood, 1968; The Mathematical Association, 1976; Cornish and Wines, 1980). Although it is by no means always clear what is meant by creativity in the context of school mathematics in such taxonomies, it is universally seen as being a higher category of behaviour than such categories as the learning of facts, the mastery of skills and techniques, the understanding of concepts and the application of these. The Assessment of Performance Unit (APU) in their first Primary Survey Report (1980) started their work on monitoring mathematics by establishing a curriculum framework. The two dimensions of this framework were (a) types of performance or outcomes in mathematics, and (b) content categories. The headings of the first were concepts, skills, applications, generalisations and proof, investigations and creativity, and attitudes. The later headings in this list reflect the concern of many mathematical educators that too much attention is given in the mathematics curriculum to what might be termed the lower categories of behaviour, such as the learning of facts, skills and techniques. The report of the prestigious Committee of Inquiry into the Teaching of Mathematics in Schools (Cockcroft, 1982) significantly underlined this concern by including in a catalogue of six elements which need to be present in successful mathematics teaching to pupils of all ages, "problem-solving...and investigational work" (paragraph 243). Although the word 'creativity' does not appear at this point in the report, it is clear that the discussion related to investigational work is concerned with giving pupils opportunities for thinking creatively in mathematics.

If creative thinking is to be encouraged and fostered, then mathematical educators need ways of recognising and assessing it. This is one of the concerns of the present study. It is relevant to note at this point that the APU's first round of testing in mathematics did not include items under the investigations and creativity section. In fact in the second Primary Survey Report (1981) this heading is subsumed in a more general heading of 'problem-solving and investigating'. Significantly, in the testing programme reported in the third Primary Survey Report (1982) the categories of assessment items used under this heading seem to suggest that the creativity element has been almost lost: processing information, formulating problems, strategies and methods of solution, generalising solutions, proving and evaluating. This loss is acknowledged by the authors to some extent (page 113) in stating the need for more open, investigatory situations to be given to pupils to "enable individual and creative approaches to mathematics to be assessed". One of the concerns of the present study then is to seek in some measure to fill this partial vacuum in our assessment of mathematical ability, and to suggest principles upon which assessment instruments for creativity in mathematics might be devised.

Two Specific Aims for the Study

The investigation into aspects of mathematical creativity in schoolchildren to be outlined in this study was devised therefore with two aims in mind.

The first of these was to produce a battery of tests which could be used with 11 - 12 year old pupils to assess aspects of mathematical ability which could justifiably be associated with the term 'mathematical creativity'. This would involve the clarification of principles upon which such tests might be constructed and the specification of criteria by which they might be accepted as valid.

The second aim was to investigate the relationships between certain characteristics of 11 - 12 year old pupils and their performances on the mathematical creativity tests devised in response to the first aim. The characteristics to be considered would be selected from those aspects of personality and attitudes which previous research has indicated may be particularly related to creativity in general. The study would concentrate on those which seemed likely to be most significant in terms of creativity operating within school mathematics.

The Initial Approach Adopted in the Study

There is no universally or even widely accepted definition of creativity. Consequently there is no obvious clear definition of mathematical creativity which can be taken as a starting point for this investigation. MacKinnon (1970) has argued that creativity is best conceived of as a multifaceted phenomenon rather than as a theoretical construct to be defined precisely. It is rather like the title of a book under which a number of related topics quite naturally fall. This view of creativity reflects the stance adopted in the present research into mathematical creativity. To start by defining precisely what is meant by mathematical creativity might prove to be a rather limiting and somewhat uncreative procedure. Consequently the initial approach adopted is to examine the key ideas which recur in the general creativity literature and to explore their potential relevance to the aims of the study.

Selection of Key Ideas

Essentially three different ways into the subject of creativity can be perceived in the literature. These are the creative process, the creative product and the creative person. Some authors (e.g. Ghiselin, 1952) have concentrated on considering the nature of the cognitive processes which contribute to creative thinking, that is, the transformation of information by the mind to find new and unexpected relations. Others (e.g. Jackson and Messick, 1965) have set out to specify criteria by which a product may be recognised as being creative. Such criteria as flexibility, originality and appropriateness are often used for assessing the level of creativity of the responses of students to so-called creativity tests (e.g. Torrance, 1966). Others (e.g. Roe, 1952; MacKinnon, 1962) have used both biographical data and a variety of assessment techniques to focus on identifying the characteristics of the creative personality. Important insights into the nature of creativity have been gained by each of these approaches. Therefore in the present study each of these headings, creative process, creative product and creative person is taken as a possible stimulus for considering mathematical creativity.

Creative Process

The creative process, particularly in problem-solving, has often been seen as involving four stages: preparation, incubation, illumination and verification (derived from Wallas, 1926). Many mathematicians have recognised these as being relevant to their own experiences of creative thinking in mathematics (e.g. Poincare, 1952; Hadamard, 1954; Littlewood, 1967; Parr 1974). Much of the interest here lies in the transition from the incubation stage to the illumination stage, where an insight into the solution of a problem is gained. Often this insight fails to take place because the person concerned is subject to a mental set. The person's thinking is fixated along inappropriate lines (Duncker, 1945; Wertheimer, 1959). Fixation in problem-solving is the counterpart to flexibility, a key aspect of creative thinking. The ideas of breaking from mental sets (or mind sets), overcoming fixations or mental rigidity are frequent themes in discussions of the creative process. Mathematics educators will immediately perceive these ideas as being relevant to children learning and doing mathematics. All will no doubt have had experience of children showing stubborn adherence to inappropriate methods or algorithms when tackling mathematics problems. Balka (1974b) includes in a list of six criteria for creative ability in mathematics: "the ability to break from established mind sets to obtain solutions in a mathematical situation". This ability to break from established mind sets in mathematics is the aspect of the creative process which is selected for investigation in the present research.

Krutetskii (1976) identified "flexibility of mental processes" as one of the major components of mathematical ability in school-children. Some of the most significant ways in which this flexibility was shown in his research was by the overcoming of fixations, sometimes referred to as "self-restrictions", or the breaking away from a stereotyped method of solution. Elsewhere Krutetskii (1969, p.117) emphasises this ability to break from stereotypes and to show flexibility in mental processes in a description of true mathematical ability: "Mathematical ability appears in varied approaches to the solution of a problem and in easy and free switching from one mental operation to another. The talented student is able, when necessary, to leave the patterned stereotyped means of solving a problem and find a few different ways of solving it... This is the real appearance of mathematical creativity".

Although this ability to break from mental sets and to overcome fixations is only one aspect of the creative process it does appear then that it is likely to be of particular significance in terms of mathematics. Consequently it is one of the key ideas used in devising a framework for the construction of the battery of mathematical creativity tests undertaken in this study and considered further in Chapter 2 of this report.

Creative Product

Assessment of creativity has concentrated mainly on the use of divergent production tests. The most elaborate batteries of these have been developed by Guilford (1959a) for identifying various factors in his structure of the intellect model, and Torrance (1966), who produced a set of divergent production tests for assessing creative thinking in both verbal and figural domains. The common element in all such tests is that the subject is given a problem with many possible solutions. For example, the problem might be, "Think of all the uses you can for a brick", or "Draw as many objects as possible which contain a circle as a main part". Such problems are designed to allow divergent thinking. This is contrasted with convergent thinking in which the subject must seek (i.e. converge upon) the one and only one correct solution. The creativity of the subject is conventionally assessed from a divergent production test by evaluating responses in terms of such measures as fluency (the number of responses), flexibility (the number of categories of response) and originality (the statistical infrequency of the responses).

Although such tests have been subject to many criticisms on the grounds of validity and reliability, they seem nevertheless to have held a certain fascination for educational researchers and many claims about creativity are based upon them. Wallach and King (1969), for example, claim that creativity as measured by ideational productivity and uniqueness on such divergent production tests is a better predictor of talented accomplishments outside of the classroom than intelligence.

One of the lines of approach in the present study is to explore the use of divergent production tests in mathematics. As has been suggested earlier it seems more natural to associate convergent rather than divergent thinking with school mathematics. The student is usually required in mathematics to find the one correct solution to a problem. However it has been noted that many modern mathematical educators have urged that children's experiences of mathematics should be broadened to include, for example, investigations in mathematics. This would then put them into situations where there may not be just one correct response or line of enquiry. This is seen as being desirable. Flexibility of mental processes has been noted already to be a major component in Krutetskii's description of mathematical ability in schoolchildren. A number of mathematical educators have seen the potential relevance of divergent production tasks, which allow for such flexibility, to the assessment of mathematical ability. Bishop (1968) questions the value of teaching a person to be creative in mathematics, a possible objective of modern mathematics teaching, if every assessment question he is faced with has one and only one right answer. Bishop goes on to suggest some ways in which divergent production might be assessed in a mathematical context.

A number of researchers have developed and used assessment instruments of this sort for investigating divergent production in mathematics, or mathematical creativity, though most of this work has been done in the United States. The present author (Haylock, 1978) devised and used two such tests, one numerical and one geometrical, with 14 - 15 year olds, in an earlier piece of research. The responses obtained from the pupils in such open-ended tests allowing divergent production give support to the view that some worthwhile mathematical ability is being tapped. Consequently this is the second key idea to be incorporated into the framework for the production of a battery of tests of mathematical creativity: the ability for production of many, varied and original responses in open-ended situations in mathematics. This is considered further in Chapter 2.

Creative Person

Much of the literature of creativity is concerned with the characteristics of creative persons. There is general agreement that personality factors are important in creative achievement, and there are many indications that creative thinking abilities as assessed by creativity tests are strongly related to aspects of personality. In a summary of research findings, Thompson (1982) asserts that creative persons are observant, express half-truths, see things as others do not, are independent in cognitive faculties, are motivated by their talents and values, can hold many ideas at once, have greater sex drive, see a complex world and have strong egos. They are often unpopular with their teachers, find it difficult to conform in institutional settings, live with anxiety, tend to make deviant scores on personality tests, are self-reliant, individualistic and independent. One of the intentions in the present study is to produce a similar description of some significant characteristics of mathematically creative schoolchildren. Such a profile could give valuable insights into the nature of mathematical creativity and possibly indicate why some children are more creative in their mathematical thinking than others.

Torrance (1962) lists no less than 84 characteristics found in one or more studies to differentiate highly creative persons from less creative ones. Inevitably, therefore, in the present study a selection had to be made of those aspects of personality and attitudes which are likely to be significant in terms of mathematical creativity in schoolchildren. Such a selection was made on the basis of hunches. Consideration was given to the many characteristics which have been judged significant in terms of creativity in general but for children in particular, and also to the nature of mathematics as it is learned and performed by children in ordinary classrooms in Primary and Middle schools. From these considerations emerged five hunches, which are formulated as six hypotheses in Chapter 3 of this report. Each of the five hunches is outlined below with a brief description of the rationale underlying it. A fuller survey of related research literature is given in Chapter 3.

Risk-taking. The first hunch is that willingness to take risks in mathematics is likely to be an important factor in relation to mathematical creativity. A number of authors have indicated that risk-taking performances of individuals may be related to measures of creativity. For example, Anderson and Cropley (1966), in a study of 13 year old children, report their conclusion that the one factor which inhibits originality more than any other is an internalized "stop-rule" which can be summarised as "don't take risks". Some authors (e.g. Pankove and Kogan, 1968) have suggested that creativity can be equated with cognitive risk-taking. Getzels and Jackson (1962) suggest that an essential difference between the two groups in their study of gifted adolescents, the highly intelligent and the highly creative students, was the creative adolescent's ability to produce new forms and to risk joining together elements usually seen as independent or dissimilar. If willingness to take risks is significant in terms of creativity in general it seems likely that it will be just as significant in terms of creativity in mathematics in particular. Success in mathematics as perceived by schoolchildren is unlikely to be associated with taking risks, but rather with taking care and being accurate. This is a subject in which pupils' responses are judged almost always as unequivocally right or wrong. To respond creatively to an open-ended situation in mathematics therefore may well require a pupil to be adventurous. The pupil must take the risk that the unusual or divergent responses will be marked wrong. Particularly for pupils who are used to getting things right in mathematics that may be painful or unwelcome. Pupils may succeed in conventional assessment in mathematics by following safe, predictable, learnt procedures. To break away from the safety of the stereotype may well involve a willingness to take risks. Such speculations as these then were the basis for the first hunch. It should be noted however that the emphasis in the present study is on the pupil's willingness to take risks specifically within a mathematical context not risks in general.

Nonconformity. It seems reasonable to conjecture that an *individual who shows a predisposition to conform is unlikely to* think divergently and to produce responses which are novel or unusual compared to the peer group. Thus the second hunch in this study is that nonconformity in a mathematical context might be related to performance in mathematical creativity tests. The notion of non-conformity used here is derived from Crutchfield (1955, 1962). An individual is nonconformist if prepared to express a judgment which is at variance with the group consensus. Anderson and Cropley, in the study mentioned above, investigated the relationship between nonconformity defined in this way and creativity in young adolescents. In fact they report no significant difference between high and low creatives in terms of the conformity/nonconformity construct. However, because of the particular nature of children's experiences of doing mathematics in school it was considered in the present study that it was worth pursuing the relationship between nonconformity and creativity specifically in the context of doing mathematics. Aiken (1976) reports that research evidence suggests that children who do well in conventional mathematics assessments tend to be conformist in school. The objective in many mathematics lessons can be interpreted as achieving conformity, that is, getting all the pupils to produce the same (correct) responses to a particular set of mathematical questions. Situations in which pupils may deviate from the rest of the group and still produce acceptable responses are all too infrequent in mathematics. Consequently the more open-ended style of divergent production test envisaged for the assessment of one aspect of mathematical creativity, with the possibility of a wide range of acceptable responses and credit for the more unusual of these, may well favour the pupil who shows a predisposition to be nonconformist and to trust personal judgments in mathematics. This then was the rationale of the second hunch. Again it should be noted that the emphasis in the present study is on the pupil showing nonconformity in a mathematical context, not nonconformity in general.

Category width. Individuals vary in the degree to which they show a tendency to code information received from the external world in broad rather than narrow categories, grouping together items which are roughly similar rather than categorizing them narrowly on the basis of their differences. Maslow (1954) claims that creative potential is associated with a predisposition to accept a wider range of attribute values required of an instance before it can be admitted to a category. The third hunch therefore is that mathematical creativity might be related to broad categorization. Significant differences between high and low creative children in terms of category width were found by both Anderson and Cropley (1966) and Wallach and Kogan (1965). Both studies found that higher creatives tend to be broader categorizers. These results were based upon performance in general creativity tests. The emphasis in the present study is on any relationship which might exist between category width and performance in mathematical creativity tests. The projected relationship is based upon the conjecture that a pupil who shows a tendency to think in narrow categories will be less open to the possibility of a wide range of ideas or procedures being relevant to a given mathematical situation. A tendency to think in narrow categories then could well inhibit the production of creative or divergent responses. Conventionally, mathematics teaching emphasises learnt algorithms and standard procedures. Exercises are grouped according to subject matter or mathematical topic. Such emphases are both necessary and dangerous. They are necessary for the pupil to master mathematical skills and techniques. But they are dangerous in that they can encourage narrow discrimination between various situations and instances of mathematical concepts or principles, thus reducing the range of applicability of the skills and techniques learnt. In particular mathematical tasks requiring overcoming fixation, by breaking from stereotypes, or flexibility in calling upon a wide range of mathematical ideas, may favour the pupil who is more predisposed to code mathematical information in broader categories. This then is the rationale for the third hunch that mathematical creativity may be related to category width.

Self-concept. The fourth hunch is that a high self-concept in mathematics might be significantly related to mathematical creativity in schoolchildren. Torrance (1965) has argued that a pupil's self-concept is important in the development of creative behaviour. Elsewhere (1962, p.77) he reports a study which found that highly creative children were rated significantly higher than less creative ones on strength of self-image. This hunch is based on the same sort of reasoning which related to risk-taking and nonconformity in mathematics. It would be expected that pupils who have high self-concepts in mathematics, who expect to do well in the subject and to succeed with the mathematical problems presented to them, would be more prepared to risk novel ideas or approaches and to trust their own judgments. So it seems reasonable to conjecture a relationship between self-concept and mathematical creativity. Again it should be noted that the study is concerned with the relationship between mathematical creativity and high self-concept specifically in mathematics, not a more general notion of self-image.

Anxiety. The fifth hunch is that performance in mathematical creativity might be related to the pupil's level of anxiety. A number of researchers have considered the relationship between anxiety and creative performance, though their conclusions are by no means unaminous. But it seems that anxiety can be linked with both the aspects of creativity which it is proposed to explore in relation to mathematics in the present study. There is evidence that high anxiety inhibits flexibility (Beier, 1951), increases rigidity (Cowen, 1952) and is sometimes associated with poor performance in divergent production tests (Krop, Alegre and Williams, 1969). Callahan (1966), with reference to the generally observed phenomenon that stress tends to increase rigidity in problem-solving, suggests that the stereotype procedure offers security, and hence anxious, insecure subjects tend to revert to it. Hadley (1965) found that high test anxiety in 12 - 13 year olds was associated with low performance on tests of divergent production, and produced evidence that teaching methods which induce anxiety curtail creative production.

It seems reasonable then to suppose that faced with a task in mathematics which requires breaking from a mental set the pupil who is less anxious about that task would be more likely to move away from the security of the stereotype or the familiar. Similarly, given a problem requiring divergent production in mathematics such a pupil would be more likely to experiment with a wider range of ideas, rather than adhere to those which are familiar, undemanding and consequently less stressful. This then is the basis of the fifth hunch, that lower levels of anxiety would be related to higher creativity in mathematics.

Previous research has indicated that high anxiety towards mathematics is associated with low mathematical achievement (Callahan and

Glennon, 1975). The concern in the present study is whether such an association can be found with mathematical creativity. Most of the results relating children's anxiety with creative performance in general (e.g. Wallach and Kogan, 1965) have used measures of test anxiety. Clearly both anxiety towards mathematics and anxiety towards tests could interact with performance on the proposed mathematical creativity tests. Both these aspects of anxiety will be considered. Consequently in Chapter 3 two hypotheses referring to anxiety are formulated, one dealing with anxiety towards mathematics and the other with anxiety towards tests.

Summary of Methodologies Used in the Investigation

The first aim of the research was to produce a battery of tests of mathematical creativity, based upon the two key ideas of overcoming fixation and divergent production. In the first phase of the programme existing instruments and possible test items were explored. A number of trial items devised by the investigator were field tested with various classes of schoolchildren and college students. These trials included analysis of responses on paper-and-pencil tests and discussion with classes about their reactions to various items. The trials were used to determine the feasibility of paper-and-pencil, group-administered tests of these two aspects of mathematical creativity, and to clarify the principles upon which such tests might be constructed. Following the field testing of selected items, in the major part of the research programme, a total of eight tests related to overcoming fixation in mathematics and 14 tests of divergent production in mathematics were administered by their teachers to a sample of 283 children aged 11 - 12 over a period of ten weeks. The responses of the pupils to these tests were catalogued and analysed, and the tests evaluated according to specified criteria. On this basis five tests of overcoming fixation and ten tests of divergent production were accepted as possibly valid tests of the aspects of mathematical creativity in question.

A large sample of pupils was used because it was intended to analyse their performances in bands of mathematical attainment, particularly in relation to the second aim, the consideration of characteristics of the mathematically creative pupil. Towards the end of the ten week period a number of paper-and-pencil, group-administered instruments related to these aspects of personality and attitudes were given to the pupils, again by their own teachers. Four of these instruments, related to risk-taking and nonconformity, were of an exploratory nature, being newly devised by the investigator for the purposes of this research. The other two, related to category width and attitudes (self-concept, anxiety towards mathematics, test anxiety) were based upon existing instruments.

The hypotheses relating to these characteristics were then considered both by examination of correlation coefficients within the various bands of mathematics attainment and also by examination of the profiles of individual pupils. These pupils were selected from the sample as being high or low on the mathematical creativity measures used and the profiles were based upon their behaviours on the instruments related to aspects of personality and attitudes.

Outline of the Report

In this chapter the two main emphases of the study have been introduced: first, the recognition and assessment of mathematical creativity in schoolchildren, and secondly, the identification of significant characteristics of the pupil showing creative ability in mathematics. The need for the study has been considered on the basis of interest in higher categories of behaviour in mathematics and the lack of assessment methods available for mathematical creativity. The initial approach to the study has been to examine ideas associated with creativity in general and to select those which seem most likely to be significant in terms of mathematical creativity in schoolchildren. From consideration of the creative process and the creative product the two key ideas of overcoming fixation and divergent production were selected as the basis for the proposed construction of a battery of tests for assessing mathematical creativity. From consideration of the creative person five hunches have been introduced and the rationale behind them outlined briefly. These have conjectured relationships between mathematical creativity and willingness to take risks in mathematics, nonconformity in mathematics, category width, self-concept in mathematics and anxiety towards mathematics and tests.

The second and third chapters of this report are concerned with the background to the present study. Chapter 2 provides a review of previous research and literature dealing with mathematical creativity, overcoming fixation and divergent production. The relevance of the notion of creativity to children's learning of mathematics is considered and a summary of research which has set out to assess creative ability in mathematics is provided. The chapter concludes with an outline of the framework to be used in the main part of the investigation as a basis for the development of a battery of mathematical creativity tests, to be described and analysed in Chapters 4 and 5. Chapter 3 reviews previous research into the characteristics of creative persons, concentrating on those aspects outlined in Chapter 1: willingness to take risks, nonconformity, category width, self-concept and anxiety. The chapter concludes with a statement of six hypotheses relating these aspects to mathematical creativity. These hypotheses are the basis of the investigation described in Chapter 6.

The fourth and fifth chapters of the report deal with the part of the present research concerned with the first emphasis of the study: the recognition and assessment of mathematical creativity in schoolchildren. Chapter 4 describes the development of the battery of mathematical creativity tests. There are two major sections to this chapter. The first deals with tests of overcoming fixation in mathematics and the second with tests of divergent production in mathematics. For each of these constructs there is a description of the development phase of the tests, the principles which emerged for the construction of such tests, and the criteria which were specified for the inclusion of a particular test as a valid assessment of the construct in question. Detailed descriptions of the tests administered and the responses obtained from the sample of 11 - 12 year olds used in the main part of the research are given. These are analysed and evaluated. The outcome of the chapter is a battery of tests for assessing mathematical creativity in 11 - 12 year olds, five tests dealing with overcoming fixation and ten related to divergent production. Chapter 5 then describes how the scores obtained by the pupils in the main part of the research were combined into overall scores for overcoming fixation in mathematics (OF) and divergent production in mathematics (DP). Relationships between OF, DP and mathematics attainment (MA) are considered. Some consideration is also given to performance on items dealing with numerical and spatial domains separately, and the question of boy/girl differences in performance on the mathematical creativity measures is dealt with.

Chapters 6 and 7 describe the investigation into the six hypotheses concerning characteristics of the mathematically creative pupil which were formulated in Chapter 3. The first part of Chapter 6 deals with the first two hypotheses, risk-taking and nonconformity, which were investigated by means of newly devised instruments. The second part deals with the other four hypotheses, category width, self-concept, anxiety towards mathematics and test anxiety, which were investigated by means of existing instruments. The hypotheses are tested by means of correlation coefficients within various bands of mathematics attainment. Chapter 6 concludes with a tentative profile of the mathematically creative pupil drawn from group data. Chapter 7 then considers the hypotheses stated in Chapter 3 by looking at profiles of individual pupils of high mathematical creativity and low mathematical creativity, arising from their performances on the instruments used and described in Chapter 6. Differences in characteristics found between high and low mathematically creative pupils are illustrated by detailed analysis and comparison of the responses of two particular pupils.

The final chapter of the report summarises the results of the investigation into mathematical creativity in children aged 11 - 12. Some conflicts between creativity in mathematics and other desirable behaviours are discussed, and some implications for the teacher of mathematics are considered.

CHAPTER 2

MATHEMATICAL CREATIVITY: OVERCOMING FIXATION AND DIVERGENT PRODUCTION

This chapter reviews the literature and research associated with creativity in mathematics. It is seen that creativity is considered to be a relevant and important notion in terms of children doing mathematics in school, and various definitions and descriptions of mathematical creativity are reviewed. A summary of research which has purported to assess creative ability in mathematics is provided. The two key aspects highlighted in Chapter 1, the ability to overcome fixations and the ability for divergent production, are then considered separately, with summaries of relevant research findings. From these considerations a framework for assessing mathematical creativity in the context of school mathematics emerges as the basis for the investigation to be described in Chapter 4.

Creativity in School Mathematics

In a paper given as the presidential address to the Mathematical Association, Tammadge (1979) suggests that there is an urgent need for teachers of mathematics to identify, encourage and improve creative mathematical ability at all levels. He argues that mathematics teaching has for too long been dominated by a rational thought/rote learning model, with an emphasis on cumulative learning of existing knowledge. The alternative imagination/intuition model allows for leaps in the learning process, the establishment by the learner of new relationships, and the possibility of creativity in the mathematics classroom. Creativity in mathematics, according to Tammadge, is not fundamentally different from creativity in other areas of the curriculum. It includes the ability to see new relationships between techniques and areas of application, and to make associations between possibly previously unrelated ideas. Deakin (1975) provides examples to support the view that such creativity in mathematics can occur at every level of schooling. Vallee (1975) similarly argues that it is as important to stress intuition and reasoned guessing in mathematics teaching as to stress deduction. Poincare (1952) suggests that there may be two different types of mathematical mind, the logical and the intuitive. Vallee concurs that both logic and intuition are recognisable tendencies in the mathematical mind, but insists that both are necessary in creative mathematics. Helson and Crutchfield (1970) identified five different types of mathematical mind within a group of adult creative mathematicians and suggest that the question is more complex than Poincare's simple logical/intuitive description. In his study of mathematical ability in schoolchildren, Krutetskii (1976) argues that mere mastery of mathematical material is not a sufficient criterion for mathematical giftedness, but needs to be extended to an "independent creative mastery of mathematics under the conditions of school instruction" (p. 68). Mathematical creativity according to Krutetskii manifests itself in "the independent formulation of uncomplicated mathematical problems, finding ways and means of solving these problems, the invention of proofs and theorems, the independent deduction of formulas, and finding original methods of solving nonstandard problems". Krutetskii clearly associates creativity in mathematics with mathematical giftedness. Others, such as Tammadge, in the paper referred to above, and Hollands (1972), discuss creativity in terms of behaviours which can be exhibited by pupils of all levels of ability and all ages, though it is clear that greater creativity can be demonstrated by pupils with greater mastery of mathematical material.

Wood (1965) in discussing 'inventiveness', the highest category of behaviour in his framework for objectives in the teaching of mathematics, defines it as the assembling of elements and parts so as to form a pattern or structure not clearly visible before. It is the quality of originality or uniqueness which makes the behaviour differ from application and comprehension. The notion of rearranging or combining the elements of mathematics in ways new to the student is commonly associated with discussions of creativity in mathematics. Cornish and Wines (1980) include creativity as one dimension in the mathematics profile assessed by their battery of tests. Their objectives are process rather than content oriented. So creativity is defined for students in the Piagetian concrete operational stage as "extending patterns with numbers, shapes, etc.; rearranging models, networks, maps, plans; transforming familiar conventions in practical situations and predicting effects." For students in the formal, abstract stage of reasoning, their definition is "extending, combining or rearranging relationships, changing and inventing conventions of systems and predicting resulting relationships."

Aiken (1973), in a review of research and literature related to mathematical creativity, indicates that definitions of this concept are usually based on either the notion of an underlying process, or of a manifest product. The process/product distinction has already been discussed in Chapter 1 as the basis for two aspects of creativity in mathematics to be considered specifically in this study.

First there are definitions or descriptions of creativity in children doing mathematics which concentrate essentially on the presumed nature of the cognitive processes involved, considering the particular quality of thinking which qualifies for the description 'creative'. Krutetskii (1969) talks of the easy and free switching from one mental operation to another. Laycock (1970) specifies an ability to analyse a problem in many ways, observe patterns, see likenesses and differences. Romey (1970) defines mathematical creativity as the combining of ideas, things, techniques, or approaches in a new way. McNulty (1969) seems to identify mathematical creativity with the abilities for pattern-spotting and for insightful solutions. In some ways these two aspects of problem-solving in mathematics are opposed to each other. Pattern-spotting and generalizing are important in mathematical thinking, but sometimes the need in problem-solving is to break from the stereotype or to avoid over-generalizing. Flexibility as opposed to rigidity is an essential ingredient of creative problem-solving in mathematics. In Chapter 1 it was the ability to break from mental sets, to overcome fixation, which was identified as a key aspect of the creative process to be considered in this study.

Secondly, there are definitions of mathematical creativity which concentrate essentially on the product. It is, after all, the products of thinking which the teacher sees, and criteria for recognising these as being creative for the pupil in question are often suggested. Spraker (1960) defines mathematical creativity as the ability to produce original or unusual, applicable methods of solutions for problems in mathematics. Jensen (1973) uses an operational definition of mathematical creativity, which emphasises problem-posing rather than problem-solving: "the ability to give numerous, different and applicable questions when presented with a mathematical situation in written, graphic or chart form." It is significant that both these last two definitions include the word 'applicable', used in the sense of 'appropriate'. Production of many, varied and original ideas in mathematics can only be judged to be creative if by clear mathematical criteria these ideas are appropriate to the situation in question. Appropriateness as a criterion of the creative product is put forward by Jackson and Messick (1965) in their consideration of conceptual problems in the assessment of creativity. It is clear that many authors and researchers have approached the subject of creativity in mathematics via the notion of divergent production. It is this idea which was highlighted in Chapter 1 as the second key aspect of mathematical creativity to be considered in this study. In a detailed analysis of categories of objectives in the teaching of mathematics Hollands (1972) specifies creativity in mathematics, which he suggests is the most neglected aspect of the teaching of the subject, in terms of divergent production. Behaviours associated with this category of objectives include: flexibility, shown by the student varying the approach or suggesting a variety of methods; elaboration, shown by the extending or improving of methods; fluency, the production of many ideas in a short time; originality, the student trying novel or unusual approaches; and sensitivity, shown by the student criticising standard methods constructively. These parameters are clearly derived from the criteria used for assessing creativity in divergent production tests such as the Minnesota Tests of Creative Thinking (Torrance 1962, p, 213ff).

Any definition of mathematical creativity must refer to both mathematics and creativity. It is clear that in the definitions and descriptions discussed above some emphasise one of these rather more than the other. Some authors (e.g. Krutetskii) clearly are most concerned with the nature of mathematical thinking and mathematical processes and emphasise these in their discussions of creativity in mathematics. In doing this they may raise the question in the reader's mind as to what is particularly creative about the behaviour identified. For example, references to pattern-spotting and making generalizations occur in some discussions of mathematical creativity (e.g. Prouse, 1964; Balka, 1974), but it is questionable as to whether these behaviours, important as they are, should be described as creative. The thinking of others about mathematical creativity is clearly dominated by the creativity part of the term. This is particularly true of the researchers who have considered divergent production in mathematics and emphasise originality and novelty. What some of these approaches have lacked is consideration of the extent to which the products obtained are valid in terms of the mathematics of the situation. It is essential that both the mathematics and the creativity are clearly present in any behaviour to be labelled as mathematically creative.

As explained in Chapter 1, it was not considered necessary in the present study of mathematical creativity in schoolchildren to begin with a clearly formulated definition. Rather, various ideas associated with creativity in general were examined and a selection made of those ideas considered to be most relevant to children doing mathematics in school. The two key ideas which have been selected are the ability to break from mental sets in mathematics, an important aspect of the creative process, and the ability for divergent production in mathematics, which can be recognised by criteria commonly applied to the creative product. The review of research into creativity in mathematics which follows shows that a number of authors have considered the second aspect, but the first has been rather neglected.

Assessment of Creative Ability in Mathematics

Table 2.1 gives information about all the instruments used by researchers in the last twenty years for assessing creative ability in mathematics which have been located by the present author.

Table 2.1

Summary of Instruments Used by Previous Researchers for Assessing Creative Ability in Mathematics

Author(s)	Year	Country	Age range	Construct/criteria used
Prouse	1964	USA	12 - 13	Characteristics of creative mathematicians: makes up problems; generalizes by induction and analogy; imagination, offers many solutions. Convergent and Divergent items.
Evans	1964	USA	10 - 13	Divergent production, fluency, flexibility, originality.
Bishop	1968	UK	Secondary	Divergent production.
Baur	1970	USA	College	Divergent production.
Hiatt	1970	USA	Secondary	Convergent thinking in solving non-routine word problems; divergent production.
Meyer	1970	USA	6 - 7	Observable behaviour in problem solving: introducing new goal, new properties; seeking new relationship, generalization; reaching elegant solution; modifying task.
Foster	1970	USA	9 - 11	Divergent production, fluency.
Mainville	1972	USA	College	Divergent production, fluency and originality.
Jensen	1973	USA	11 - 12	Divergent production: posing numerous, different and applicable questions for a given mathematical situation.
Maxwell	1974	USA	Secondary	Divergent production.
Balka	1974	USA	Secondary	Divergent thinking: formulating hypotheses, evaluation of unusual ideas, sensing what is missing, splitting general problems into specific subproblems. Convergent thinking: determining patterns, breaking from mind sets.
Dunn	1976	UK	12 - 13	Divergent production, fluency.

Table 2.1 cont.

Krutetskii	1976	USSR	Primary/ Secondary	Problem-solving model. Flexibility shown by producing many solutions, breaking from stereotypes, overcoming self-restrictior
Haylock	1977	UK	14 - 15	Divergent production, fluency, flexibility, originality.
Zosa	1979	USA	12 - 13	Divergent production, fluency, flexibility, originality.
Brandau & Dossey	1979	USA	Secondary	Observation of verbal and non-verbal actions in open-ended mathematical situations, scored for fluency, flexibility, originality, organisation.
Cornish & Wines	1980	Australia	12 - 15	Extending, combining, rearranging patterns and relationships; transforming conventions and predicting effects.

The work of Cornish and Wines (1980) was actually the production of a battery of tests for providing mathematical profiles of secondary school students, of which creativity is just one dimension. Krutetskii's (1976) major research programme was concerned with the identification of components of mathematical ability in schoolchildren, based on a problem-solving model. One of these components is flexibility of mental processes. It is on the strength of this and the fact that Krutetskii seems to use the terms 'mathematical giftedness' and 'creative ability in mathematics' synonymously, that this work is included in the catalogue in Table 2.1.

Of the 17 entries in this table, 11 refer to work undertaken in the USA. All these are doctoral dissertations submitted to American Universities, and the majority of them have worked with pupils of secondary school age or college students. Except for Prouse (1964) and Balka (1974), who include components of convergent thinking in their criteria, the dominant notion in these studies is divergent production. Most include user-developed instruments similar in style to the standard divergent production tests for general creative thinking ability, but with a mathematical content. The mathematical validity of these tests is very variable. Prouse's criteria are derived from the characteristics of outstanding creative mathematicians identified by Carlton (1959): they make up or see problems in data or situations which arouse no particular interest in others; they tend to generalise particular results either by finding a common thread of induction or by seeing similar patterns by analogy; they have vivid imaginations concerning the way things are in space; they offer more than one acceptable solution to a problem, with the solutions clever or uncommon. Balka uses six criteria for mathematical creativity selected by a panel of eminent mathematics educators. These include such criteria as "the ability to formulate mathematical hypotheses concerning cause and effect in a mathematical situation" and "the ability to split general mathematical problems into specific subproblems." It is clear that these criteria, like those of Prouse, are dominated by the mathematics part of the term 'mathematical creativity', and it is a little difficult to appreciate why all of them are justifiably included as showing specifically creative ability. However, Balka is the first researcher into mathematical creativity to include a reference to the ability to break from established mind sets to obtain solutions in a mathematical situation.

Four studies in the United Kingdom are included in Table 2.1. Bishop (1968) proposes the use of divergent production items in extending the range of question-types used to assess pupils in secondary schools, and gives some examples. Foster (1970) undertook an exploratory study of creativity in mathematics by trying two divergent production tasks with 9 - 11 year olds. Haylock (1977) used two divergent production tests, one numerical and one geometric, with 14 - 15 year olds. Apart from these small investigations, the only instrument developed in the United Kingdom is a set of six divergent production tasks in mathematics devised by Dunn (1976) and used with 12 - 13 year olds in Northern Ireland.

Krutetskii's flexibility component is identified by means of problems in four categories which support the dual emphasis on divergent production and overcoming fixations in the present study. These are: problems with changing content, problems on reconstructing an operation, problems suggesting self-restriction, and problems with several solutions. Of these, the last is clearly linked with divergent production, in that flexibility is assessed by the production of many and varied responses. The first three are concerned with different aspects of overcoming fixation. In the first case two similar-looking, but different problems are given to the student and an assessment is made of the extent to which the pattern of thought used in the first interferes with the solution of the second. In the second case a series of problems is used to establish a stereotype method of solution which is then inappropriate for a later problem in the series. An assessment is made of the student's ability to break from the mental set established in the earlier problems. In the third case it is judged that certain problems are not solved by students because they impose restrictions on the elements of the problem which are not necessary, such as the assumption that a four-sided figure must be convex.

Mental Sets, Overcoming Fixation and Rigidity

The idea of breaking from mental sets, or overcoming fixations, referred to by Balka and Krutetskii in their investigations into creative ability in mathematics has its roots a long way back in psychological literature. By 1941 Gibson could report eight different uses of the term 'set' in psychological research. These are:

(i) A prearoused expectation of stimulus objects, qualities or relations.

(ii) A conceptual scheme, not expected, but aroused by the stimulus pattern.

(iii) An expectation of stimulus relationships either prearoused or acquired during repeated stimulations (sometimes referred to as conditioning).

(iv) An intention to react by making a specific movement or not so to react.

(v) An intention to perform a familiar mental operation.

(vi) A mental operation or method, not intended, but aroused by the problem.

(vii) A tendency to complete or finish an activity.

(viii) A tendency to go on performing an activity after the occasion is over.

Luchins (1942, 1951) investigated in particular the 'Einstellung' effect. This German word has the meaning of an adjustment made beforehand, or an allignment, and is used in psychology in the sense of a relatively rigid attitude or predisposition. Luchin's experiments consisted of a series of problems designed to establish in the mind of a subject a particular algorithm or stereotype method of solution. The Einstellung effect is shown when the subject continues to apply that method or process even when inappropriate, inefficient or unsuccessful. Cunningham (1966) points out that this is an example of what is sometimes termed an 'objective' set, in that the set is established by the materials or sequence of events within the experiment as opposed to a 'subjective' set, which would be a set of attitudes, intentions or presuppositions brought to the experiment by the subject. This distinction is difficult to maintain. The

Einstellung phenomenon may be developed by the sequence of events within the experiment, but it may also be the case that the subject comes to the experiment with a predisposition to think in terms of algorithms for solving series of problems or in terms of looking for patterns and generalizations, because such strategies have been successful in the past. This would be particularly true of mental sets operating in mathematics. It is clear then that although there is an underlying idea common to discussions about mental set (mind set, fixation), the phenomenon is by no means susceptible to a unified description or explanation. The central idea behind the use of the term is that the subject demonstrates a set form of behaviour in problem-solving which has been proved to be appropriate in many instances but is continued to be used even when inappropriate. (Even this summary does not include all of Gibson's categories, for example vii and viii.) The set may be a result of the conditions of the experiment, or a set of assumptions brought to the problem by the subject; that is it may be aroused or prearoused, objective or subjective. Alternatively the set may be the result of a complex interaction of both the conditions of the experiment and the previous experiences of the subject. Some subjects may show a greater predisposition to set. It thus becomes difficult to determine whether an experiment is measuring susceptibility to set or ability to overcome set. Another useful distinction in the discussion of set is between fixations on particular processes and fixations on the use of particular entities. Some authors (e.g. Duncker, 1945) refer to a phenomenon called 'functional fixedness' in which a subject shows a fixation on the way an entity should be used, thus limiting its applicability in the solution of a problem. Scheerer (1963) suggests three reasons for failure to solve problems due to fixations.

"A person may start with an implicit but incorrect premise.

He may fail to perceive an object's suitability for a solution because it must be used in a novel way or because it is embedded in a conventional context. Or he may by unwilling to accept a detour that delays the achievement of his goal."

The first two of these are clearly related to Krutetskii's problems suggesting self-restriction. A student may fail to solve a mathematics problem because of some restriction which the student imposes on the elements which may or may not be used.

Much of the early work on overcoming fixation and mental sets was undertaken with adults. However the relevance of these ideas to children's learning and problem-solving has not been overlooked. In a review of rigidity in children's problem-solving, Cunningham (1966) concludes that the behaviour labelled as rigid is potentially the result of the interaction of personality and situational factors. Some children may show a predisposition to think in rigid ways, as a result of a general trait of rigidity which underlies many of their actions and attitudes. Rokeach (1960) describes how adults who are found to possess a general trait of rigidity in their outlook and attitudes to the organisation of their own lives also show greater difficulty than more flexible individuals in overcoming various levels of beliefs or self-imposed assumptions in problem-solving. This trait of rigidity may well be related to other traits such as a tendency to conformity and intolerance of ambiguity. On the other hand Cunningham asserts that 'drill' and the learning of fixed procedures in school may contribute to attitudes which favour the development of rigid behaviours. This suggestion had been put forward by Luchins (1942), and was further investigated by Kellmer-Pringle and McKenzie (1965) with 10 - 11 year olds. They compared the performances of children in a progressive school and a traditional school on the Luchins Water Jugs test (this test is described in

Chapter 4 of this report). The hypothesis that teaching style might influence children's rigidity was not supported on the whole. No overall difference was found between the two schools, nor between sexes in terms of ability to overcome fixation. The only clear indication arising from this study was that the lower streams in both schools showed more rigidity, and to some extent the lower stream pupils in the progressive school showed less rigidity than those in the traditional school. Cunningham (1966) reports that the average correlation between intelligence and rigidity as shown by the Water Jugs test has been computed as about -0.17. He suggests that there is some evidence that children of higher intelligence may show less rigidity, but that the picture is complicated by the different classroom treatments to which brighter and duller pupils may be conventionally subjected. With regard to age, Cunningham found an inverse relationship between age and rigidity, and in discussing the fact that this is contrary to the earlier findings of Luchins (1942) postulates that changes in the school curriculum may have contributed to pupils becoming less rigid rather than more rigid as they grow older. Nearly all these findings are related to performances on the classic Water Jugs test. Whether such findings would be replicated with other problems, particularly in mathematics, involving fixations and rigidity, is an open question.

It has already been noted that Balka and Krutetskii have seen these ideas of mental set and fixation to be particularly relevant in discussing creative ability in mathematics. This is clearly because mathematical problem-solving does sometimes require the breaking away from stereotype procedures or from the use of conventional and expected elements. Yet much mathematics learning necessarily contributes to the formation of standard procedures, algorithms and stereotypes. It seems appropriate therefore to label as 'creative', behaviour which shows an ability to break from such mental sets in mathematics. Allinger (1982a, 1982b) again highlights the fact that in mathematics mind sets can be sometimes positive in effect, by implanting useful skills and techniques, but can also be negative and harmful, by forming a mental obstruction preventing the correct or most appropriate solution to a problem. He gives examples of three types of negative mind sets in mathematics, both at elementary and secondary school levels. These he calls 'visual perception', Einstellung effect and functional fixedness. In the case of a visual perception mind set, the pupil may, for example, show fixation on the way certain geometric figures should be alligned in space, or on whether a figure represents a two or three-dimensional shape. This is allied to Krutetskii's notion of self-restriction. Allinger asserts that the Einstellung effect, the fixation on a particular algorithm, is very common in mathematics, particularly in the repeated application of successful procedures in arithmetic in less than optimal situations. Functional fixedness can also occur in mathematics. For example, a student may have learnt to use a particular Dienes block to represent a unit, and will then have difficulty in using it to represent anything else, such as a 'ten'. Allinger's discussion and examples of this third type of fixation are less convincing than the other two. Cunningham sees functional fixedness as being a particular problem in science teaching, rather than in mathematics.

From this review of the use of the term 'mental set' and associated ideas such as rigidity and fixation, it is clear that there is an application to children doing mathematics in schools. From the many examples, types and formulations of rigidity and mental set discussed, it seems to the present author that two key ideas emerge as being of most relevance. First it is clear that children may show fixation in mathematics by the continued use of an initially successful algorithm, even when this becomes inappropriate or less than optimal. This algorithm may be a process which has been learnt beforehand, (such as the standard algorithms of arithmetic), or it may be one established by a series of examples within the problem in question. Secondly, the fixation may be some sort of self-restriction related to the content universe of the problem. The pupil may restrict inappropriately the range of elements which may by used or related to the given problem. In the investigation described in Chapters 4 - 7 of this report the terms 'algorithmic fixation' and 'content-universe fixation' are used to refer to these two aspects. The first of these is derived from Gibson's categories (v) and (vi), Krutetskii's problems on reconstructing an operation, and the Einstellung effect defined by Luchins and applied to mathematics learning by Allinger. The second aspect is derived from Gibson's category (i), Krutetskii's problems suggesting self-restriction and Scheerer's references to implicit but incorrect premises, and related to Allinger's examples of visual perception set in mathematics.

Divergent Production in Mathematics

It has been noted already that much research into mathematical creativity has centred on the use of divergent production tests in a mathematical context. Dunn (1975, 1976b) provides useful summaries of much of this research. The findings from these investigations and other findings relating divergent production and mathematics will now be reviewed.

Richards and Bolton (1971) found that for 10 - 11 year old pupils divergent thinking ability contributes little to mathematical ability as assessed by conventional tests. They suggest that teaching fostering divergent thinking will produce minimal beneficial effects in terms of performance on mathematics achievement tests. Pamboukian (1972) also found a low relationship between general creativity and mathematics achievement in elementary schoolchildren. These conclusions are supported by McGannon (1972). He showed that scores on High School mathematics contests were highly negatively correlated (-.727) with scores on the Remote Associates creativity test (Mednick, 1962). He argues that conventional criteria for identifying gifted mathematics students, such as school mathematics grades and performances in mathematics contests, may not be good predictors of the mathematically creative student. This conclusion is open to the criticism that it is based on the assumption that the Remote Associates test might itself be a good predictor of mathematical creativity. In fact there is evidence to show that mathematical creativity as measured by divergent production tests in mathematics is not necessarily related to general creativity as measured by general divergent production tests. Haylock (1978) reports highly significant correlations between two general creativity tests and highly significant correlations between two mathematical divergent productions tests used with 14 - 15 year olds. But for the group of students of relatively high mathematics attainment it was found that the correlations between general and mathematical creativity were near zero. This suggests that creativity in mathematics may be a specific ability and not just a combination of general creative ability and mathematics attainment.

Dirkes (1974), using the Torrance (1966) figural and verbal tests of creative thinking as pre-and post-tests, investigated the effects of a training programme for children which emphasised divergent thinking in mathematics (and other areas) and also transfer of methods between areas of learning. She found that there was no improvement in transfer from general to mathematical divergent thinking as a result of the programme, and some gains in terms of overall performance only on the verbal tests of creative thinking. Judging the figural tests to be more related to mathematics, Dirkes's conclusion is that mathematical creativity is not related to general creativity.

Evans is the first researcher in Table 2.1 to use divergent production tests based upon mathematical situations with schoolchildren. He selected ten tests from sixteen which were found to intercorrelate. Each of these presents the pupil with a mathematical situation to which he must respond in as many ways as possible. Evans reports significant positive correlations between mathematical creativity scores and IQ, arithmetic achievement, mathematics grades, mathematics attainment and general creativity. The correlations between mathematical creativity and other mathematics scores is not entirely surprising, since it is to be expected and generally found to be the case that the number and range of responses a pupil can make in an open-ended mathematical situation will be related to the level of mastery of mathematical skills and knowledge. But there is clear support (Hiatt, 1970) for the view that such divergent production tasks do measure some aspects of mathematical ability not assessed by conventional attainment tests. Mainville (1972) found that for prospective elementary school teachers there was no significant correlation between scores on a mathematics achievement test and a mathematics divergent production test. This investigation found, as did that of Baur (1970), that college students' performances on mathematical creativity tests can be improved by an appropriate training programme emphasising divergent thinking. By contrast, Meyer (1970) found no such gains with first grade schoolchildren following a programme which emphasised creative approaches to mathematics.

Another study relating divergent thinking and mathematics was undertaken by Maxwell (1974). She gave secondary school students six geometric problems, three classified as convergent and three as divergent, and on the basis of their performances on these, separated out a group of high convergent thinkers and a group of high divergent thinkers in mathematics problems. Maxwell then observed the behaviour of these pupils in solving a mathematical problem involving the arrangement of some coloured blocks on a grid so that no two of the same colour appear on any row, column or diagonal. Her conclusion is that, since the high divergent students used fewer generalizations in their problem-solving than the high convergers, divergent thinking plays a minor role in mathematical thinking. Of course, the assumption here that the behaviour on the one block problem can be equated with mathematical thinking in all its complexity is very questionable.

Dunn (1976a) undertook a factor analysis of the scores of 12 - 13 year old pupils on a variety of tests, including six mathematics divergent production tests, some general divergent thinking tests, IQ and mathematics attainment. The mean correlation between the six mathematics divergent production tests was low (0.26) and they certainly did not group together as an identifiable factor. Dunn's analysis suggests: a convergence factor, high on IQ and mathematics attainment; a verbal-divergence factor, high on some of the verbal divergent thinking tests; and a numerical fluency factor, high on mathematics attainment and two numerically oriented divergent production tests. MacLean (1975) also used Dunn's six mathematical creativity tests, and found that some correlated with mathematics attainment and others with general creativity scores.

Jensen (1973) investigated for 11 - 12 year old pupils the relationships between mathematical creativity, as measured by her own tests of divergent production in mathematics, numerical aptitude, and mathematical achievement in terms of computation and problem-solving. She used an operational definition of mathematical creativity in terms of the ability to give numerous different and appropriate questions when presented with a mathematical situation. She reports that the two conventional assessments of mathematical ability, numerical aptitude and mathematical achievement, showed moderately high positive correlations. But the correlations between the mathematical creativity scores and the conventional measures were low (though positive and significant). She also found that the girls in her group overall scored better for the divergent production tasks than the boys. Jensen recommends that some sort of assessment of mathematical creativity would be a useful addition to the profile of a pupil's performance in mathematics.

The evidence from previous research using divergent production tests in mathematics is somewhat inconclusive, but there seems to be general support for the following standpoint. Such tests do seem to tap abilities in mathematics not necessarily tapped by conventional assessments of mathematics attainment. But mathematical creativity, as measured by such tests of divergent production, will not be independent of other performance variables, such as, in particular, mathematics attainment scores. The ability for divergent production in mathematics is not necessarily related to the ability for divergent production in nonmathematical situations.

Different Styles of Divergent Production Tests in Mathematics

The various tests using divergent production in mathematics referred to in Table 2.1 actually show a great variety in terms of both construct and administration. The majority of instruments are group-administered, paper-and-pencil tests. But other approaches have been used.

Foster (1970) administered one of his tests individually to the pupils in his exploratory study. Each pupil was presented with a pack of playing cards and required to sort out subsets of six cards, giving a reason for the subset to the investigator. Meyer (1970) videotaped children tackling some problems related to tessellations. Their level of creativity in these problems was assessed by observing how frequently they introduced a previously unspecified goal, identified an appropriate unstated property of the task, sought a relationship between a property of the task and other properties, sought a generalization, reached a mathematically elegant product or modified the task. Both the style and the construct of this approach are unusual. It is only similar to a conventional divergent production test in that the criterion used for scoring is the number of different ideas produced. But the emphasis on this approach to assessing mathematical creativity is more in the direction of the mathematics than the creativity. Brandau and Dossey (1979) also used observation of pupils' behaviour as a means of assessing mathematical creativity. The pupils were required to think aloud while considering a series of open-ended situations in mathematics. Fluency was measured by the number of verbal mathematical statements used, originality by the relative infrequency of these, flexibility by the number of different categories of statements, organization by the number of instances of specializing, generalizing and conjecturing. Mathematical creativity was then assessed by summing the scores for fluency, flexibility, originality and organization. Brandau and Dossey found that all these variables were highly significantly correlated.

Such approaches as these to assessing mathematical creativity may be contrasted with the instruments developed by such as Prouse, Evans, Baur, Jensen, Maxwell, Balka, Dunn, Haylock and Zosa. These are all group-administered, paper-and-pencil tests, modelled on the style of the conventional divergent thinking tests. The pupils are given mathematical situations or problems and required to produce many and varied responses. These are then assessed variously for fluency, flexibility and originality. With such a variety of styles and approaches to assessing mathematical creativity it is clear that some caution needs to be exercised in attempting to draw together the results of all this previous research.

There are, however, three recurring themes which can be discerned by an analysis of many of the divergent production tests used in mathematics. In the investigation to be described in Chapter 4 these are referred to as problem-solving, problem-posing and redefinition.

Problem-solving. A number of the tests used by previous researchers are simply mathematical problems with many solutions. A typical task would be (Maxwell, 1974): given the set of numbers, 3, 21, 2, 10 and the symbols for addition, subtraction, multiplication and division, to make up as many combinations as possible which are equal to 17. Other examples would be (Bishop, 1968): "If (p + q) (r + s) = 36, what possible values could p, q, r and s take?" and (Dunn, 1976a): "Find all the ways you can of cutting a four-by-four square grid in half by cutting along the grid lines."

Problem-posing. Prouse (1964) and Balka (1974) both made use of questions in which the student is presented with a paragraph containing numerical information (for example, about wages or costs) and then is required to write down as many questions as possible about the situation so described which could be answered from the given information. Jenson (1973) used a similar approach, but the situations presented to the pupils were in written form, graphic form and chart form. So, for example, the pupils would be presented with a bar chart and asked to write down all the questions they could think of which could be answered from the graph. It is significant that Krutetskii (1976) used problems with unstated questions as one way of assessing the student's ability for formalised perception of mathematical material, one of his main components of mathematical ability.

Redefinition. One way in which divergent production in mathematics has been assessed is by giving pupils situations to which they can respond in many, varied and original ways, only by continually redefining the elements of the situation in terms of their mathematical attributes. For example, it could be suggested that Foster's card sorting test requires the pupil continually to redefine the cards in terms of such attributes as colour, suit, even numbers, greater than 7, and so on. Haylock (1978), using an idea taken from Krutetskii, gave pupils a geometric figure and asked them to write down as many different statements as possible which could be made about one particular line segment in the diagram. Success in this task requires continually redefining the function of the line in terms of its relationship to other parts of the figure.

In the construction of a battery of divergent production tests for the investigation reported in this present study, these headings, problem-solving, problem-posing and redefinition, are used, not as hard and fast categories, but as stimuli for the production of ideas for tests of mathematical creativity.

A Framework for Investigating Mathematical Creativity

The review of literature concerned with creative ability in mathematics undertaken in this chapter lends support to the decision to include both assessment of the ability to break from mental sets in mathematical problem-solving and the ability for divergent production in mathematics in the construction of a battery of tests of mathematical creativity stated as one of the aims of the present study.

Consideration of the relevance of the notions of mental sets and fixations to children solving mathematics problems has suggested that two key aspects may be involved: fixation in terms of the repeated use of a successful algorithm or procedure even when this becomes inappropriate or unsuccessful; and fixation in terms of the range of elements appropriate for application to a given problem. These have been referred to as 'algorithmic fixation' and 'content-universe fixation'.

Consideration of the ways in which divergent production in mathematics has been assessed has suggested that three possible ways of constructing tests of this sort are: via a problem-solving situation; via a problem-posing situation; and redefinition, in which the pupil is required continually to redefine the elements of the situation in terms of their mathematical attributes.

These then are the components of a framework which emerged in the present study for the assessment of mathematical creativity in 11 - 12 year old children. Figure 2.1 gives a schematic representation of this framework. In order to relate the assessment of creativity to the school mathematics curriculum it seemed desirable that tests should include examples of both numerical and spatial situations. Hence the framework also includes reference to these two domains. The framework in this form was the basis for the investigation undertaken in the present study and described fully in Chapter 4 of this report.

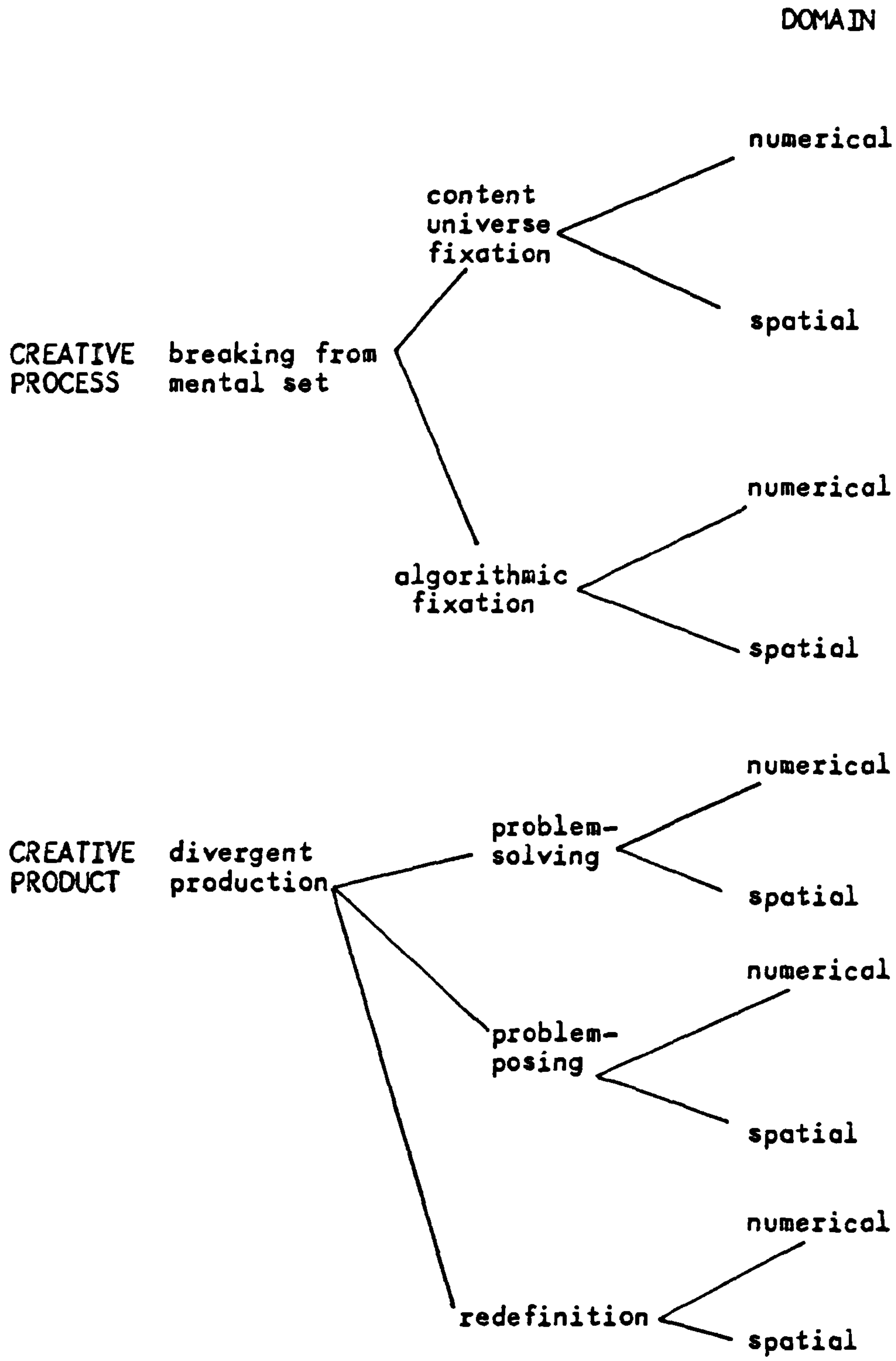


Figure 2.1. A framework for investigating creativity in the context of school mathematics.

CHAPTER 3

SOME CHARACTERISTICS OF THE CREATIVE PERSON

In the introduction to this chapter it is seen that much attention has been given to the study of characteristics of creative individuals, that some rather fragmentary evidence is available about personality and attitudinal factors which might be associated with a liking for or an ability in mathematics, but that apparently no investigation has been undertaken specifically into the characteristics of individuals who show creative ability in mathematics. The literature and research dealing with the aspects of risk-taking, non-conformity, category width, self-concept and anxiety in relation to general creative ability is then surveyed, with detailed accounts of three significant studies (Wallach and Kogan, 1965; Anderson and Cropley, 1966; Allen and Levine, 1968). Particular attention is given to research dealing with willingness to take risks, because of the very varied nature of the approaches taken to indentifying this behaviour. The chapter also includes reference to an interesting suggestion by Torrance (1982) concerning hemisphericity and creative thinking. The chapter concludes with the formulation of six hypotheses, concerning characteristics of mathematically creative pupils, which form the basis for the investigation to be described in Chapters 6 and 7 of this report.

Introduction

The characteristics of creative individuals have been the subject of many studies. Creative persons are usually identified in these studies either by means of creativity tests or by reference to creative achievements. There is so much evidence concerning the characteristics which are found to be typical of creative persons that Torrance and Khatena (1970) have produced an instrument for identifying creatively gifted adolescents and adults on the basis of personality traits. In each item of this instrument the subject is required to state which of two given characteristics they consider best describes themselves. Each pair of characteristics offered includes one of 60 traits found to be more typical of the creative person (Torrance, 1965, p. 226-7), such as "courageous in convictions", "independent in judgment", "curious", "intuitive", "willing to take risks". Halpin (1974) provides a table of means and standard deviations obtained on this instrument by students gifted in eight different subject areas, arranged in order from the highest mean score in terms of showing creative personality traits, to the lowest. Mathematics comes right in the middle of this ordering: Social Science, Art, Science, English/Mathematics (equal), Drama, Music Foreign Languages. This indicates that the traits which have been found to be typical of creative persons in general may not necessarily be typical of mathematically gifted students. Although it is not intended that creativity in mathematics should be used synonymously with giftedness, as some authors (e.g.Krutetskii) appear to do, there is at least some support here for the aim of this present study to investigate the characteristics of pupils who are creative specifically in the context of school mathematics.

The assumption in the present study - as in most of the investigations into mathematical creativity surveyed in Chapter 2 - is that creativity in mathematics is an aspect of mathematical ability which can be demonstrated to some degree by pupils at all levels of achievement. So the focus in the present study will be on identifying some of the characteristics which distinguish pupils showing higher levels of creativity in mathematics from those of their contemporaries with similar levels of mathematics attainment but who show less creativity. Although, as has been noted in Chapter 2, there have been a number of studies of creativity in mathematics, none has addressed itself to this particular question.

There has been some attention given to the personalities of mathematically gifted children, which would nevertheless be relevant to the present study. Aiken (1973, p.19) summarises these results by suggesting that the mathematically gifted child tends to be curious, persistent, highly intelligent, with a good memory, reluctant to accept the obvious, with a dislike for repetitiousness and routine, highly independent, frequently unconventional, flexibile and well-adjusted. Aiken also provides a summary (p.20) of the findings regarding the personalities of famous mathematicians identified as creative in terms of their achievements, though no particularly clear picture emerges from these studies. Head (1981) draws attention to the fact that by far the greatest emphasis in research into mathematical education has been in the area of cognitive psychology, and consideration of personality variables in mathematical performance has been neglected. In a review of work undertaken in this area he concludes that there is some evidence that certain personality characteristics (such as having a greater sense of responsibility, more independence, introversion, being less sensitive to the feelings of peers, having a tendency to be syllabus-bound) may be associated with a liking or an ability in mathematics, but the evidence is very fragmentary.

Rather more interest has been taken in the area of attitudes to mathematics, no doubt because of the phenomenon of "mathematics anxiety" observed by many teachers, confessed to by many adults and associated with the particular nature of school mathematics with its emphasis on exercises which are answered either "right" or "wrong". In a survey of literature related to attitudes and mathematics, Reyes (1980) indicates that although mathematics anxiety may be related to other types of anxiety, many capable people who are not generally anxious are anxious about mathematics. Not suprisingly, high mathematics anxiety is generally found to be associated with low achievement in mathematics, and at the other end of this dimension of attitudes, high self-confidence is found to be typical of high achievers in mathematics. Where sex-related differences in achievement in favour of boys are found these are usually accompanied by parallel sex-related differences in this confidence/anxiety dimension of attitudes.

The background to the present study then is first that much research has been undertaken into the characteristics of creative persons in general and this has provided valuable insights into the nature of creativity and the creative process. Secondly, it appears to be the case that there is only fragmentary evidence relating personality variables to mathematics, and this is in connection with mathematical giftedness or mathematics achievement in conventional terms. Thirdly, although the relationships between attitudes, particularly in terms of anxiety/self-confidence, and mathematics have been investigated in a number of studies, a literature search in this area revealed no study which has given any consideration to the personality or attitudinal characteristics of the children identified by the approaches of these studies as showing more mathematical creativity than others. One of the aims of the present work is to provide an initial exploration of some characteristics, which, on the basis of the hunches outlined in Chapter 1, might be expected to be typical of mathematically creative schoolchildren: willingness to take risks, nonconformity, broad category width, high self-concept in mathematics, low anxiety to mathematics and tests in general. The remainder of this chapter provides a survey of the relevant literature and research into these characteristics, which leads to the formulation of six hypotheses concerning the characterisics of mathematically creative individuals.

Three Significant Studies

This section describes mainly the procedures and conclusions of three studies particularly relevant to the present research since each deals with characteristics of schoolchildren in relation to creativity. The research of Wallach and Kogan (1965) was undertaken with a sample of 151 children aged 10 - 11 years, that of Anderson and Cropley (1966) with 320 children aged 13 years, and that of Allen and Levine (1968) with 164 children aged mainly 10 - 11 years.

Wallach and Kogan's study included particular consideration of the relationships between category width and creativity, and between anxiety and creativity. Creativity is considered by Wallach and Kogan, on the basis of the associative theory of Mednick (1962), to be an ability to form associative elements into new combinations which meet criteria specified in the given situation or which are in some way useful. Five divergent production tests based on this construct and including both visual and verbal stimuli, were administered to the children, in a game-like atmosphere. With tests of this sort, scored for number and uniqueness of associates, and administered in this way, they were able to conclude that creativity is "independent of the conventional realm of general intelligence, while at the same time being a unitary and pervasive dimension of individual differences in its own right". (p. 65) They assert that the ability of a child to display creativity as they conceive of it is unrelated to the ability to earn high scores on measures of general intelligence.

The 70 boys and 81 girls in their sample were thus divided on the basis of high IQ/low IQ and high creativity/low creativity in order to study some of the characteristics of children in four groups: high IQ/high creative, low IQ/high creative, high IQ/low creative and low IQ/low creative.

Wallach and Kogan hypothesised a relationship between category width and creativity. The notion of category width is derived from consideration of the ways in which individuals code the information which they receive from the external environment (Bruner, 1957). Cognitive strain is reduced by seeing data not as unique occurrences but as being related to past data which they resemble (Bruner and Olver, 1963). This process of connecting is referred to as coding, and is the way in which, according to Bruner, the learner renders events meaningful. In any given culture coding becomes highly stereotyped, so that a given event will be coded by most people in the same way. But some individuals appear to have a capacity for making novel or unusual codings which show themselves as creative thinking (Cropley, 1967, p.39). The argument then is that the more a person tends to treat data as though they are related, that is to think in broad categories, the more likely that person is to think creatively. By contrast, people who make very fine discriminations between inputs from the external world are unlikely to make numerous or unusual associations. Willingness to accept as roughly equivalent elements which are in some ways different would seem to be an attribute favourable to thinking creatively.

To investigate this hypothesis Wallach and Kogan used an adaptation of Pettigrew's (1958) Category Width test. (This is described more fully in Chapter 6 of this report). The children were presented with a series of statements about the central tendencies of certain measurement variables, such as, "most windows are about 34 inches wide". They were then asked to make guesses about the maximum and minimum possible values of these variables, such as the width of the smallest or largest windows. The rationale of this instrument is that narrow coders will tend to make guesses close to the given central tendency, because of the predisposition to think in narrow restricted terms about, for example, what might constitute a window, whereas broad coders will make guesses a long way from the stated central value. In this way the deviation from the central tendency in the individual's guesses becomes the basis for a measurement of category width.

The results of the investigation show support for the hypothesis, for both boys and girls, but more strongly in the case of girls. The authors suggest that the association of creativity with broad categorization which they found is because the latter involves tolerance of deviant instances, which, interpreted as an acceptance of the possible, is clearly likely to contribute towards a capacity for divergent production.

Wallach and Kogan also investigated the relationships between anxiety and creativity in their sample of children. Using a questionnaire which assessed general anxiety, test anxiety and defensiveness (the tendency to keep a disposition to anxiety in check by means of a powerful set of personal defence systems), they were able to analyse these personality variables as functions of creativity and intelligence, and vice versa. Significant interactions were found with both general and test anxiety, but only in the case of the boys. These findings and other work concerned with anxiety and creativity are considered later in this chapter.

Anderson and Cropley (1966), working with 13 year old pupils, also investigated the relationship of category width to creativity, using the same instrument as Wallach and Kogan, in a study of correlates of originality. Other variable they considered included: nonconformity, derived from the operational definition of Crutchfield

(1955), that is, a willingness to stay with one's personal judgment even when it is at odds with that of some larger group; willingness to take risks; and impulse expression, that is, a willingness to express impulses and emotions even when such behaviour is at odds with the accepted conventions of the individual's culture. They hypothesised that creative students would be broad categorisers, nonconformist, willing to take risks and high on impulse expression. Except for the conjectured relationship with nonconformity, their hypotheses were supported. Significant differences were obtained in the expected directions in terms of category width, risk-taking and impulse expression between high and low creatives, defined as the top 10% and bottom 10% in performance on divergent production tests.

The authors suggest that these three attributes can be considered to be different manifestations of what they refer to as 'stop-rules'. The developing child is seen to acquire control of his own behaviour by internalizing adult verbalisations of various rules which channel behaviour into what is considered right or acceptable. These 'stop-rules' emcompass the conventions of the child's cultural setting. To think in broad categories requires tolerance of deviants and an acceptance of possibilities which may not be generally accepted into certain categories. Impulse expression runs the risk of one's behaviour being judged unacceptable or unconventional. Anderson and Cropley sum up their results by suggesting then that there is a single stop-rule in operation which bears upon creativity: "don't take risks!" A preference for safe, stereotyped, conventional behaviour involves a low degree of risk, but is unlikely to produce creative thinking. By contrast, creativity will be facilitated by a willingness to risk expressing one's impulses and judgments, even if these might appear to flout conventions. Hudson (1968) also suggests that failure to think divergently is a consequence of a rule-obeying behaviour. The pupil assumes that certain rules are applicable and conforms to these. Head (1981) sees this notion of rule-obeying behaviour as being particularly relevant to mathematics, in which pupils are continually learning and applying rules.

Anderson and Cropley report that the boys in their study performed as a group significantly higher in the creativity tests than the girls. But they also found that the girls showed a much greater predisposition to narrow category width and they suggest that this factor contributed most to the observed sex-related differences in creativity. They also discuss the possibility that girls as a group emerge as narrow coders because they are continually reinforced to be cautious and careful.

Risk-taking in this study was assessed by means of an instrument devised by Brim (Brim and Hoff, 1957), which is based on a 'desire for certainty' construct. The subject who shows less desire for certainty is considered to be a greater risk-taker. The instrument contains 32 items consisting of statements like: "The chances that an American citizen will believe in God are...in 100". The subject is purported to show greater desire for certainty by making an estimate close to 0 or 100. In addition the subjects are asked to indicate how sure they are of their estimates, on a five-point scale. Thus a combination of a high degree of desire for certainty in the estimate and a high level of certainty in backing that judgment would produce a high overall score for 'desire for certainty', which it is suggested indicates a low willingness to take risk, and so on. Cropley (1967) claims that a high score on this instrument indicates that the subject is more willing to take intellectual risks by having a guess in a problem situation and then backing that guess in the absence of any better information, rather than playing safe by making a neutral estimate and expressing no confidence in it. And this behaviour was found to be typical of those students found to be highly creative thinkers as assessed by divergent production tasks. Other approaches to measuring willingness to take risks are discussed later in this chapter.

Allen and Levine (1968) investigated the relationship between nonconformity and creativity, which they claim is well known. This assertion is based upon the work of Crutchfield (1962) who found nonconformity to be typical of creative adults. Yamamoto and Genovese (1965) investigated with 10 - 11 year old pupils the conjecture that creativity is correlated with lack of conformity to group pressure but their results were rather inconclusive. Allen and Levine investigated the possibility of a causal link between creativity and low conformity in 10 - 11 year old children. A control and an experimental group were established, matched for IQ. The experimental group were then given a creativity training programme designed to increase their problem-solving skills. The programme was found to be successful in this respect. The two groups were then assessed for conformity. Allen and Levine devised an interesting procedure for this assessment, based on the notion of conformity to group pressure. The children were presented with a number of questions to answer, in a multi-choice format. One hour later they were given the questions again but told for each question what the most common answer had been. In one third of the cases this information was the truth (neutral items), but in the remainder (critical items) this was not the case. Conformity was measured by the extent to which the individual changed answers on the critical questions in the direction of the purported most common answer. Some of the questions were related to attitudes and opinions, and therefore did not have a correct answer. Others were questions with one definite correct answer, such as facts, spellings, synonyms, numerical items and so on. The results of the investigation show that in the case of subjective questions related to attitudes and opinions there was slightly greater conformity shown by the experimental group than the control group, particularly by the boys. The creativity training programme did not reduce conformity in terms of attitudes and opinions, but may even have increased it. However in the case of the objective questions, those with one correct answer, the experimental group were found to be significantly less conformist than the control group. It seems that training in creative thinking contributed to the pupils' determination to stick with their answers to objective questions and not be swayed by the opinions of others. It is interesting to note that for the control group it was found that conformity was inversely related to IQ, but this was not the case in the experimental group. The creativity training programme could therefore have enabled the less intelligent pupils particularly to become less dependent upon the opinions of others.

The results of these three investigations contributed to the decision to investigate in the present study the relationships between creativity in mathematics and risk-taking in mathematics, nonconformity in mathematics and category width. The Wallach and Kogan work also suggested that questions of anxiety towards mathematics and towards tests would be worth pursuing in connection with creativity in mathematics. Other relevant research in this direction is surveyed later in this chapter. A great variety of approaches to investigating risk-taking in particular have been used in various studies. These will now be surveyed.

Approaches to Assessing Willingness to Take Risks

Kogan and Wallach (1964) describe a number of different approaches to assessing risk-taking which they used with adult students in a study of the role of risk-taking as it operates in a motivational context, and the influence of personality in steering people towards either risk-taking or conservatism.

Some of their risk-taking instruments were 'cognitive-judgmental' tasks, in which the risk element is covert and emerges only in the strategy employed to solve a problem in order to meet the overt requirements of the task. The risk element is in the subject's own tolerance of the likelihood of error. In this category of risk-taking, Kogan and Wallach include the Brim instrument (as used also by Anderson and Cropley) and the Pettigrew category width instrument. This interpretation of category width as a function of risk-taking is not uncommon. Wallach and Caron (1959) had assessed risk-taking in children by an instrument clearly related to the notion of category width, in which they were required to decide whether ambiguous geometric figures were or were not similar to a given key figure. Risk-taking was identified as a greater willingness to include the ambiguous figures within the class of similar figures. Bruner and Tajfel (1961) distinguish broad and narrow categorizers in terms of their willingness to risk errors of inclusion or exclusion respectively. When presented with more varied examples of a particular class of objects, the narrow categorizer reacts to the contrasts or differences by forming new categories. The broad categorizer tends not to react to stimulus change. Bruner and Tajfel suggest that these are simply different strategies for coping with the consequences of error. The narrow categorizer by reacting runs the risk of making an error of exclusion, whereas the broad categorizer by not reacting risks an error of inclusion. In spite of this analysis, Kogan and Wallach have chosen to interpret the score on Pettigrew test as a measure on a risk-taking continuum in this cognitive-judgmental domain.

The second approach to risk-taking which they identify is in terms of decision-making. In this case the risk element is explicit rather than implicit, in terms of the subject's own assessment of the probabilities of success or failure. In the Choice-Dilemmas Procedure, for example, subjects are presented with a choice dilemma between a risky and a safe course of action. They are then required to state what probability of the risky alternative's success would they accept in order to be prepared to choose this option.

In addition to the cognitive-judgmental/decision-making distinction, Kogan and Wallach also distinguish between risk-taking in hypothetical conditions (such as the Choice Dilemmas Procedure) and pay-off conditions (i.e. when a monetary reward is involved), and also between risk-taking in chance and skill conditions. For example, the Skill Bets instrument, involving a shuffleboard game of skill, with the possibility of a monetary reward and the subject setting the level of difficulty, would be categorized as a decision-making, skill task, under pay-off conditions.

The value of this analysis by Kogan and Wallach is that it draws attention to the fact that there is little evidence for regarding risk-taking willingness as a single, consistent style of behaviour. They cite evidence (p.9) that individuals who show willingness to take risks in one sort of condition will not necessarily do so in another. It is clear that any investigation of risk-taking must not read anything too general into results based upon the use of a single instrument. Consideration of such aspects as the nature of the rewards for success or failure, the probabilities of success or failure, the extent to which the risk element is overt or convert, and the distinction between chance and skill in decision-making, is clearly important.

Strum (1971), in a study with 9 - 11 year old children, defined risk-taking as a tendency to guess in a classroom setting even when there was a penalty. This was measured by an instrument in which pupils were presented with a verbal ability multi-choice test, for which they could themselves choose how many points were available for each question (i.e. 1, 2, 3, or 4 points). The points would be lost for wrong answers. Risk-taking on this test was assessed by means of the proportion of wrong answers which a pupil marked with the maximum 4 points. Strum's finding that there is no support from her study for linking creativity with risk-taking must be countered by questions about the very limited nature of the risk-taking behaviour assessed by this single instrument. It has also been suggested (Hoffman, 1962) that creative students have very negative attitudes to multi-choice tests, because these tests favour so strongly a convergent style of thinking. However, Horber and Geisinger (1983) have examined this suggestion empirically and found no evidence that this is the case. They also report that risk-taking is not correlated with attitudes to multi-choice tests. However it is clear that in certain circumstances the behaviour of a pupil on a multi-choice test could be related to willingness to take risks. This possibility is used in the investigation described in Chapter 6 as the basis for an exploratory instrument for assessing risk-taking in mathematics.

The fact that risk-taking behaviour varies from one situation to another is supported by the results of Pankove and Kogan (1968), who administered three different risk-taking instruments to 162 children aged 10 - 11 years. One of these was the shuffleboard game used by Kogan and Wallach. The second instrument was a guessing game in which children are offered a sequence of clues and a monetary prize for the first child to identify correctly the hidden object. The third was an instrument suggested by McClelland (1961) in which children are required to place a cross in the centre of a circle they have drawn. The size of the circle drawn is presumed to be inversely related to the degree of risk involved. It is clear that if risk-taking is involved in all three of these procedures then it is taking very different forms: there are considerable differences in terms of the pay-off involved, the balance between skill and chance, and the probabilities of success or failure. Not surprisingly then, Pankove and Kogan found that these three risk-taking instruments produced scores which were not correlated at a level of any statistical significance. Pankove and Kogan's intention in their study was to investigate the claim that risk-taking was associated with creativity, a claim which they point out though often made is unsupported by empirical evidence, particularly in the case of children. Their own findings did little to clarify the purported relationship. Only in the case of the shuffleboard game, and then only for boys, did they find that creativity (as measured on two divergent production tests) was associated with a preferred level of risk-taking. This relationship was most strong amongst low defensive boys, leading the authors to suggest that self-confidence is a possible mediating link in the creativity/risk-taking relationship, particularly in view of the nature of the task in which this relationship was found.

McClelland (1958) describes a number of other games - involving ring-tossing, tilting maze boards, dot connections and word memory - in which the subject sets the level of difficulty, which he used as instruments for measuring risk-taking in children aged five and eight years. He found that moderate risk-taking was associated with a high need for achievement. His interpretation of this finding is that the child with high achievement need, wanting strongly to do well, is prepared to take a risk, but, because failure is more painful to such a child, the desire to take a risk is moderated.

Glover (1977) reports that college students identified as high risk-takers were found to be significantly more flexible and original in their responses to the Torrance tests of creative thinking than those identified as low risk-takers, though they showed less elaboration in their responses. He investigated the relationship between risk-taking and creativity further by consideration of the so-called 'risky shift' phenomenon. This refers to the tendency for group discussion about risk-taking to result in a shift towards greater willingness to take risks. Two groups, one control and one experimental, were given pre- and post-tests of creative thinking and risk-taking. In the intervening period the control group relaxed while the experimental group engaged in group discussion based on the risk-taking instrument. The risky shift phenomenon occurred as expected within the experimental group. But this was also accompanied by significant increases in flexibility and originality scores on the divergent production tests. Elaboration scores decreased. The control group showed no changes of significance on any of the measures. Glover suggests that these results serve to strengthen the relationship between risk-taking and creativity, and speculates that the encouragement of greater willingness to take risks may be used in a classroom setting to increase the level of creative responding of students.

In another investigation with college students, Eisenman (1969) assessed their willingness to take risks by offering them the opportunity to let their grades on the first test decide their grades for the course. He also used a money-making/losing game. He found that some creative students were adventurous and willing to take risks. Others however were more skilled in impersonal problems, were introverted and quite unwilling to take risks. He warns against expecting any creative individual to possess any characteristic found to be typical of creative groups.

The picture which emerges from this review of research into risk-taking and creativity is far from clear. This is almost certainly because of the very varied nature of risk-taking behaviour associated with different instruments and tasks (Kogan and Wallach, 1964). But theoretically risk-taking is seen as a very pervasive phenomenon in cognition and personality and is considered by some authors to be involved in broad coding, nonconformity and impulse expression (Anderson and Cropley, 1966). The relationship with creativity has not been established strongly in empirical studies with children, the results being very dependent on the style of instrument used for assessing risk-taking.

Hemisphericity and Creativity

An interesting relationship between hemisphericity and creativity is suggested in some recent work by Torrance (1982). The left hemisphere of the brain is associated with logical, sequential processing of information, dealing mainly with verbal, analytical, temporal and digital materials. The right hemisphere is associated with nonlinear, holistic processing, dealing with nonverbal, spatial, analogic, emotional and aesthetic materials. Torrance suggests that creative functioning involves both types of activity, the right hemisphere generating divergent, creative ideas which the left hemisphere evaluates. This suggestion would be consistent with Vallee's (1975) argument that mathematical creativity calls upon both intuition (a right hemisphere activity) and deduction (a left hemisphere activity). It is also clearly related to Krutetskii's (1976) identification of the type of mathematically capable pupil referred to as the analytic-synthetic type. Such a pupil demonstrates the ability to switch freely between analytic thinking (left hemisphere activity) and synthetic thinking (right hemisphere activity). Torrance investigated the relationship between creativity and hemisphericity with college students. A self-report instrument was used to determine which students showed a preference for an information processing style based on left hemisphere activity, which on right, and which adopted an integrated style. They were also assessed for creative style (using the instrument developed by Torrance and Khatena, (1970), based on personality characteristics of creative individuals), and creative ability (using divergent production tests). Torrance reports that creative style was found to be related positively and significantly to right hemisphericity, and to be related negatively and significantly to left hemisphericity, with inconclusive results for the integrated style. Less significantly he found that creative ability was negatively related to left hemisphericity, and to some extent positively related to right hemisphericity.

These results seem to indicate that in terms of creative style the right hemisphere is predominant, and also in terms of creative production to some extent. The suggestion that left hemisphere activity is necessary to evaluate creative products is consistent with Torrance's findings. The notion of hemisphericity seems to be related to discussions of category width and coding. Narrow coding would appear to be associated with predominantly left hemisphere activity, concentrating on analytical, sequential thinking, with an awareness of differences between inputs. Broad coding would likewise be most strongly associated with right hemisphere activity with a tendency to think synthetically, holistically and in terms of similarities. Hence there is theoretical support for the association of broad coding with creative thinking in this analysis. This analysis is also particularly relevant to mathematical creativity. Mathematical activity in schoolchildren is conventionally seen as predominantly calling upon the left hemisphere, particularly in material dealing with numerical and algebraic entities. To be creative in mathematics might therefore involve a greater degree of right hemisphere activity. A connection between broad coding and mathematical creativity would support this suggestion.

Self-Concept, Self-Confidence and Anxiety in Relation to Creativity

The notions of self-concept, self-confidence and anxiety can be clearly related theoretically to creative functioning. It would be expected that a person with a high self-concept, that is a person who expects to succeed, who has a high opinion of his or her capabilities would show greater self-confidence in breaking away from safe, predictable paths in problem-solving. A person showing anxiety as a general attitude towards working in a particular field would similarly be unlikely to move away from safe, secure ground and produce original or novel ideas. Prince (1973) asserts that each person's most fundamental enterprise is the development, enhancement and protection of their self-concept. He argues further that creative activity contributes to a person's self-esteem. Thus it can be suggested both that creative behaviour is dependent upon a high self-concept and also that it contributes to a high self-concept. Since threats to self-esteem are experienced as anxiety, it could likewise be suggested that anxiety would inhibit creativity on the one hand and be produced by failure to act creatively on the other. This theoretical relationship between creativity and self-confidence/anxiety has been explored empirically by a number of researchers from different angles.

MacKinnon (1961) reports that the highly creative people (in terms of creative achievement in various fields) which he and his associates studied were characterized by strong self-concepts and an unwillingness to accept anything on the mere say-so of others in authority. He suggests that the creative person is not preoccupied with the opinions of others on his products because he has a high self opinion, and consequently is freer to be himself than others. Torrance (1962, p.76) cites the findings of studies undertaken by Weisberg and Springer in 1961 with 9 - 10 year old children, which found, amongst other things, that the most highly creative children were rated significantly higher on strength of self-image in psychiatric interviews.

Beier (1951) investigated the effects of induced anxiety on flexibility of intellectual functioning in adults. The experimental group were given a Rorschach test with interpretation to induce anxiety about the next test to be administered. This was a sorting test requiring flexible thinking for successful solution. The control group, who had not been subjected to the anxiety-inducing experience, showed greater flexibility than the experimental group who showed greater rigidity in their thinking. Cowen (1952) investigated the effect of varying degrees of stress upon college students' performances on the Luchins Jugs tests. Three groups of students were compared: one group (the control) being subjected to no stress in the way the test was administered, one being subjected to mild stress, and the third to strong stress. Cowen reports that in terms of both problem solution time and the number of rigid solutions, the control group showed least rigidity and the strong stress group showed greatest rigidity. Krop, Alegre and Williams (1969) used disturbing film to induce anxiety in students and again found that the induced stress diminished performance in divergent production tasks, but not in a convergent thinking test. It is clear from these studies that, at least for adults, rigidity in thinking is influenced strongly by the conditions and implications of the testing situation. The more anxiety associated with the test the less flexibility is likely to be evident. Cunningham (1966) suggests that stress can make the subject somewhat defensive, insecure and no longer able to feel completely free in exploring the test situation.

The findings relating creative performance to anxiety assessed as an attitude usually by means of a questionnaire approach are less clear-cut. Some researchers have concentrated on general anxiety, that is an attitude which shows itself as generally worrying about things, and others specifically on test anxiety. Hadley (1965) suggests that both psychoanalytic and multiplicative drive theories would conclude that excessive anxiety would inhibit intellectual performance, but that there is a theoretical basis for expecting some minimum amounts of anxiety to be necessary for the highest performance to take place. This suggestion would be supported by the findings of Hargreaves (1974) that the removal of time limits in divergent production tasks actually depresses test scores in children. Hargreaves suggests that this phenomenon is due to the way in which the child's motivation interacts with the test situation. Hadley therefore investigated the possibility of a curvilinear relationship between anxiety and creativity in 12 and 13 year old children. Both general and test anxiety were assessed by means of a questionnaire. The hypothesised curvilinear relationship was exhibited to some extent. Those with very low anxiety tended to show moderate levels of creativity, those with a slightly higher level of anxiety were the most creative group, and then those with medium to very high anxiety showed decreasingly less creativity with increasing anxiety level as the highest level of anxiety is approached. Hadley also found that divergent production tests involving less structured tasks required a minimum level of anxiety for successful performance. But on those tests involving more structured tasks even very low anxiety appeared to interfere with highest performance. These results have implications for the investigation into mathematical creativity, since the tasks involved in the assessment of this ability or set of abilities would inevitably , because of the mathematical content, contain a fair degree of structure. In a second part of his study, Hadley reports findings that show again that performance on creativity tests is inhibited by stressful test conditions.

Klein, Fredrikson and Evans (1969) found that an intermediate level of test anxiety was associated with low levels of production of many responses in creativity tests with college freshmen. They suggest that the subjects with high test anxiety are impelled to produce many responses because of their concern to do well and worries about doing badly in the test. Furthermore, subjects with low test anxiety are less inhibited in their responses because they do not fear ridicule or embarrassment if these are judged to be of poor quality, so thus they are also enabled by such an attitude to produce many responses. But the subjects with an intermediate level of test anxiety are fearful of reporting responses which might not be of high quality, but not so fearful of failure that they feel obliged to report their mediocre responses.

In another study undertaken with college students, White (1968), on the other hand, found that low anxiety was associated with high levels of divergent production, with the descriptors "restrained" and "apprehensive" emerging as the most significant factors related to poor performance in creative thinking tasks.

In a study undertaken with 71 Israeli children aged 6 - 7 years, Strauss (1981) used the children's responses to a Rorschach test to estimate levels of repression and anxiety. Some aspects of creative ability were found to be related to decrease in repression and to some extent with decrease in anxiety, but this finding was only in the case of non-verbal tests. Performance in verbal creativity tests, particularly at that age, would be more likely to be related to intelligence, and Strauss found no relation between intelligence and the variables of repression and anxiety. In interpreting these findings it is important to note that Dudek (1974) has shown that scores on creativity measures do not stabilize until around the age of 10 years.

In their investigation with 10 - 11 year old children, Wallach and Kogan (1965) considered both the possibility that personality traits might be a function of creativity and intelligence, and also the possibility that such traits as anxiety and defensiveness, assessed by means of a questionnaire, might exert a causal influence in intelligence and creativity. With personality considered as a function of the modes of thinking their analysis found significant interactions for both general and test anxiety, but only in boys. The high creative boys showed intermediate levels of anxiety both in general and towards tests. The lowest levels of anxiety were shown by boys of high intelligence and low creativity, which the authors suggest may be because their competencies match most closely the demands of conventional school assessments. The highest levels of anxiety were exhibited by low intelligence, low creative boys. Treating intelligence and creativity as consequents of anxiety and defensiveness, Wallach and Kogan found that for boys test anxiety was inversely related to intelligence, and defensiveness inversely related to creativity. No significant interactions with creativity were found for girls, except for the fact that the pattern of results for creativity was markedly different from that for intelligence. Girls high in test anxiety and defensiveness were found to be exceptionally low in intelligence.

This survey of previous research into the relationships between creativity and self-concept/anxiety suggests that significant but complex interactions between these variables might be expected. There is evidence that in some cases anxiety can be associated with poor performance both in Einstellung tests and in divergent production tests, though this finding is not universal. Different conclusions have been obtained by different researchers depending on the instruments used for assessing creative ability and the personality aspects, and also depending on the age of the subjects under consideration.

Six Hypotheses to be Investigated

In this chapter a survey has been provided of previous research into the relationships between creativity and aspects of personality. These aspects have been selected for consideration on the basis of five hunches discussed in Chapter 1 about likely interactions between personality and mathematical creativity, based on consideration both of the nature of creative thinking and the nature of mathematics as it is learnt in schools. Six hypotheses can now be formulated as the basis for the investigation into personality and attitudinal factors in relation to mathematical creativity which is described in Chapters 6 and 7 of this report.

It should be noted first that since some of the factors being considered, such as self-concept and anxiety towards mathematics, are known to be related to mathematics attainment (Reyes, 1980), and furthermore performance in mathematical creativity tasks would be expected on both theoretical and empirical grounds (e.g. Dunn, 1976a) to be limited to some extent by the pupil's mastery of mathematical skills and knowledge, then it will be necessary in the final analysis to consider pupils in bands of similar levels of mathematics attainment. This condition is built into the hypotheses framed below.

Hypothesis 1. "Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to be more inclined to take risks in mathematics."

The survey of risk-taking research has shown that risk-taking can be conceived of as a pervasive phenomenon in cognition and personality, that there are indications that it is associated with creative thinking, (Anderson and Cropley 1966) but that there are many different categories of risk-taking behaviour (Kogan and Wallach, 1964). This hypothesis refers specifically to taking risks in the context of doing mathematics. It will be necessary to consider different ways in which such risk-taking might occur, varying such elements as the nature of the reward or penalty for success or failure, the probabilities of obtaining the reward or the penalty, and the balance between chance and skill. The rationale of the hypothesis is that conventional school mathematics does not encourage risk-taking, but inclines more towards safe, learnt procedures, with an emphasis on care and accuracy. Such emphases if carried over into performance in mathematical creativity tests would theoretically induce rigidity and inhibit divergent production.

Hypothesis 2. "Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to be more nonconformist in their behaviour in mathematics."

There is evidence from both empirical (Allen and Levine, 1968) and biographical (Barron, 1963) studies that creative individuals are less conformist in the manner defined by Crutchfield (1955, 1962).

That is to say they are more inclined to trust their own judgments even when these are at variance with the opinions of a larger group. The rationale of the hypothesis is that children who demonstrate that, in the context of doing school mathematics, they are prepared to stick with their own judgments, even when these are seen to be at variance with others, are more likely to be creative and to show originality in their responses.

Hypothesis 3. "Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to be broader categorizers."

There is clear evidence that category width is positively related to creative ability (Anderson and Cropley, 1966; Wallach and Kogan, 1965), as assessed by divergent production tasks. It would be expected that a predisposition to think in broad categories would be favourable for pupils tackling mathematical creativity tasks as well. To think in flexible ways about the range of mathematical concepts or procedures which might be applicable to a particular situation would seem to require that the information perceived from the situation and the information available in the pupil's cognitive structures should not be coded in narrow or restricted categories.

Hypothesis 4. "Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to have higher self-concepts in mathematics."

Creative individuals tend to be characterized by high self-concepts (MacKinnon, 1961). In order to move away from safe, predictable territory in mathematical problem-solving it would seem to be necessary that pupils should be confident in their own abilities to cope with whatever turns up. This is the rationale of the hypothesis that pupils with higher self-concepts, specifically in terms of expecting to do well in mathematics, would show more creativity in their performance in mathematics.

Hypothesis 5. "Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to have lower levels of anxiety towards mathematics."

There is much evidence that induced anxiety inhibits divergent thinking (Hadley, 1965) and promotes rigidity in problem-solving (Beier, 1951; Cowen , 1952). An attitude of anxiety assessed by questionnaires has also been found to interact with creative performance, though the survey of research in this area suggests that the relationship is complex. Anxiety towards mathematics has been regarded as a particular problem in school learning (Reyes, 1970). The hypothesis formulated here is based on the hunch that anxiety towards mathematics would have a particularly inhibiting effect upon a pupil's ability to break from mental sets and to think in divergent ways in mathematics.

Hypothesis 6. "Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to have lower levels of test anxiety."

Many of the studies concerned with anxiety and creativity have dealt specifically with test anxiety (Wallach and Kogan, 1965; Klein et al, 1969). This hypothesis has been formulated in addition to hypothesis 5 because it is suspected that anxiety specifically about mathematics may be more significant in inhibiting creative thinking in mathematics than anxiety about assessments in general. Some associations between test anxiety and creativity have been found in previous research. The hypothesis implies that pupils with lower levels of anxiety towards tests will be in a more favourable position for tackling mathematical creativity tasks in a test situation.

It will be interesting to see which of hypotheses 5 and 6 is most strongly supported, if at all, or whether both mathematics anxiety and test anxiety are significantly related to mathematical creativity performance.

Chapter 2 of this report has surveyed relevant literature dealing with mathematical creativity, in terms of overcoming fixation and divergent production. Chapter 3 has now considered the background to the conjectures about the relationships between personality characteristics and mathematical creativity, by surveying research into some particular aspects of the creative person.

Chapter 4 of the report now describes the development and administration of a battery of tests for assessing mathematical creativity in 11 - 12 year old children, based on the framework outlined at the end of Chapter 2. The results obtained from this battery of tests are analysed further in Chapter 5. Chapters 6 and 7 then describe the investigation undertaken into the hypotheses formulated at the end of Chapter 3. This leads to some con-clesions about the characteristics of pupils who are found to be high mathematically creatives.

CHAPTER 4

THE DEVELOPMENT OF A BATTERY OF TESTS FOR THE ASSESSMENT OF ASPECTS OF MATHEMATICAL CREATIVITY IN CHILDREN AGED 11 - 12

This chapter describes the development, administration and analysis of a battery of tests designed to assess the two major aspects of mathematical creativity highlighted in Chapter 1: (a) the ability to break from mental sets (overcoming fixations) in mathematics, and (b) divergent production in mathematics. The chapter begins with an outline of the methodology employed with a description of the test situation used for the main part of the research programme. This is followed by two substantial sections, the first dealing with the tests related to overcoming fixation, and the second with the divergent production tests. Each of these two sections will consider the background and development of the tests in question, the criteria evolved for devising such tests, and an analysis in detail of the responses of the pupils in the main part of the research to each test in turn.

Methodology

The initial phase of the development of the battery of tests involved the analysis of the way in which the two key ideas had been investigated in previous research and consideration of tasks which might be given to 11 - 12 year old pupils for which they could use the mathematics they knew to demonstrate their abilities for overcoming fixation and divergent production. As ideas emerged selected tasks were field tested by the researcher with various classes of schoolchildren and students. These trials were designed to test the feasibility of group-administered, pencil-and-paper tests for assessing these two aspects of mathematical creativity.

The trials included both analysis of the responses of pupils on such pencil-and-paper tasks and discussion with them about the tasks they had undertaken and their reactions to them. These analyses and discussions served to confirm whether or not the tasks in question were to any extent valid assessments of the constructs, and also to refine the wording of the instructions given to the pupils. The classes used in the trials contained on average thirty pupils and were taken from situations readily accessible to the researcher. They included, for example, 10 - 12 year old pupils in Middle schools, a class of 12 - 13 year olds in a High school, and first year College students training for teaching. The responses obtained from the piloting of these tests helped to crystallise the criteria which must necessarily be satisfied for tests assessing the two aspects of mathematical creativity being considered. The major part of the research involved the administration and analysis of a battery of pencil-and-paper tests to 283 pupils aged 11 - 12 in a test situation described below. A large sample was decided on to allow for analysis of pupils' performances within fairly narrow bands of mathematical attainment. The age range 11 - 12 was chosen because it was presumed that by that age most pupils would have the necessary mastery of some basic mathematical skills and knowledge to tackle tests concerned with the two aspects of creativity in question, but they would not yet be learning mathematics under the constraints of a rigid public examination syllabus. About half the tests included in the battery of mathematical creativity tests had been developed from the trials and the others were newly devised on the basis of theemerging notions of what constituted a valid test of these particular abilities. Tests were designed to cover both numerical and spatial domains. The battery included eight tasks related to overcoming fixation in mathematics, of which five were eventually considered to be successful in terms of the construct, and fourteen tests of divergent production in mathematics, of which ten were judged to meet the criteria which had been determined in the course of development. This evaluation was done by means of the detailed analysis of the pupils' responses to these mathematical creativity tests which forms the major part of this chapter.

Also included in the battery were a standardised norm-referenced test of mathematics attainment (the NFER EF test), a mathematical problem-solving test, and six tests related to personality traits. The mathematics attainment test would be used to put the pupils into bands of similar levels of attainment in mathematics for much of the analysis to be undertaken. This would be necessary because inevitably a pupil's performamce on the mathematical creativity tests would be affected by the level of mastery of skills and knowledge available. The problem-solving tests contained a number of unusual problems derived from Krutetskii (1976) and would be used to give some degree of validation to the mathematical creativity tests. The instruments related to personality traits were designed to test the hypotheses formulated in Chapter 3. These are described in detail in Chapter 6. Two of them, a category-width test and an attitude questionnaire, were modifications by the researcher of existing instruments, for 11 - 12 year old English schoolchildren. The other four were newly devised in an attempt to assess pupils' willingness to take risks or to show nonconformity specifically in the context of doing mathematics.

Further analysis of the pupils' scores on the mathematical creativity tests included consideration of the relationship between mathematical creativity and mathematics attainment, numerical and spatial aspects, and boy/girl differences in performance. These are dealt with in Chapter 5.

The hypotheses related to personality traits were investigated both by analysis of correlations between scores on mathematical creativity measures and personality-based measures within various bands of mathematics attainment (Chapter 6), and also by consideration of profiles of high mathematically creative and low mathematically creative pupils (Chapter 7).

<u>The Test Situation for the Major Part of the Research</u>

A total of 283 pupils, aged 11 - 12 years, took part in the test programme, though because of inevitable absences not all pupils took all tests. The pupils were the entire top years of three 8 to 12 Middle Schools, with the exception of a handful of pupils who were judged by their teachers to be incapable of following the simplest instructions in mathematics. Thus the sample used covered a wide spectrum of ability, but did not include many of the lowest attainers in mathematics. In fact the sample is shown to be top-heavy in terms of the level of mathematics attainment by the distribution of their standardised scores obtained on the NFER EF test of mathematics attainment shown in Table 4.1.

The three schools were situated on the outskirts of Norwich, England. Norwich is, by English standards, a somewhat isolated city in the middle of the mainly rural county of Norfolk. Greater Norwich has a population of less than $\frac{1}{4}$ million. It is both an historic, cathedral city and an industrial city with large areas of terraced housing, much of it council housing of a high standard. Most of the suburban growth of owner-occupied housing has been around the edges of the city boundary, and it was in such areas that the research was carried out. Two of the schools used were in an established suburb, largely of inter-war semi-detached private housing, with a fairly settled population of skilled working class and middle class residents. The third school was in a mainly middle class outer suburb of postwar housing. The three schools had changed from Primary (7 - 11) to Middle (8 - 12) in the late 1970's. They were still organised mainly on Primary school lines, with class teachers responsible for much of the teaching, but with setting for mathematics in at least the top two years.

Table 4.1

Distribution of Standardised Scores on the NFER EF Test of Mathematics Attainment for the 283 Pupils Used in the Main Part of the Research

NFER EF score	Number of pupils
130 - 140+	56
115 - 129	99
100 - 114	103
85 - 99	19
70 - 84	3
not obtained	3

The schools involved in the major part of the research all followed very similar mathematics syllabuses, based mainly on the same text book series (Beta Mathematics, published by Schofield and Sims), with a strong emphasis on high standards of basic skills in mathematics. The tests were administered to the pupils by their own teachers over a period of ten weeks in a Summer Term, according to the timetable given in Appendix 1. The battery also included the tests related to aspects of personality which are discussed in Chapters 6 and 7. The teachers were given precise instructions for administering the tests in an attempt to obtain similar testing conditions across the three schools (see Apendix 2). Three schools were used in order to provide a sample large enough to allow analysis of the performances of pupils within fairly narrow bands of mathematical attainment.

Assessment of Pupils' Ability to Overcome Fixation in Mathematics

Background

Flexibility of mental processes is a key idea in all discussions of creativity. Krutetskii (1976) has identified this as one of the principal components of mathematical ability in schoolchildren. In mathematics this flexibility involves combining in unexpected ways methods and knowledge already understood in order to solve a problem. In the sense that in problem-solving the student is conventionally seeking one solution we are dealing here with an aspect of thinking which might be thought of as 'convergent' - on the other hand it is 'divergent' in the sense that the student must break away from the predicted or expected path in the search for an appropriate solution.

The starting point for the development of the assessment instruments to be described below was then the assumption that *a significant aspect of being flexible in problem-solving in mathematics is the ability to break from an established mental set.* From the responses of people involved in mathematics education to a list of some 25 criteria associated with the measurement of general creativity, Balka (1974) identified six criteria considered to be relevant to measuring creative ability in mathematics. These included "the ability to break from established mind sets in mathematical situations." Previous work by the present author (Haylock, 1977) had also identified this as a significant aspect of creative problem-solving in mathematics. It is not surprising that mathematics educators should select this as being important in the consideration of creativity in mathematics. Anecdotal evidence from mathematics teachers recalls many situations where pupils' thinking seems to be set along particular, inappropriate lines. For example, at a simple level, all mathematics teachers will have observed pupils using the long multiplication algorithm for a calculation like 20 x 10, or the addition of fractions algorithm for, say, $\frac{1}{2} + \frac{1}{4}$, when away from the context of a set of arithmetic exercises requiring the algorithms concerned the pupils would be able simply to state the answers required. Gibson (1941), in a review of the concept of 'set' as it is used in psychological theory showed that it was a concept used freely but with much variation in the meaning. The notion of 'mental set' as used in the present research is essentially derived from the concept of set in problem-solving as used by Luchins (1951). This refers to a factor which facilitates or encourages responses for which a subject has been prepared, possibly by a combination of previous experiences and the context and instructions of the problem as presented, and at the same time inhibits any competing response. Thus a student attempting to solve a mathematics problem might fail to do so appropriately because of rigidity of thought, as opposed to flexibility. Hence the particular feature which the research set out to highlight would be a pupil's failure to solve appropriately a mathematics problem which requires nothing more than elementary mathematical skills within the pupil's grasp, because of the action of such a mental set.

Development of Instruments

Luchins (1942) had devised a classic example of a problem demonstrating this feature. Students were required to measure out a given quantity of water using three jugs of varying capacity. The first five examples could be solved most efficiently by using the same method: fill up jug B, pour off into jug A, and then pour off twice into jug C. For example, with A = 21, B = 127, C = 3, the problem would be to measure out 100 units. In this case, B - A - 2C = 100. The sixth example required that 20 units be measured out, where A = 23, B = 49 and C = 3. Once again the problem can be solved by the method 'B - A -2C', but this would be to overlook the simplest solution, 'A - C'. Luchins found that many students failed to find this simplest solution because of the previously established method of thinking about the problem. In the development stage of the present research this jugs problem was tried out informally and discussed with several classes of both schoolchildren, in the age range 11 - 13 years, and college students. Failure to find the most appropriate solution to the last part of the problem was very common, in fact as common with the students as with the schoolchildren. The action of the mental set persisted even when the test had an extra part added requiring the measurement of 5 units, with A = 50, B = 65, and C = 5! Discussion about why the more efficient solutions had not been found for the last two parts invariably produced comments such as, "I thought, 'There's a pattern here' and it worked every time", and "I thought I should always start by filling up the largest jug first". It appeared that, for many students, once a way of solving these problems emerged this would exclude the possibility of any other approach being considered. These trials indicated that this could form an appropriate test of overcoming fixation for 11 - 12 year olds, since no more than elementary arithmetic and spatial ideas were involved. The test in its final version appeared as Test 10 (Jugs) in the main part of the research programme. (See Appendix 1)

Krutetskii's series of problems used to identify components of mathematical ability in schoolchildren included a number of items in which failure to solve the problem might be due to what he terms 'self-imposed restrictions'. This is a form of mental set where the pupil imposes some restriction upon his thinking about a problem which is not inherent in the situation and consequently he is unable to solve it. For example, one problem requires the pupil to draw a quadrilateral intersected by a single straight line so that the resulting diagram contains four triangles. Many pupils fail to solve this problem because they restrict their thinking to convex quadrilaterals. The solution requires breaking from this self-imposed restriction by drawing a concave quadrilateral as shown in Figure 4.1.

Prior to the current research, the present author gave this problem to 128 pupils aged 14 - 15 years (Haylock, 1977) and found that only 15% of them solved it successfully. The majority of the others showed a fixation on using convex quadrilaterals. In the development of the current research several possible fixations about quadrilaterals were considered as the basis of an assessment of the ability to overcome fixation. It was noted that, when asked to draw a four-sided figure, almost invariably Primary school pupils will draw a rectangle with the sides parallel to the edges of the paper and the base as the longer side. This suggested the following possible self-imposed restrictions about a four-sided figure: (a) that the sides must be horizontal and vertical, (b) that the angles must be right angles, (c) that the longer side must be the

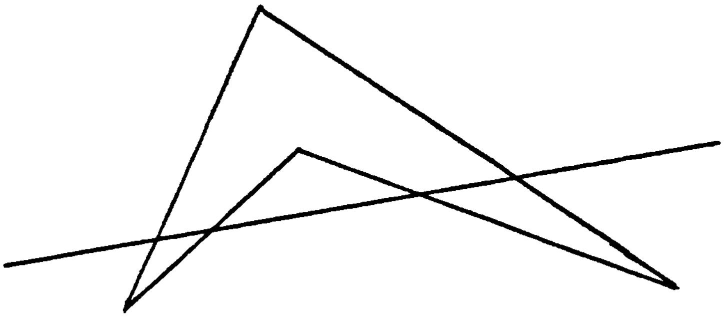


Figure 4.1. To draw a quadrilateral intersected by a single straight line so that the resulting diagram contains four triangles: the solution requires overcoming fixation on convex quadrilaterals.

"This is one side of a four-sided figure:

Draw the other three sides so that the area of the figure is 8 cm^2".

Figure 4.2. A problem requiring overcoming fixation on horizontal and vertical sides.

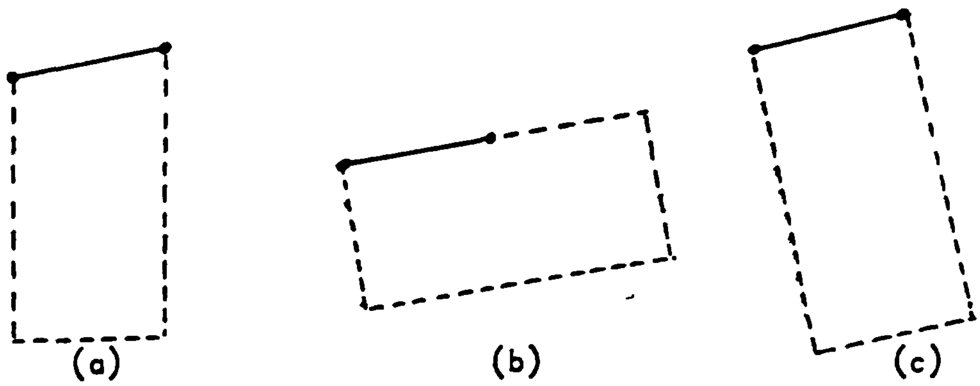


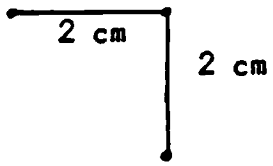
Figure 4.3 Three responses to the problem shown in Figure 4.2.

base, and (d) that it must be convex. In the context of a lesson on area a class of 10 - 11 year old pupils were asked to complete the drawings of various four-sided figures in order to make their areas equal to, greater than, or less than given quantities. It was planned that to solve these problems they would need to overcome the restrictions listed above, and this was borne out in the incorrect responses and failures. For example, given the problem, shown in Figure 4.2, a number of pupils showed that they restricted their thinking to rectangles with horizontal and vertical sides by drawing figures like the one shown in Figure 4.3(a). Others demonstrated their fixation on the idea that the longer side must be the base by drawing a figure like the one shown in Figure 4.3(b) rather than the correct solution which is shown in Figure 4.3(c).

Another problem used with this class in this series is shown in Figure 4.4. Again the self-imposed restrictions were demonstrated in the incorrect solutions, such as the one shown in Figure 4.5(a), and the comments of the defeated pupils: "I thought you had to draw an oblong." and "I didn't know that was allowed", when presented with a correct solution like the one shown in Figure 4.5(b). However a number of pupils in the trial succeeded with these problems, and the trial indicated that 11 - 12 year old pupils should be able to cope with the mathematical ideas required in this series of problems, particularly if given the areas of some basic squares and rectangles as reference points, but that they might fail because of the self-imposed restrictions listed above. The test was refined and appeared in its final form as Test 1 (Areas) in the major part of the research. (See Appendix 1).

The various problems related to overcoming fixation used with pupils in the development phase of the current research

"These are two sides of a four-sided figure:



Draw the other two sides so that the area of the figure is greater than 4 cm^2."

Figure 4.4. A problem requiring overcoming fixation on right angles.

(a) (b)

Figure 4.5. Two responses to the problem shown in Figure 4.4.

drew attention to the fact that there were essentially two different kinds of fixation involved. The terms 'algorithmic fixation' and 'content universe fixation' were coined to describe them. Sometimes, as in the jugs problem, the mental set established is an algorithm, a process which leads to a solution, and the pupil continues using the algorithm even when inappropriate. This is the so-called Einstellung effect. On other occasions the stumbling-block for the pupil is a self-imposed restriction about the content of the problem, as in the examples about quadrilaterals described above. It was decided therefore to try to develop a battery of assessment items incorporating these two features. Furthermore, in order to reflect the two major areas of the primary mathematics curriculum, number and geometry/measurement, it was decided to aim for tests in both numerical and spatial domains. The set of problems about areas of quadrilaterals which had been developed was clearly in a spatial domain, so the next task was to devise a test involving content universe fixation in a numerical domain. Two tests were devised, and tried with classes of pupils, which incorporated problems which required the use of simple fractions (halves) in their solution, but which might not be solved because of fixation on the use of whole numbers. The first of these was a problem involving the completion of a series of magic squares. The first three of these could be done by trial and error using whole numbers, but the last of them required entries involving halves. The magic square problems (exactly as they appear in Test 20, question 6 in Appendix 1) were given to a top set of 12 - 13 year old pupils in a comprehensive school. Disappointingly not one pupil was able to solve the fourth magic square, although nearly all succeeded with the first three. One or two pupils tried using negative numbers, but none considered the possibility of fractions. Once the suggestion of using fractions was made, however, most of them could solve the problem quickly. This experience suggested that although the arithmetic involved is fairly straightforward the task might be too complex for 11 -12 year olds, but nevertheless the problem was included in the main research as question 6 in the Problems Test no. 20. (See Appendix 1). A test using the same idea of fixation on whole numbers, but in a less complex setting, was devised and found to be more within the compass of 11 - 12 year old pupils. The pupils were given a series of problems of the form, "Find two numbers whose sum is...and whose difference is...". Provided they were reminded of the meanings of the words sum and difference beforehand most 11 - 12 year olds could tackle this type of question successfully. The series of problems would then include one example which required the use of 'halves' - "Find two numbers whose sum is 9 and whose difference is 2". Pupils who were stuck on this one problem would protest vociferously, "But I've tried every possibility - it can't be done!"

<u>Principles for constructing tests</u>. The trials led to the following set of principles which were used for the construction of tests of overcoming fixation to be included in the battery of assessment instruments for the main part of the research:

1. It would be expected that on each test some pupils aged 11 - 12 years would succeed, and some would fail, and the main reason for some failing should be demonstrably the failure to break from a mental set.

2. The mental set might be either an algorithmic fixation or a content universe fixation. The battery of tests should include examples of both.

3. Tests should be devised in both numerical and spatial domains.

4. The mathematical skills and knowledge required in the tests should be well within the grasp of the majority of 11 - 12 year old pupils.

These principles indicate that measurement of the ability to overcome fixation would be based upon success or failure in certain critical items. It should be noted that an alternative approach, particularly for assessing algorithmic fixation, is to measure the ratio of the average time taken to solve a set of problems using one particular algorithm to the time taken to solve a subsequent problem requiring the abandonment of this algorithm. This is an approach which has been used by Krutetskii in a series entitled "Problems on reconstructing an operation". (Krutetskii, 1976, pp 139-141). Unfortunately the test situation envisaged for the main part of the research precluded the use of such tests which can only be used with individuals or possibly small groups of pupils.

<u>Description and Evaluation of Tests Used in the Main Part of the Research Programme for the Assessment of Pupils' Ability to Overcome Fixation in Mathematics</u>

<u>Criteria</u>. Arising from the principles which guided the construction of this battery of tests the following criteria were used in determining, on the basis of the responses obtained from the 280 11 - 12 year old pupils concerned, whether to include the test as a valid assessment of the ability to overcome fixation in mathematics.

1. For a test discriminating between pupils on the basis of their ability to overcome content universe fixation: (a) some pupils should be successful in solving the given problem; (b) some should fail, and the responses of the pupils should provide internal evidence that this failure was due principally to pupils working

within an insufficient and restricted universe.

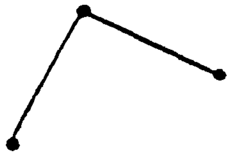
2. For a test discriminating between pupils on the basis of their ability to overcome algorithmic fixation (such a test would contain a number of similar items some designed to establish an algorithm and one or more critical items having a more appropriate solution without the use of the algorithm): (a) the responses of the pupils should indicate that the algorithm had been established by successful completion by most pupils of most of the non-critical items; (b) some pupils should succeed in obtaining the most appropriate solutions to the critical items; (c) the responses of the pupils who fail on the critical items should provide internal evidence that this failure was due to the continued application of the algorithm.

Tests used. The eight tests shown in Table 4.2 were devised and used in the main part of the research, of which five were eventually judged to meet the criteria laid down above for valid tests of the ability to overcome fixation in mathematics. The tests are reproduced in Appendix 1.

Test 1: Areas. Prior to the test the pupils revised the calculation of areas of relevant rectangles and a right angled triangle. The four questions that followed were constructed to assess pupils' ability to overcome fixation on a particular content universe in a spatial domain. They are shown in Figure 4.6.

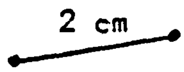
Parts 1 and 2 required overcoming fixation on horizontal and vertical sides for a rectangle. Part 2 also required overcoming fixation on the 'base' of a rectangle being the longer side. Part 3 required overcoming fixation on right angles and part 4 on convex quadrilaterals. The examples used in the revision prior to the test were, of course, chosen to confirm such fixations. Some of the typical incorrect responses are shown in Figure 4.7. Like

<u>Part 1</u>: These are two sides of a four sided figure.



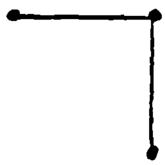
Draw the other two sides so that the area of the figure is 4 cm^2.

<u>Part 2</u>: This is one side of a four sided figure.



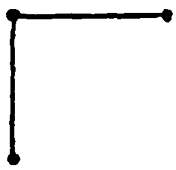
Draw the other three sides so that the area of the figure is 8 cm^2.

<u>Part 3</u>: These are two sides of a four sided figure.



Draw the other two sides so that the area of the figure is more than 4 cm^2.

<u>Part 4</u>: These are two sides of a four sided figure.



Draw the other sides so that the area of the figure is less than 2 cm^2.

Figure 4.6. Questions used in Test 1 (Areas).

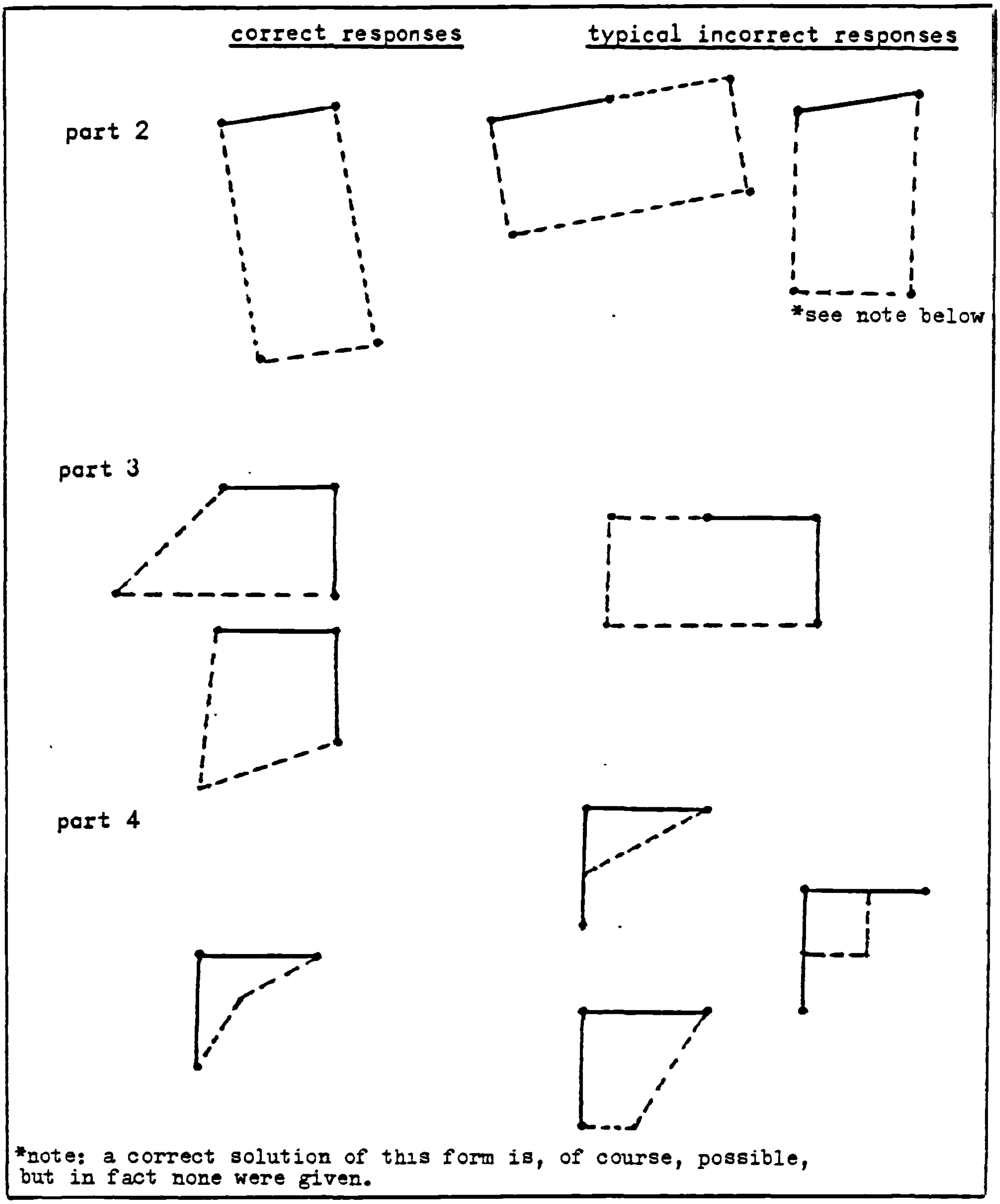


Figure 4.7. Examples of responses to Test 1 (Areas).

those obtained in the field testing of these problems they clearly reflect these fixations.

Table 4.2

Tests of Overcoming Fixation Used in the Main Part of the Research

Name	Number	Domain
	Content universe fixation	
Areas	1	spatial
Sum and difference	14	numerical
Magic square	20 (qu. 6)	numerical
	Algorithmic fixation	
Jugs	10	numerical
Cuts	24	spatial
	Not used in final analysis (all algorithmic)	
Multiplication	8	numerical
Fraction	9	numerical
Double and add	15	numerical

A total of 257 pupils took this test. The four parts to the question produced a good spread of results with 9.7% of the pupils able to manage only the first part, and 24.5% all four parts successfully. The order of the questions correctly anticipated the order of difficulty, as shown in Table 4.3. The range of scores was therefore from 0 to 10. Table 4.4 shows the means and standard deviations of scores for this test.

Test 14: Sum and difference. Prior to the test the pupils revised the meanings of the word sum and difference with a simple example. The ten questions in the test required them to find two numbers with a given sum and a given difference. The first nine questions could be solved using trial and error methods with positive integers. The solution of the last question required overcoming fixation on this particular numerical domain, by using simple fractions.

Table 4.3

Performance of Pupils on the Four Questions in Test 1

Part	Number (%age) of correct solutions	Marks awarded
1	255 (99.2%)	2
2	221 (86.0%)	2
3	129 (50.2%)	3
4	86 (33.5%)	3

Table 4.4

Summary of Scores on Test 1 (Areas)

	Number	Mean	Standard deviation
Boys	140	6.40	2.77
Girls	117	5.97	2.61
All	257	6.17	2.72

Each question was of the form: "Find two numbers whose sum is x and whose difference is y". In the critical question (x), the numbers x and y were 9 and 2, with solution $5\frac{1}{2}$, $3\frac{1}{2}$. A total of 157 pupils out of the 240 taking the test got enough questions correct (six or more) to indicate that they had sufficient understanding and a successful strategy for solving these questions.

The performance of these 157 pupils on the ten questions in the test was as shown in Table 4.5.

Table 4.5

Performance of 157 Pupils on the Ten Questions in Test 14

Question number	Numbers used in question x	y	Correct solution	Number of pupils making an error	%age of pupils making an error
i)	10	4	7,3	1	0.6%
ii)	12	8	10,2	15	9.6%
iii)	23	1	12,11	21	13.4%
iv)	19	17	18,1	28	17.8%
v)	25	15	20,5	38	24.2%
vi)	16	0	8,8	16	10.2%
vii)	14	10	12,2	32	20.4%
viii)	10	10	10,0	13	8.3%
ix)	22	14	18,4	43	27.4%
x)	9	2	$5\frac{1}{2},3\frac{1}{2}$	66	42.0%

Table 4.5 indicates that apart from question (x) the most difficult questions tended to be those involving larger numbers, such as (ix), (v), (vii) and (iv). In fact for the first nine questions the coefficient of correlation between the sum of x and y and the percentage of these 157 pupils making an error for each question is as high as 0.87. On this basis the expected percentage of errors for question (x) would be as low as about 3%. However this did prove to be the most difficult question with 42% of these pupils failing. This analysis suggests that there is some justification in the assertion that there was a difficulty in overcoming fixation on whole numbers in part (x) of the test.
It is interesting to note that there was no particular difficulty

in the two questions involving zero, namely parts (vi) and (viii).

The overall performance of all 240 pupils on the test was as given in Table 4.6.

Table 4.6

Overall Performance of 240 Pupils on Test 14

Category	Description	Score	Frequency	%age
A	at least 6 correct, (x) solved successfully	10	92	38.3%
B	at least 6 correct, (x) not solved correctly	0	65	27.1%
C	less than 6 correct	0	83	34.6%

The pupils in category A (38.3%) in Table 4.6 scored 10 points for successfully overcoming the fixation on whole numbers in part (x). The pupils in category B (27.1%) demonstrated the anticipated behaviour of the fixation construct. The number in this category was disappointingly small, mainly because so many pupils (34.6%) failed to demonstrate sufficient competence in the first nine questions to be in a position to tackle question (x) from the basis of a mental set. Of these 83 pupils in category C, 25 pupils showed a tendency just to give the sum or difference as one solution and one of the given numbers as the other (e.g. for $x = 14$ and $y = 10$ they would give the solution 10,4), 35 pupils just gave the sum and difference of the two given numbers in each case, and the answers of the remaining 23 pupils just could not be categorised. Table 4.7 summarises the results of this test.

Test 20 qu. 6: Magic squares. This test was designed to assess the same fixation as Test 14 (Sum and Difference), but in a more complex setting. The test proved to be more successful in producing the anticipated behaviour of the fixation construct, and only a small number of pupils succeeded in overcoming the fixation.

Table 4.7

Summary of Scores on Test 14 (Sum and Difference)

	Total	Category A	B	C	Mean score
Boys	136	56	34	46	4.1
Girls	104	36	31	37	3.5
All	240	92	65	83	3.8

In the test the pupils were first given an example of a magic square. This is an array of nine numbers in which the rows, columns and diagonals all sum to the same answer. They were then required to complete the magic squares shown in Figure 4.8.

In each case the sum of the rows, columns and diagonals is 9. The first three examples can be completed by trial and error strategies using whole numbers, but the fourth square requires a break from this mental set and the use of fractions to obtain the solutions shown in Figure 4.9.

The majority of the 253 pupils who took this test were able to develop a successful strategy for the first three parts. In fact, 127 (50.2%) of them managed to complete all three successfully, and a further 87 (34.4%) solved two out of three (this will be counted as 'successful' in this context). However, of these 214 pupils only 21 solved the critical fourth question. There was also one pupil who tried using halves but was finally unable to obtain the correct solution. It appears then that in the context of these magic squares the fixation on the domain of

(i)

	3	
1	3	5
4		

(ii)

	2	
4	3	2

(iii)

	2	
0	3	6

(iv)

	6	
1	3	5

Figure 4.8 Four magic squares to be completed in Test 20 (qu. 6).

$2\frac{1}{2}$	6	$\frac{1}{2}$
1	3	5
$5\frac{1}{2}$	0	$3\frac{1}{2}$

Figure 4.9. Solution to final part of Test 20 (qu. 6).

integers was very strong: a total of 193 (76.3%) of the pupils demonstrated the anticipated behaviour of the fixation construct, namely developing a successful strategy for parts (i) - (iii), but failing in part (iv). There was just one pupil who succeeded in part (iv) but made an error elsewhere. The full analysis of the results of this test are given in Table 4.8.

Table 4.8

Distribution of Various Categories of Responses to Test 20 (qu.6)

Category	Description	Score	Frequency	%age
A	No correct solutions	0	5	2.0%
B	Just one of (i)-(iii) correct, failed in (iv)	0	33	13.0%
C	Just two of (i)-(iii) correct, failed in (iv)	0	86	34.0%
D	All of (i)-(iii) correct, failed in (iv)	0	107	42.3%
E	All four solved correctly	10	20	7.9%
F	(iv) correct, error elsewhere	10	1	0.4%
G	(iv) incorrect, but attempted to use fractions	5	1	0.4%

In Table 4.9 "success" refers to categories E and F in Table 4.8, "partial success" to category G, and "failure" to categories A, B, C and D.

Test 10 Jugs. This test was designed to establish an algorithmic fixation in a numerical domain and then to assess the pupil's ability to overcome this fixation when appropriate. The administrator of the test gave the pupils careful instruction about the problems and worked through an example with them. They were then given six problems in which they were required to find the best way of measuring out a given quantity of water using three jugs of specified capacities. The six questions used are shown in Table 4.10.

Table 4.9

Summary of Scores on Test 20 qu. 6 (Magic Squares)

	Success (score 10)	Partial success (score 5)	Failure (score 0)	Mean score
Boys	11	1	130	0.8
Girls	10	0	101	0.9
All	21	1	231	0.85

Table 4.10

Six Questions Used in Test 10

Question	Measure out	Jugs A holds	Jug B holds	Jug C holds
1	52	10	64	1
2	14	100	124	5
3	3	10	17	2
4	100	21	127	3
5	20	23	49	3
6	5	50	65	5

This test is a modification of the classic test designed by Luchin (1942) and discussed earlier. Each of the examples can be solved by using the algorithm represented by B - A - 2C. To assist in the establishment of this algorithm the administrator of the test gave the pupils the solution to example 1 with an explanation after one minute. A number of pupils, presumably because of their inadequate numerical skills, were unable to achieve this precise solution in each example, but nevertheless demonstrated a clear consistent approach which consisted essentially of the algorithm: fill up the biggest jug (B) first and then pour off into the smaller jugs. The interest lies in questions 5 and 6. Although these can be solved by the same process, B - A - 2C, question 5 is best done by breaking from this mental set and using the simple process A - C. Question 6 has the trivial solution, C.

A large percentage of pupils (55.6%) gave the classical response essentially using the algorithm B - A - 2C consistently in all six questions. A further 15.2% demonstrated the less precise algorithmic fixation of always filling up the biggest jug first. The fact that as many as 70.8% of the pupils taking this test continued to use their inappropriate algorithms in question 5 and 6 supports the assertion that the 10.8% who found the alternative solutions in these last two questions were demonstrating the ability to overcome algorithmic fixation. One surprising outcome was that a further 9.6% were successful in question 5 but then reverted to their original algorithm for question 6!

The full analysis of the pupils' responses to this item is given in Table 4.11.

In Table 4.12 "success" refers to category A in Table 4.11 "partial success" to categories B and C, and "failure" to categories D, E and F.

This test illustrates well an underlying conflict between the notion of overcoming mental sets and mathematical problem-solving. It could be argued that, having found a method which works in every case, it is actually most efficient to continue using it blindly until it fails. This is typical mathematical behaviour for pupils in mathematics lessons: learn and master an algorithm - recognise a problem as requiring that algorithm - apply the algorithm to obtain the solution. To actively seek an alternative, more elegant method of solution is possibly a behaviour which conflicts with the pupil's previous experience of what constitutes successful behaviour in mathematics lessons, and clearly may involve some element of risk-taking.

Table 4.11

Distribution of Various Categories of Responses to Test 10

Category	Description	Score	Frequency	%age
A	Successful breaking of mental set in 5 and 6	10	27	10.8%
B	Successful breaking of mental set in 6, not 5	6	2	0.8%
C	Success in 5 but reverted in 6	6	24	9.6%
D	Essentially used B-A-2C in all six questions	0	139	55.6%
E	Used process of filling jug B first throughout	0	38	15.2%
F	Not able to cope with these problems at all	0	20	8.0%

Table 4.12

Summary of Scores on Test 10 (Jugs)

	Success (score 10)	Partial success (score 6)	Failure (score 0)	Mean score
Boys	21	15	100	2.2
Girls	6	11	97	1.1
All	27	26	197	1.7

<u>Test 24: Cuts</u>. This test was designed on similar lines to Test 10, but in a spatial domain: first an algorithm was established and then the pupil's ability to break from it when appropriate was assessed.

The pupils were given a rectangle, as shown in Figure 4.10(a), and asked to find out how many lines would be needed to cut the rectangle into a specified number of equal parts. For example, to cut it into two equal parts, one line is required.

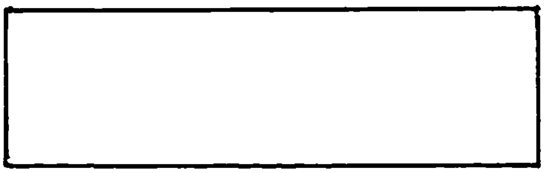
The first three questions given to the pupils established the following pattern, using the same process of vertical lines:

3 parts	2 lines needed
5 parts	4 lines needed
7 parts	6 lines needed.

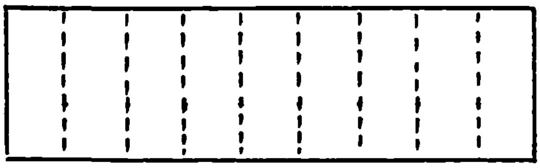
The final question asked for nine parts. Using the above pattern, that is, by generalizing - a behaviour to be encouraged and commended in most mathematical problem-solving - the pupils were 'expected' to arrive at the solution "8 lines". In fact 234 of the 245 pupils taking the test demonstrated the anticipated behaviour of the fixation construct with the solution shown in Figure 4.10(b).

Just two of the pupils managed to break from the process established in the first three parts of the test and gave the solution using just four lines which is shown in Figure 4.10(c). The remaining nine pupils were unable to solve these problems at all. Table 4.13 gives a summary of the results on this test.

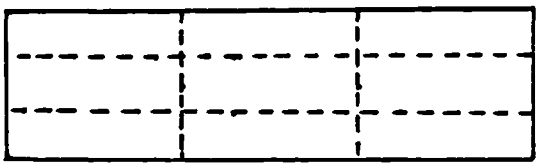
Note: it could be suggested that the pupils were given insufficient encouragement to seek the non-algorithmic solution in the final part of this test. For example, the idea of using the least possible number of lines could have been emphasised more strongly in the introduction. Also the addition of another



(a) The rectangle to be cut into a specified number of parts.



(b) An algorithmic solution for nine parts.



(c) A solution for nine parts using only four lines.

Figure 4.10. Diagram related to Test 26 (Cuts)

part at the end, to cut the rectangle into say four or six parts may have given them more chance of breaking from the mental set of vertical cuts. A fine balance needs to be maintained in items in this category between establishing an algorithm on the one hand, and giving on the other hand sufficient indication that breaking from an established algorithm may be a desirable solution.

Table 4.13

Summary of Scores on Test 24 (Cuts)

	Success in over-coming fixation (score 10)	Failure in over-coming fixation or unable to do the questions (score 0)	Mean score
Boys	2	132	0.15
Girls	0	111	0
All	2	243	0.08

Tests Related to Overcoming Algorithmic Fixation not Used in Final Analysis

Test 8: (Multiplication) and Test 9 (Fractions). In these two tests pupils were presented with series of five long multiplication and addition of fractions questions as shown in Table 4.14. The fourth question in each case was a particularly simple question which would not require the application of the algorithm. It was agreed by the pupils' teachers that most of the children would be capable of writing down the answers to 20 x 10 and $\frac{1}{2} + \frac{1}{4}$ without any working. The interest of the test lay in whether in this context they would do that or whether they would continue to use the full algorithm. The results of the tests are shown in Table 4.15.

Table 4.14

Questions Used in Tests 8 and 9

Test 8	Test 9
21 x 12	2/5 + 1/3
32 x 22	1/4 + 2/3
34 x 23	1/3 + 2/7
20 x 10	1/2 + 1/4
16 x 21	1/3 - 1/8

Table 4.15

Distribution of Various Categories of Responses to Critical Questions in Tests 8 and 9

Category	%age of Pupils (out of 248)	
	Test 8	Test 9
Wrote answer straight down, no working	18.5%	10.8%
Used the algorithm to some extent	79.9%	69.5%
Could not do these questions	1.6%	19.7%

Although these results are interesting, it was decided to reject these as valid tests of overcoming fixation. Unlike the other tests used, the questions were familiar exercises to the pupils and it was the opinion of several teachers that their own previous instructions about "showing all the working", together with the "space for working" provided on the test-paper, would lead the children to justifiable setting down of unnecessary working in the critical question.

Test 15: Double and add. In this test a machine was described into which could be fed pairs of numbers. The output is always double the first number, plus the second. For example, (4, 3)-----> 11. The pupils were required to find the missing numbers in the following examples: (, 4)-----> 10, (, 7)-----> 9, (, 12) -----> 16, (, 13) -----> 23, (, 8) -----> 30, (3,)-----> 15, (8,)-----> 16.

The rationale of the test was to establish a mental set in the first five parts (e.g. a subtract and halve process) which might then need to be broken from in order to solve the last two questions. It was expected therefore that a significant percentage of pupils would get the first five questions correct and the last two incorrect. In fact only 9 (3.8%) of the 240 pupils taking this test got the last two questions wrong, with a further 9 (3.8%)making one error rather than two. The majority, 216 pupils in fact (90.8%), were successful in these last two questions.

The failure to establish a fixation through this test is easily explained. Discussion with one class of children about how they solved these questions revealed a trial and error strategy rather than an algorithmic approach. Such a strategy is still appropriate in the last two questions so there is no mental set to be broken.

Summary of Results for Assessment of Pupils' Ability to Overcome Fixation in Mathematics

Five items have now been considered as possibly valid tests of the ability to overcome fixation. Three of these (Tests 1, 14 and 20 qu. 6) were concerned with content universe fixation, and two (Tests 10 and 24) with algorithmic fixation. Three have been in a numerical domain (Test 14, 20 qu. 6 and 10) and two in a spatial domain (Test 1 and 24). The five items, each scored out of a maximum score of 10, show a range and order of difficulty as indicated in Table 4.16.

It is intended in the final analysis of creative behaviour in mathematics to combine the pupils' scores on these five items into an aggregate score for "overcoming fixation".

Table 4.16

Means for Five Tests of Overcoming Fixation

(Maximum score, 10 in each case)

Test	Mean score
1 (Areas)	6.2
14 (Sum and Difference)	3.8
10 (Jugs)	1.7
20 qu. 6 (Magic Squares)	0.85
24 (Cuts)	0.08

Assessment of Pupils' Ability for Divergent Production in Mathematics

Background

The second major aspect of creativity in mathematics to be investigated was the ability for divergent production. The tests devised in this section of the research were designed to encourage divergent thinking, so that situations were provided in which not just one but many solutions or responses are possible. The tests were modelled on the traditional divergent production tests developed by such as Guilford and Torrance, but in a mathematical context. As before both numerical and spatial domains would be used.

Divergent production is one of five categories of operations in the model of the intellect put forward by Guilford (1959). These operations are cognition, memory, convergent production, divergent production and evaluation. The essential difference between convergent and divergent production is that the first is concerned with seeking logical necessities whereas the second is concerned with seeking logical possibilities. Creative thinking will often involve both these aspects, but much research into creativity has concentrated on tests using a divergent production paradigm. The essence of such tests must always be that the situation presented will allow many possible, different and acceptable responses. The creativity of the respondents will then be indicated by such parameters as fluency (the number of appropriate responses), flexibility (the number of different categories of responses) and originality (the relative infrequency of the responses). Such situations as these, encouraging divergent production are not traditionally associated with the assessment of mathematical ability, where the conventional test poses a problem with one and only one right answer upon which the student's thinking must converge. However, as has been seen in Chapter 2, some researchers have used divergent production tests with a mathematical content, such as Prouse (1967), Bishop (1968), Foster (1970), Balka (1974) and Dunn (1976). The present author had also devised two such tests (Haylock, 1978), and used them in an investigation with 14 - 15 year old pupils. This work suggested that performance in divergent production tasks in mathematics was not related to performance in similar tasks with a non-mathematical content for pupils of roughly the same level of mathematical skills and knowledge.

Development of Instruments

For the purposes of the present research the task was to devise a battery of tests of divergent production in mathematics suitable for use with 11 - 12 year olds, to analyse their responses and to judge whether they indicated that the tests were assessing an aspect of mathematical ability which could be recognised as being worthy of association with the term 'mathematical creativity'. As with the development of the tests associated with overcoming fixation, the approach which was used was first to examine previous work in this area, then to try out a number of possibilities informally with classes of pupils, to clarify the principles upon which such tests would be constructed, and finally to devise and administer a battery of such tests within the main testing programme with the large sample of 11 - 12 year olds. Criteria would be specified against which the tests would be evaluated on the basis of the pupils' responses.

Examination of attempts to assess divergent production in a mathematical context suggested three different styles of question which might be particularly productive in terms of bringing to bear a wide range of mathematical ideas in pursuit of logical possibilities. These are referred to in this research as problem-solving, problem-posing and redefinition, although these labels should be seen as essentially catalysts for the construction of test items, rather than as categories for classification of those items.

Problem-solving. Bishop (1968) describes and advocates for the assessment of some important aspects of mathematical ability the use of such problem-solving items as: "$(p + q)(r + s) = 36$; what possible values could p, q, r, s have to make this true?"

This item captures the essence of the problem-solving style of divergent production test being sought in the present research. A problem is posed for which there are many possible acceptable solutions. To produce many different solutions the pupils must think widely and in an unrestricted way within the constraints of the stated problem. This item was tried in the form given above with a group of first year students in a College of Education, during the development stage of the present research. Several self-imposed restrictions which limited the number of possible solutions provided by the students were demonstrated. These included limiting p, q, r, s to natural numbers, excluding zero, excluding fractions and decimals, excluding the possibility that (p + q) or (r + s) might be less than 1, using only positive numbers, and so on. All these ideas were well within the students' grasp, as demonstrated by their ability to produce such answers when the appropriate suggestion was made. This informal experience suggested that this might be a useful test item for the main part of the research, but for 11 - 12 year olds it would need to be presented in a form which did not assume familiarity with algebraic notation. Also the trial showed that marking of the responses would be extremely difficult because there were perhaps too many possibilities. Consequently the task was modified for use with 11 - 12 year olds to: "Three cards that I will call A, B and C have numbers written on one side of them. If you add the number on card A to the number on card B and multiply the answer by the number on card C you get 9.

$$(\boxed{A} + \boxed{B}) \times (\boxed{C}) = 9$$

What do you think the numbers on the cards A, B, C might be? List as many different possibilities as you can think of."

It appears in this form as Test 18 (3 Cards) in Appendix 1.

The need for obtaining the right balance between setting a task which, on the one hand, is sufficiently open so that divergent production of many possible responses can be achieved, and, on the hand, has sufficient inbuilt constraints to make it possible to discriminate between pupils in terms of the originality and mathematical appropriateness of their responses, began to emerge as an important principle in the development phase of the research.

For example, the following was tried with a class of 12 - 13 year olds: "Find as many different shapes as you can by joining up dots on a nine-dot centimetre square grid, and work out their areas."

It was found that because there were so many possible solutions to this problem there was nothing mathematically significant about many of the less frequently found shapes. Consequently the task was changed to: "By joining up dots on a nine-dot centimetre square grid, with straight lines, draw as many different shapes as you can with an area of 2 cm^2."

The inclusion of the extra constraint made this a much more successful item for discriminating between pupils and, with an appropriate introduction, it was used in this form as Test 7 (Nine-dot Areas) in the main part of the research. (See appendix 1).

Problem-posing. A number of researchers into divergent production have made use of some form of Make-Up-Problems Test, for example, Getzels and Jackson (1962), in their study of highly creative and highly intelligent students. Also, Krutetskii, in identifying the component of mathematical ability in schoolchildren which he refers to as "the ability for formalised perception of mathematical material", used a number of problems with an unstated question. Mathematically capable pupils were found to formulate the appropriate question immediately, being able to grasp the relationships between the constituent parts of the problem at once. This notion of problem-posing then seemed to be a potentially fruitful stimulus for the construction of items for the assessment of divergent production in mathematics. It seemed likely that situations of this sort might produce a variety of responses of mathematical interest and significance. The principle which guided the development of the tests in this category was that a situation must be presented in which a number of appropriate mathematical questions, using a range of mathematical ideas within the grasp of the pupils, could be posed.

Such a situation was presented by the investigator to a class of 12 - 13 year old pupils in the development stage of this research. Information about the number of boys and girls in their families was collected by the class, the data were organised and then displayed on a scattergram, as shown in Figure 4.11, each pupil providing one cross in one of the squares. The next step in a conventional teaching situation might be for the teacher to pose various questions which could be answered from the graph by the pupils, to test their ability to interpret the diagram. On this occasion, however, the pupils themselves were challenged to write down as many different questions as they could think of which could be answered from their graph. They were encouraged to aim for interesting and unusual ideas. The scattergram contained sufficient components for these to be related together in many different ways. Questions were posed which required analysis of individual squares on the graph, comparison between squares, consideration of rows, columns, diagonal and regions, and so on. There were, of course, many obvious questions, like, "How many families had so many boys and so many girls?", which were used by most pupils and by some pupils exclusively. But the less frequently used and original questions, such as, "How many families had less than

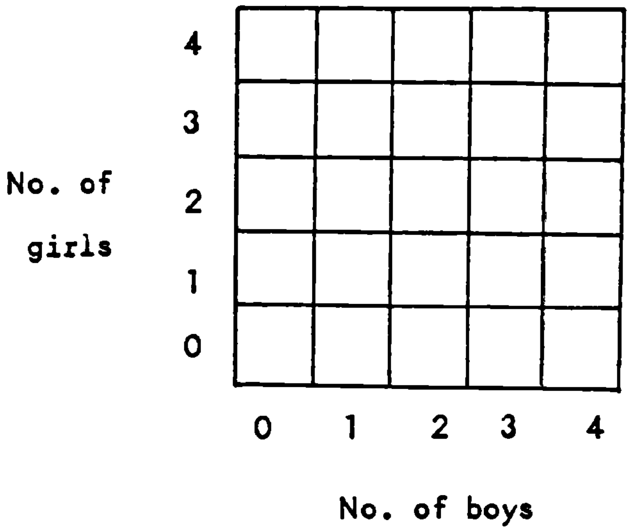


Figure 4.11. Scattergram for a divergent production problem-posing task.

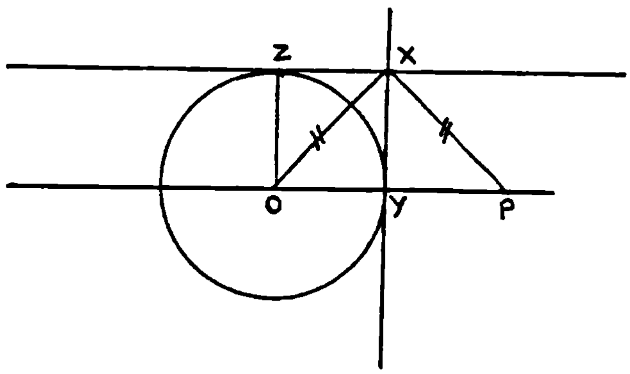


Figure 4.12 Diagram for a divergent production redefinition task.

three children?" and, "How many families had more than two boys and less than three girls?" showed divergent thinking which could be recognised as involving the manipulation of significant mathematical ideas. The main difficulty experienced in assessing the pupils' responses to this task was in interpreting some of their questions. Consequently, to assist in this, when this item was used as Test 12 (Scattergram) in the main part of the research, it was decided to ask the pupils to answer their own questions.

Redefinition. The term 'redefinition' was coined by Guilford, in discussing his model of the intellect, to describe the ability to give up old interpretations of familiar objects in order to use them in some new ways. Often, for the student of mathematics, however the problem is that the component parts of a given situation have to be continually reinterpreted. For example, in solving a geometric problem a particular line segment may have to be interpreted sometimes as the side of a triangle, sometimes as a radius of a circle, sometimes as half the diameter, and so on. In previous research mentioned earlier with 14 - 15 year olds (Haylock, 1978), the pupils were asked to write down as many different statements as they could about the line XY in the diagram shown in Figure 4.12. As many as 60 different statements were produced, involving a wide range of mathematical ideas, as the pupils continually reinterpreted the role of the given segment.

A number of general divergent production tests contain this feature, that the student must continually reinterpret the elements in the given situation. Wallach and Kogan (1965) used an instrument called 'Similarities' in which children were asked questions like, "Tell me all the ways in which a potato and a carrot are alike". To produce many and varied responses to such items the child must continually reinterpret 'a potato' and ' a carrot' in

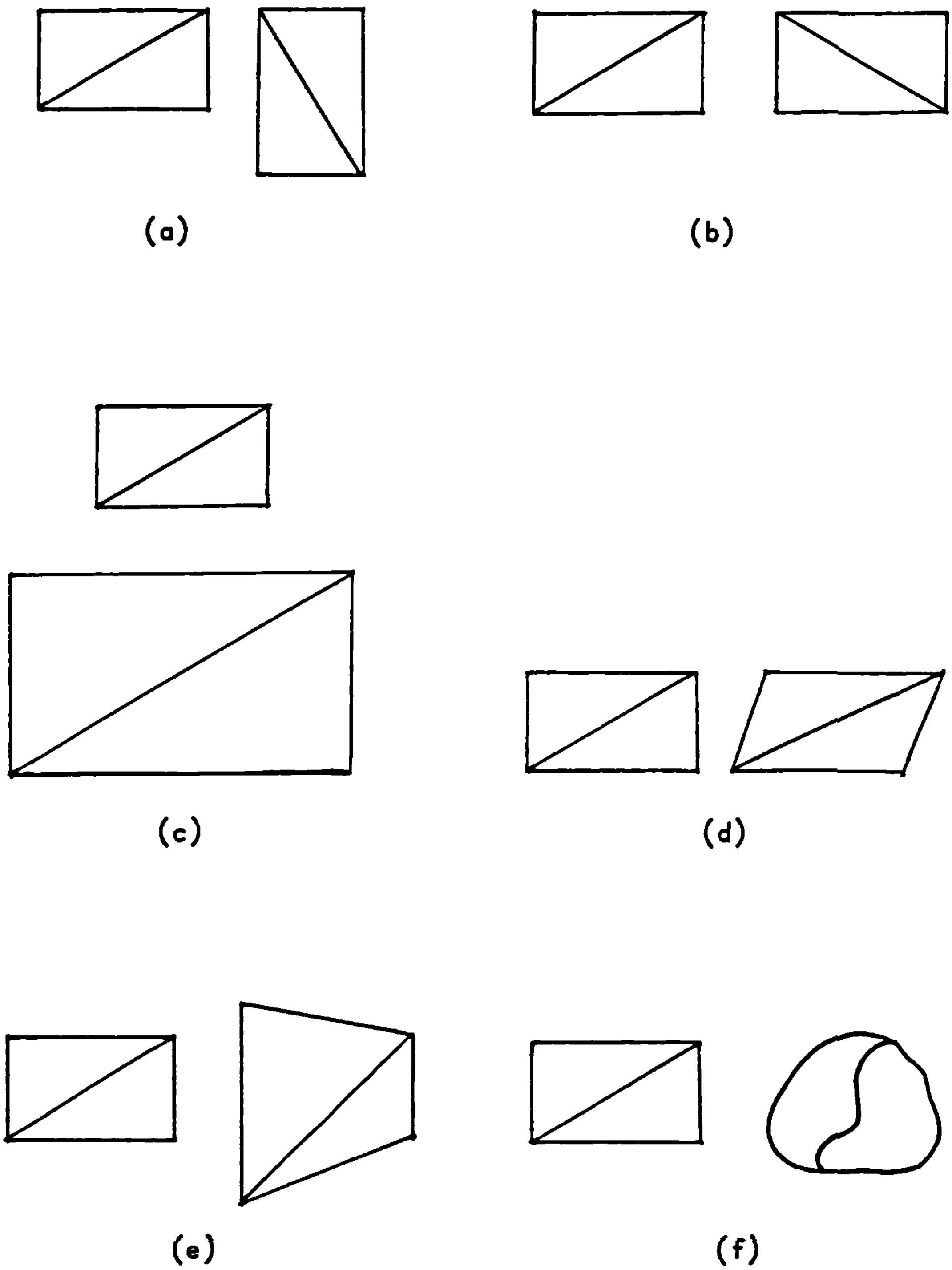


Figure 4.13. Sequence of shapes used for discussion of similarities and differences.

terms of their attributes and functions. This suggested the development of a similar test in a mathematical context. Making statements about the similarities between two given numbers or two given geometric figures would involve essential mathematical thinking ranging over all the possible attributes and functions of the numbers or shapes, thereby giving opportunity for worth-while divergent production. The investigator discussed with a number of classes of students and schoolchildren the sequence of pairs of shapes, shown in Figure 4.13 in terms of their similarities and differences. It was found that most students would immediately recognise what was different about the second shape in each case, but to make statements about the ways in which the two shapes were the same proved more difficult. Pair (d) was the most productive source of ideas, with reference to such attributes as angles, sides, parallelism, diagonals, bisectors, height, width and so on, and this became the basis for an assessment item in the major part of the research. It appears as Test 5b (Similarities: shapes) along with a parallel item, Test 5a (Similarities: numbers) in Appendix 1.

From the experience provided by these trials, a battery of tests of divergent production based upon the principles of problem-solving, problem-posing and redefinition, covering both numerical and spatial domains, were devised for use within the main part of the research programme for use with 11 - 12 year olds. The type of assessment instrument which was sought in this development would allow an 11 - 12 year old pupil to bring to bear a wide range of mathematical ideas upon a given situation, and would discriminate between pupils on the basis of their ability to use their mathematical background in many different and varied, original ways. The tasks set should be such that all pupils should be able to make some appropriate responses, but there would need to be sufficient mathematical constraints within the tasks so that original responses will be non-trivial and have some degree of face validity for associating them with the notion of mathematical creativity.

Description and Evaluation of Tests Used in the Main Part of the Research Programme for the Assessment of Pupils' Ability for Divergent Production in Mathematics

Criteria. Arising from the principles which emerged in the development of these tests the criteria listed in Table 4.17 were used, on the basis of the responses of the 280 11 - 12 year old pupils concerned, to judge whether or not to include a particular test in the final analysis as a valid assessment of divergent production in mathematics.

Tests used. Fourteen tests of divergent production were devised and used in the main part of the research and they are reproduced along with other tests in the programme in Appendix 1. The tests based upon the notion of divergent production are listed in Table 4.18.

By the criteria, laid down in Table 4.17 ten instruments were included in the final analysis. Three of these were essentially problem-solving situations, in which a problem is posed which has many possible solutions. Three further items were essentially problem-posing tests, in which a situation is presented and the pupil is invited to pose as many and as varied as possible mathematical problems related to the given situation. The remaining four items involved some element of redefinition, that is reinterpreting in a number of different ways the mathematical components or relationships in a given situation.

Table 4.17

Criteria for Tests of Divergent Production

1. The pupils' responses should use a range of mathematical ideas.

2. At least 20 appropriate responses are possible for these pupils.

3. The pupils' responses should show a fairly consistent interpretation of the instructions in the test.

4. There should be several obvious responses, so that most pupils would obtain these: such responses would be zero-rated for originality.

5. There should be a number of appropriate responses which are obtained by relatively fewer ($<10\%, <5\%, <2\%$) pupils, so that credit for originality can be given accordingly.

6. These original responses should have some degree of face validity for indicating mathematical creativity: in other words, the original responses should not be mathematically trivial.

Scoring tests of divergent production. Tests of divergent production have conventionally been scored for fluency, flexibility and originality. Fluency is the total number of acceptable responses, flexibility the number of different categories of responses, and originality marks are awarded for responses which are relatively infrequent for the peer group. Several authors have pointed out inbuilt difficulties in this approach to scoring these tests. Clark and Mirels (1970) showed that flexibility and originality scores are inevitably dependent upon fluency scores. Torrance (1966) admits correlations of about 0.8 between fluency and each of the other two variables in relation to his tests of divergent thinking.

Table 4.18

Divergent Production Tests Used in the Main

Part of the Research

Name	Number	Domain
	Problem-solving	
Nine-dot areas	7	spatial
Results	16	numerical
Three cards	18	numerical
	Problem-posing	
Counters	2	numerical
Cross-number	11	numerical
Scattergram	12	spatial
	Redefinition	
Subsets	3	numerical
Shape-finding	4	spatial
Similarities (numbers)	5a	numerical
Similarities (shapes)	5b	spatial
	Items not used in final analysis	
Nine-dot routes (problem-solving)	6	spatial
Classroom (problem-posing)	13	num/sp
What can you see? (redefinition)	17	spatial
Factory (problem-posing)	19	numerical

Such results are confirmed by the work of Nuttall (1971) who also found correlations in excess of 0.7 between fluency, flexibility and originality in a number of tests of divergent thinking, including some in mathematical contexts. Seddon (1982) has argued the need for a composite measure of divergent thinking - rather as momentum is a composite of mass and velocity - and has attempted to find an acceptable way of combining fluency, flexibility and originality into a single parameter.

With this background it was decided not to score pupils' performances on the divergent production tests administered in the present research for the three separate parameters, fluency, flexibility and originality, in the conventional way. For the statistical analysis envisaged it would be more appropriate to use a single score for each pupil on each test to indicate how well they had done in it. Another factor in this decision is that there is a particular problem in considering fluency in items with a numerical domain. Vast numbers of responses can be obtained in open-ended questions simply by varying the numbers used or by repeating the same principle over and over again. It was found that pupils would often respond in this way, in spite of encouragements to produce different and varied answers. For example, in Test 16 (Results) pupils were required to deduce as many different results as possible which can be obtained easily from the given result , $23 \times 35 = 805$. One pupil gave the following set of responses: $23 \times 350 = 8050$, $23 \times 3500 = 80500$, $2300 \times 350 = 805000$, $23 \times 35000 = 805000$, $230 \times 35 = 8050$, $2300 \times 35 = 80500$, $23000 \times 35 = 8050000$, etc, etc..., a total of 25 similar results, ending with $23 \times 35000000000000000 = 805000000000000000$!

Clearly there would be no justification in awarding this pupils 25 marks for fluency - or, at least, such a score would not indicate anything of significance in terms of mathematical ability. It is necessary therefore in some tests to reward the number of mathematical <u>ideas</u> used rather than the raw number of responses. This is inevitably blurring the distinction between fluency and flexibility. In the example quoted it would seem that perhaps at most four ideas have been used: namely, the idea of adding a zero to the first number, the idea of adding it to the second number, the idea of adding zeros to both numbers, and the idea that this process of adding zeros can be continued for ever.

So the general policy arrived at for marking these divergent production tests was to produce a single score for each pupil's performance on any given test, this score to represent the number of distinct mathematical responses or ideas given, plus extra credit for original responses, determined by their relative infrequency. Originality as measured by statistical infrequency is a well established criterion of a creative product. Another important criterion which must be taken into account is the appropriateness of any given response. Jackson and Messick (1965), for example, point out that to be appropriate a product must fit into its context and make sense in terms of the limitations or demands of the situation. In a mathematical context it is often a straightforward matter to identify inappropriate responses, particularly when they are simply incorrect, for example, the response "$23 \div 805 = 35$" in Test 16 (Results). However the decision to reject a response as inappropriate in the present research was not always straightforward, and inevitably a degree of subjectivity must be allowed.

The method of scoring that was developed and adopted then was as follows:

1. Delete all inappropriate or incorrect responses.
2. Delete all superfluous responses which are merely trivial

repetitions of the same mathematical idea, e.g. those obtained simply by varying the numbers used.

3. Award one mark for each remaining response or mathematical idea - the choice between these alternatives to be determined by the context.

4. Award bonuses for original responses: three marks for responses/ideas used by less than 2% of the pupils, two marks for those used by less than 5%, and one mark for those used by less than 10%.

Test 7: Nine-dot areas. This was a problem-solving task in a spatial domain with many possible solutions. The pupils were required to find as many different shapes as possible with an area of 2 cm^2 formed by joining dots on a nine-dot centimetre grid with straight lines. The 248 pupils taking this test produced a total of 23 responses between them as shown in Figure 4.14, meeting the first two of the criteria stated in Table 4.17 for a valid test of divergent production, i.e. using a range of mathematical ideas and producing at least 20 appropriate responses. There were a number of obvious responses, such as numbers 1 - 6 in Figure 4.14, which were obtained by more than 50% of the pupils and a number of original responses, such as numbers 11 - 23, obtained by relatively fewer pupils. It is clear that the more original responses are by no means mathematically trivial. Apart from response 9 it can be seen that the non-original responses are those which can be obtained by the combination of unit squares and triangular half-units. The more infrequent responses incorporate a diagonal line joining non-adjacent dots. Another possible deduction from these pupils' responses is that those responses containing an exterior acute angle tend to be more elusive. These remarks indicate that criteria 4, 5 and 6 for a valid test of

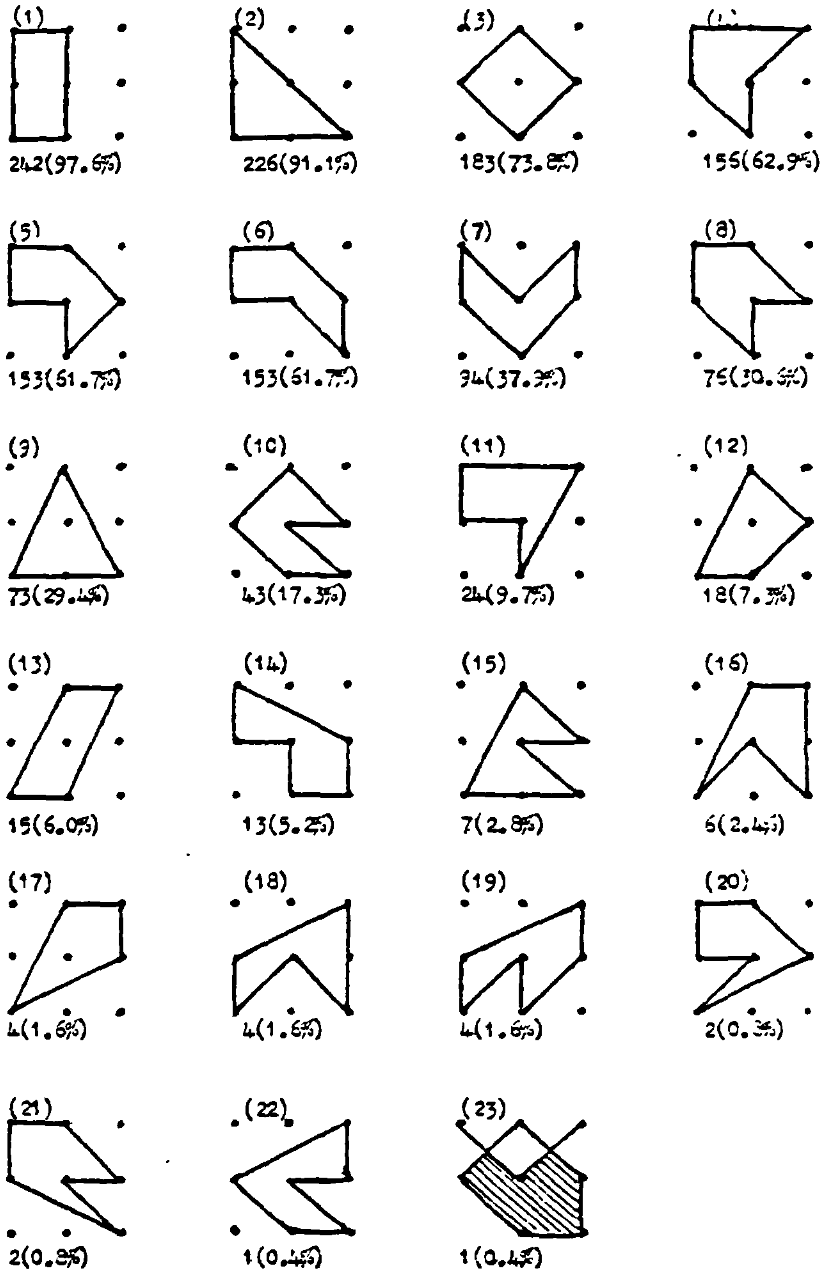


Figure 4.14. Acceptable responses of 248 pupils to Test 7 (Nine-Dots Areas). (The figures underneath each diagram indicate the number of pupils giving that response and the corresponding percentage).

Figure 4.15. Some responses to Test 7 (inappropriate).

divergent production are met. There was one problem related to criterion 3, with regard to the pupils' interpretation of the instructions in the test. Some pupils produced "shapes" such as those shown in Figure 4.15. The problem was whether to accept these as valid interpretations of the instructions to find a shape with an area of 2 cm^2. The majority of pupils may have decided, justifiably, that such responses would not be one shape with an area of 2 cm^2, but several shapes with areas totalling 2 cm^2. It was decided therefore - and this is a case where an element of subjective judgement is involved - to reject such responses as inappropriate. Other examples of inappropriate responses which had to be deleted were: (a) incorrect respcnses, i.e. shapes with an area not equal to 2 cm^2, and (b) repetitions - i.e. one shape repeated in a different position; such reflections, rotations and translations were deliberately excluded in the instructions. The remaining responses were awarded one mark each, with originality bonuses as follows: responses 11 - 14, one mark; responses 15 - 16, two marks; and responses 17 - 23, three marks.

Response 23 is worthy of some comment. This response was totally unanticipated, but perfectly valid and clearly creative. It is a good example of the way in which pupils may surprise teachers with their divergent thinking when given the opportunity to solve open-ended problems. Table 4.19 gives a summary of these pupils' performances on this test.

The number of acceptable responses given by the pupils ranged from 0 to 19, with the majority (83%) of them giving between three and eight responses, as indicated in Table 4.20. With bonuses for originality added the overall scores ranged from 0 to 39.

Table 4.19

Summary of Scores on Test 7 (Nine-dots Areas)

	Number	Mean score	Standard deviation
Boys	132	7.0	4.8
Girls	116	6.3	3.2
All	248	6.7	4.1

Table 4.20

Frequencies of Various Numbers of Responses (Fluency Scores) on Test 7 (Nine-dots Areas)

No. of responses	0	1	2	3	4	5	6	7	8	9
No. of pupils	1	3	8	30	22	40	37	48	29	12
%age	0.4	1.2	3.2	12.1	8.9	16.1	14.9	19.4	11.7	4.8

No. of responses	10	11	12	13	14	15	16	17	18	19
No. of pupils	6	6	3	0	1	0	0	1	0	1
%age	2.4	2.4	1.2	0	0.4	0	0	0.4	0	0.4

Test 16: Results. This was a problem-solving task in a numerical domain with many possible solutions. The pupils were given the result 23 x 35 = 805 and required to deduce from this other results, without engaging in substantial calculations. As an example they were given 23 x 350 = 8050. The responses of the 239 pupils taking this test showed a wide range of mathematical ideas, which were categorised as shown in Table 4.21.

The examples quoted in Table 4.21 are a selection of the actual responses produced by the pupils. They indicate that the open-ended problem posed in this test gave them the opportunity to bring to bear in it a wide range of mathematical notions. Whereas most pupils were able to produce some results, mainly in category 1 by following the principle used in the given example, but also substantial numbers in categories 2, 3 and 4, there were pupils who produced a number of highly original responses (categories 5 - 16) which clearly have some degree of mathematical substance to them. This analysis of the pupils' performances on this test indicates that it meets the criteria stated in Table 4.17 for inclusion as a valid test of divergent production.

The incidence of pupils merely providing trivial repetition of the same idea was particularly high in this test, although this occurred almost entirely in categories 1 - 4 above (i.e the most frequently used ideas). In accordance with the marking principles developed for these tests no more than four results under any one heading were accepted. Sometimes fewer than four would be accepted if the pupil showed no variety within the category. For example, in category 1 (adding zeros to the 23 and 35,) a pupil giving just the four responses 23 x 3500 = 80500, 23 x 35000 = 805000, 23 x 350000 = 8050000 and 23 x 3500000 = 80500000 would be given just two marks, whereas a pupil giving 23 x 3500 = 80500, 230 x 35 = 8050, 23 x 35000 = 805000 and 2300 x 3500 = 805000 would score four marks.

Other examples of inappropriate responses which had to be deleted were: (a) Incorrect results - sometimes a pupil used an original idea, such as negative numbers, but used it incorrectly, e.g. - 23 x - 35 =-805. It seems hard not to give credit for the idea, but in general the principle was adhered to that responses had to be mathematically correct in order to be acceptable.

Table 4.21

Acceptable Responses to Test 16 (Results)

Mathematical idea used	Examples	No. of pupils	%age
1. Adding further zeros (multiplying 23, 35 by 10's)	23 x 3500 = 80500 230 x 35 = 8050	198	82.8%
2. Decimals (dividing 23, 35 by 10's)	2.3 x 35 = 80.5 23 x 0.35 = 8.05	76	31.8%
3. Rearranging multiplication Results as division	805 ÷ 23 = 35 805 ÷ 35 = 23 8050 ÷ 23 =350 8.05 ÷ 23 = 0.35	105	43.9%
4. Commutative law applied to multiplication results	35 x 23 = 805 350 x 23 8050	102	42.7%
5. Rewriting 23, 35 using addition, subtraction	(20 + 3) x 35 = 805	12	5.0%
6. Rewriting 23, 35, 230 using multiplication	23 x 7 x 5 = 805 $(11\frac{1}{2}$ x 2) x 35 = 805 (115 x 2) x 35 = 8050	10	4.2%
7. Doubling numbers in various results	46 x 35 = 1610	5	2.1%
8. Halving numbers in various results	$11\frac{1}{2}$ x 35 = $402\frac{1}{2}$	5	2.1%
9. Adding or subtracting 1 to either 23 or 35	23 x 34 = 782 24 x 35 = 840	5	2.1%
10. Adding 2 to the 23	25 x 35 = 875	1	0.4%
11. Operating on the given result	$\frac{1}{2}$ of (23 x 35) = $402\frac{1}{2}$ (23 x 35) x 2 = 1610	5	2.1%
12. Distributive law (addition)	20 x 35 + 3 x 35 = 805	4	1.7%
13. Distributive law (subtraction)	100 x 35 - 77 x 35 = 805	2	0.8%
14. Fractions (other than halves)	1/23 of 805 = 35	1	0.4%
15. Multiplying the 23 by 3	69 x 35 = 2415	1	0.4%
16. Rearranging the equation	805 - 35 x 23 = 0	1	0.4%

(b) Results which clearly were not deduced easily from the given result , such as 8000 + 50 = 8050 and 24 x 240 = 5760. On the whole the 24 pupils who scored zero on this test gave results like this, indicating that the notion of "deducing" further results had not been grasped. With the superfluous and inappropriate responses deleted the remaining responses were awarded one mark each, with originality bonuses for using the mathematical ideas contained in the following categories: category 5: one mark; categories 6, 7, 8, 9, 11: two marks; categories 10, 12, 13, 14, 15, 16: three marks. Table 4.22 gives a summary of these pupils performances on this test.

Table 4.22

Summary of Scores on Test 16 (Results)

	Number	Mean score	Standard deviation
Boys	137	6.1	4.4
Girls	102	6.2	4.6
All	239	6.2	4.5

The number of acceptable responses given by the pupils ranged from 0 to 20, with 5.9 being the mean number of responses. Table 4.23 shows the frequencies of various numbers of responses. With originality bonuses added the scores on this test ranged from 0 to 27.

Test 18: Three cards. This test was also a problem-solving task in a numerical domain with many possible solutions. The pupils had to consider three cards, A, B and C, with numbers written on them such that if the number on card A is added to the number on card C the answer "9" is obtained. The problem was to find as many as possible values for the numbers on A, B and C.

Table 4.23

Frequencies of Various Numbers of Responses (Fluency Scores) on Test 16 (Results)

No. of responses	0	1	2	3	4	5	6	7	8	9	
No. of pupils	24	11	26	13	22	19	19	23	27	16	
%age	10.0	4.6	10.9	5.4	9.2	7.9	7.9	9.6	11.3	6.7	
No. of responses	10	11	12	13	14	15	16	17	18	19	20
No. of pupils	13	8	8	4	1	2	2	0	0	0	1
%age	5.4	3.3	3.3	1.7	0.4	0.8	0.8	0	0	0	0.4

The responses of the 242 pupils taking this test showed a wide range of mathematical ideas, including use of fractions, decimals and negative numbers. Analysis of the responses suggested the best way to categorise them in terms of mathematical ideas involved was to consider first the value given to C, since the majority of responses used C = 1, 3 or 9, with a further substantial group using C = 2. Other values for C were all infrequent enough to be counted as original and usually involved a subtle shift in the use of mathematical ideas. (See Table 4.24)

Again the criteria in Table 4.17 for a test of divergent production were met by the pupils' responses to this item, as can be seen by examining the range and variety of mathematical ideas used in the selection of actual responses quoted as examples in

Table 4.24

Acceptable Responses to Test 18 (Three Cards)

	Description of category: Value of C	Other criteria	Examples A	B	C	No. of pupils	%age
1.	3	A, B positive integers or zero, A + B = 3	2 3	1 0	3 3	200	82.6%
2.	1	A, B positive integers or zero, A + B = 9	2 0	7 9	1 1	153	63.2%
3.	9	A, B = 1 or 0, A + B =1	1	0	9	53	21.9%
4.	3	Use of halves for A, B (or .5), A + B = 3	$2\frac{1}{2}$	$\frac{1}{2}$	3	86	35.5%
5.	1	Use of halves for A, B A + B = 9	$7\frac{1}{2}$	$1\frac{1}{2}$	1	45	18.6%
6.	9	Use of halves for A, B A + B = 1	$\frac{1}{2}$	$\frac{1}{2}$	9	31	12.8%
7.	3	Other fractions, A + B = 3	$1\frac{1}{4}$	$1\frac{3}{4}$	3	27	11.2%
8.	1	Other fractions, A + B = 9	$6\frac{1}{4}$	$2\frac{3}{4}$	1	12	5.0%
9.	9	Other fractions, A + B = 1	$\frac{1}{4}$	$\frac{3}{4}$	9	7	2.9%
10.	2	A + B = $4\frac{1}{2}$ (fractions)	2	$2\frac{1}{2}$	2	67	27.7%
11.	1,2,3 or 9	Use of decimals other than .5 for A,B	1.4 4.2	7.6 0.3	1 2	13	5.4%
12.	$4\frac{1}{2}$	A + B = 2	1	1	$4\frac{1}{2}$	16	6.6%
13.	6	A + B = $1\frac{1}{2}$	1	$\frac{1}{2}$	6	8	3.3%
14.	$1\frac{1}{2}$, $\frac{1}{2}$	A + B = 6, 18	2 10	4 8	$1\frac{1}{2}$ $\frac{1}{2}$	10	4.1%
15.	18	A + B = $\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$	18	6	2.5%
16.	$2\frac{1}{4}$	A + B = 4	1	3	$2\frac{1}{4}$	6	2.5%
17.	positive	Either A or B negative	-2	5	3	7	2.9%
18.	multiple of 9 (or 18)	e.g. C = 36, 72, 144, 288	0	$\frac{1}{8}$	72	2	0.8%
19.	8, 4	A + B = 9/8, 9/4	1	$\frac{1}{8}$	8	4	1.7%
20.	1/4, 1/3, 1/10	A + B + 36, 27, 90	30	6	$\frac{1}{4}$	2	0.8%
21.	9/5, 9/8, 3/4	A + B = 5, 8, 12	1	7	9/8	2	0.8%

Table 4.24. There are a number of obvious responses, such as categories 1 and 2, which were produced by most pupils. None of the reponses in categories 1 - 10 were counted as original. The mathematical idea of using fractions for A and B was common to categories 7, 8 and 9 (with C = 1, 3, 9), so the individual percentage frequencies in categories 1 and 9 were not considered significant.

Originality marks were awarded for use of the mathematical ideas contained in all the other categories as follows: 11 - 12: one mark; 13 - 17: two marks; 18 - 21: three marks.

As in Test 16, it was necessary to delete superfluous results which were merely trivial repetitions of the same idea. However the idea of repeatedly using the same idea is, of course, a valid and useful mathematical idea itself, so, as in Test 16, it was decided to accept up to four responses in any one category. The only other responses which had to be deleted as inappropriate were the inevitable incorrect ones. One common error was to give C = 0, with A + B = 9. Table 4.25 gives a summary of these pupils' scores on this test.

Table 4.25

Summary of Scores on Test 18 (Three Cards)

	Number	Mean score	Standard deviation
Boys	136	9.0	6.5
Girls	106	7.4	4.9
All	242	8.3	5.9

The number of acceptable responses given by the pupils ranged from 0 to 25, with 7.7 being the mean number of responses.

Table 4.26 gives a summary of the numbers of responses given by the 242 pupils taking this test. With originality bonuses added the overall scores ranged from 0 to 31.

Table 4.26

Frequencies of Various Numbers of Responses (Fluency Scores) on Test 18 (Three Cards)

No. of responses	0	1	2	3	4	5	6	7	8	9
No. of pupils	18	12	16	10	15	17	18	16	18	19
%age	7.4	5.0	6.6	4.1	6.2	7.0	7.4	6.6	7.4	7.9
No. of responses	10	11	12	13	14	15	16	17	18	19
No. of pupils	15	16	13	14	2	3	5	2	3	3
%age	6.2	6.6	5.4	5.8	0.8	1.2	2.1	0.8	1.2	1.2
No.of responses	20	21	22	23	24	25				
No. of pupils	2	1	1	2	0	1				
%age	0.8	0.4	0.4	0.8	0	0.4				

<u>Test 2: Counters</u>. This test was essentially a problem-posing task in a numerical domain. The pupils were asked to consider a situation in which a child, in order to solve an unknown mathematics question, puts out twelve counters, as shown in Figure 4.16. They were then asked to write down as many different and varied questions as they could think of which the child might have been trying to work out with the counters arranged like this. To help in the interpretation of their responses they were also required to provide the answer to each question. The suggestion 6 + 6 = 12 was

given to them to get them started.

The responses of the 257 pupils taking this test showed that it gave them the opportunity to bring a range of mathematical ideas to bear upon this open-ended situation, as shown in the summary of responses for this test given in Table 4.27.

This analysis of the responses indicates that the test meets the criteria laid down in Table 4.17. A wide range of mathematical ideas is used, and whereas many pupils limited themselves to interpreting the arrangement of counters as simple multiplication and addition, and, less frequently, division and subtraction, others brought in such relatively infrequently used ideas as fractions, factors, equations and "problems". On the whole the original responses are not trivial, and are appropriate to the situation described in the test. However it must be conceded that there are difficulties in applying the appropriateness criteria to some of the responses. Some pupils produced just any calculations they could think of with the answer 12. Some of these were clearly inappropriate to the arrangement of counters given, such as $4\frac{1}{2} + 7\frac{1}{2} = 12$, $48 - 36 = 12$, $72 \div 6 = 12$, $2^3 + 6 - 2 = 12$. There were however a number of doubtful responses of this sort. For example, had the pupil who gave the response $2^2 + 8 = 12$ really related this to the arrangement of counters given, or just thought up a calculation with the answer 12? Certain combinations of addition and subtraction, likewise, are appropriate to the arrangement of counters, such as $6 - 3 + 9$, whereas other very similar combinations are not, such as $6 + 9 - 3$ (since the child would presumably have to use more than the 12 counters available). Consequently a certain degree of subjectivity was involved in assessing the appropriateness of some of the responses. In some cases the credit for originality was reduced when the level of

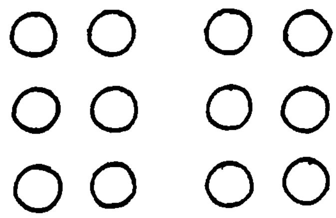


Figure 4.16. An arrangement of twelve counters for use as a problem-posing divergent production task.

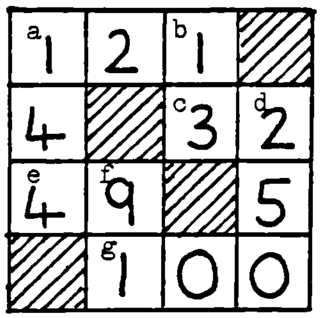


Figure 4.17. A completed cross-number puzzle used for problem-posing in Test 11.

Table 4.27
Acceptable Responses to Test 2 (Counters)

Mathematical idea used		Examples	No. of pupils	%age
1.	Multiplication of two integers	$2 \times 6 = 12$, $3 \times 4 = 12$	226	87.9
2.	Addition of two integers	$4 + 8 = 12$, $9 + 3 = 12$	167	65.0
3.	Division of two integers	$12 \div 3 = 4$, $12 \div 6 = 2$	108	42.0
4.	Subtraction of two integers	$12 - 3 = 9$, $12 - 8 = 4$	92	35.8
5.	Strings of addition	$3 + 3 + 3 + 3 = 12$ $6 + 4 + 2 = 12$	191	74.3
6.	Combination of +, x (not using distributive law)	$3 + 3 \times 3 = 12$ $4 + (4 \times 2) = 12$ $*2 + 2 \times 3 = 12$	84	32.7
7.	Subtraction by comparison	$6 - 6 = 0$	68	26.5
8.	Combination of +, -	$6 + 6 - 3 = 9$	27	10.5
9.	Strings of multiplication	$2 \times 3 \times 2$	23	8.9
10.	Use of units/objects	"If I have 3 apples and you have 9...etc". (or with pence etc)	9	3.5
11.	Clear use of distributive law	$3 \times (2 + 2) = 12$, $2(3 + 3) = 12$	9	3.5
12.	Combination of x, - or x, ÷	$3 \times 4 - 6 = 6$, $3 \times 4 \div 2 = 6$	7	2.7
13.	Square numbers	$2^2 + 8 = 12$	7	2.7
14.	Strings of subtraction	$12 - 6 - 3 = 3$	7	2.7
15.	Equations	$4 + x = 12$, $3 + \square = 12$ $? + 4 = 12$	7	2.7
16.	Rearrangement of counters	"How many ways can you arrange 12 counters...etc".	2	0.8
17.	Factors	"Find the factors of 12."	2	0.8
18.	Problems	"Divide 12 sweets between 2 children so that one has two more than the other	2	0.8
19.	Counter representing ten or hundred	$400 + 800 = 1200$	1	0.4
20.	Substitution into formula	$L = 3$, $M = 2$. Find LMM	1	0.4
21.	Fractions: "half of"	$\frac{1}{2}$ of 12, 12/2	10	3.9
22.	1/n of...	$\frac{1}{3}$ of 12, $\frac{1}{4}$ of 12	6	2.3
23.	$\frac{3}{4}$, $\frac{2}{3}$ of...	$\frac{3}{4}$ of 12, $\frac{2}{3}$ of 12	3	1.2
24.	Other fraction ideas	$1\frac{1}{2} \times 8 = 12$	1	0.4
		$3/12 + 9/12 = 12/12$	1	0.4
		$\frac{1}{2}$ of $12 = \frac{1}{4}$ of?	1	0.4

(*note: responses like this, although strictly incorrect in terms of precedence of operations, were accepted, since the sequence of operations intended was clear - the error is notational rather than mathematical.)

appropriateness of an original response was doubtful. Thus originality bonuses were awarded for the use of the mathematical ideas in the following categories: 9, 13: one mark; 10, 11, 12, 14, 15, 21, 22: two marks; 16 - 20, 23 24: three marks.

In addition to the problem of inappropriate responses there were, of course, a number of incorrect responses which were simply deleted. As would be expected with a numerical domain, the question of superfluous responses arose also, particularly in categories 2, 4, 5 and 6. For example, a pupil might give every possible pair of integers which sum to 12. The principle of accepting no more than four responses in one category adopted in previous tests of divergent production was used again here. Table 4.28 gives a summary of these pupils' scores on this test.

Table 4.28

Summary of Scores on Test 2 (Counters)

	Number	Mean score	Standard deviation
Boys	116	12.0	5.5
Girls	141	11.0	5.7
All	257	11.4	5.6

With superfluous, inappropriate and incorrect responses deleted, the number of acceptable responses given by the pupils on this test ranged from 1 to 28, with 10.8 being the mean number of responses. Table 4.29 gives a summary of the number of responses given by the 257 pupils taking this test. With originality bonuses added the overall scores ranged from 1 to 35.

Table 4.29

Frequencies of Various Numbers of Acceptable Responses (Fluency Scores) on Test 2 (Counters)

No. of responses	0	1	2	3	4	5	6	7	8	9
No. of pupils	0	2	6	4	8	8	18	20	24	18
%age	0	0.8	2.3	1.6	3.1	3.1	7.0	7.8	9.3	7.0
No. of responses	10	11	12	13	14	15	16	17	18	19
No. of pupils	26	16	26	23	8	11	8	3	10	1
%age	10.1	6.2	10.1	8.9	3.1	4.3	3.1	1.2	3.9	0.4
No. of responses	20	21	22	23	24	25	26	27	28	
No. of pupils	5	3	3	3	0	2	0	0	1	
%age	1.9	1.2	1.2	1.2	0	0.8	0	0	0.4	

Test 11: Cross-numbers. This test was also essentially a problem-posing task in a numerical domain. An obvious way of tapping divergent thinking in schoolchildren is to reverse the conventional mathematics question, by giving them the answer and inviting them to pose the question. The cross-number puzzle, a numerical version of the crossword puzzle, is a straightforward way of doing just that. The pupils were given the completed crossnumber puzzle shown in Figure 4.17 and required to make up the clues to fit the given answers. To get them started a suggested clue for 'a' across was given: "the number of pence in £1.21". The instructions overtly encouraged the use of different ideas and trying to make up the cleverest clues. Not surprisingly with this

Table 4.30

Acceptable Responses to Test 11 (Cross-Number)

Mathematical idea used	Examples	No. of pupils	%age
1. Calculations: a) addition	16 + 16	200	80.0
b) subtraction	20 - 7	164	65.6
c) multiplication	8 x 4	223	89.2
d) division	182 ÷ 2	60	24.0
e) combinations of +, -, x, ÷	20 x 10 +5x 10	86	34.4
2. Powers: a) squares	7^2	59	23.6
b) square roots	$\sqrt{169}$	15	6.0
c) cubes	$3^3 + 22$	3	1.2
d) higher powers	2^5	2	0.8
3. Special numbers: a) 13 (unlucky)	Unlucky number	80	32.0
b) 144 (gross)	gross	20	8.0
c) 12 (dozen)	twelve dozen	13	5.2
d) 100 (century)	a century	23	9.2
e) 100 (centurion)	no. of soldiers under a centurion	1	0.4
f) 100 (telephone)	dial operator	1	0.4
g) 100 (cent)	"cent" means this	6	2.4
h) 21 (age)	'key of the door' plus 100	1	0.4
i) 13 (baker's dozen)	baker's dozen	7	2.8
j) 90 (degrees)	degrees in right angle plus one	1	0.4
k) 13 (teenage)	teenager now	1	0.4
l) 20 (score)	a score times 5	3	1.2
m) 50 (bull's eye)	a bull's eye minus one	1	0.4
4. Real-life problems: a) objects/sets	4 pockets of sweets, 8 in each	9	2.6
b) money: shopping/bills	spend 51p change from £1	10	4.0
c) time	if I am 50 and you are 41 yrs older	3	1.2
d) distance/time	96 km/h...how far in 20 minutes?	2	0.8
e) snooker scores	black and a pink score in snooker	1	0.4
f) dart scores	you are on 42, score 10...what needed?	3	1.2
g) non-concrete calculations, with units	40p + 51p	16	6.4
5. Units exchanging: a) pence in £1, £x	how many p in £2.50?	15	6.0
b) ½p in £x	how many ½p in £1.25	4	1.6
c) g/kg	how many g in 0.25 kg?	1	0.4
d) ounces/pounds	how many oz in 2 lbs	1	0.4
e) mins/hours	how many mins in 1 hour 40 mins?	4	1.6
f) days/weeks/years	no. of weeks in year minus 3	2	0.8
g) mm/cm/m	how many mm in 10 cm?	10	4.0
h) ml/litres	how many ml in tenth of a litre?	2	0.8
6. Facts: a) no. in known set	how many in our class?	3	1.2
b) engine size	question about 250cc motor bike	1	0.4
c) times of events	minutes in football game with 1 minute extra	2	0.8
d) dates	1 yr before start of World War, 19..	1	0.4
e) addresses	Prime Minister's door no. times 10	1	0.4
7. Ideas of sequences: a) next in sequence	43, 45, 47,...	7	2.8
b) before	the number before	6	2.4
c) between	halfway between 30 and 34	7	2.8
d) first/smallest	first odd number more than a dozen	7	2.8
e) 2nd, 3rd,...	5th prime number squared	6	2.4
f) greater/less than	greater than 31, less than 33	5	2.0
8. Decimals:	2.5 x 100	17	6.8
9. Ratio:	decrease 132 by the ratio 12.11	3	1.2
10. Fractions: a) ½, ¼	find ¼ of 1000	73	29.2
b) others	a third of 96	13	5.2
c) %age	20% of £6.05	3	1.2
11. Big numbers: (>1000)	2100 = 2051	19	7.6
12. Zero in calculations:	50 x 5 + 0	6	2.4
13. Number bases: a) convert to base 10	1034_5 in base 10	6	2.4
b) convert from base 10	$^{4}10 = 2?$	3	1.2
c) Roman numerals	CXXI = ?	6	2.4
14. Digits: a) hundreds, tens, units	add tens to units to make 100	1	0.4
b) operations on digits	second digit is 2 more than first	25	10.0
c) anagrams	anagram of 502	13	5.2
d) upside down	16 upside down	5	2.0
15. Number properties: a) odd/even	the fifth odd no.	15	6.0
b) prime	the sixth prime no.	10	4.0
c) multiples/tables	highest in the 12 times table	9	3.6
d) factors	5 goes into it the same no. of times as 10 goes into 200	4	1.6
16. Algebraic ideas: a) formulas	x = 7, x X x =?	2	0.8
b) equations with x,y	xy = 36, xy, in numerical order...	1	0.4
c) equations with	2x + 2 = 100	3	1.2
d) "this no."	25 is a quarter of this number	9	3.6
e) reference other question	2½ times 'g' across	4	1.6

very open-ended situation there was a very wide range of ideas used by the pupils in making up their clues, so there was plenty of opportunity for original responses. However to balance this it is to be noted that the pupils are limited to 8 responses in total, since there are only eight clues required. Most of the 250 pupils taking this test used straightforward calculations in some of their clues, but many of them supplemented these with clues using almost every conceivable aspect of arithmetic within their experience, as shown in the analysis of their responses given in Table 4.30.

Again, from the analysis of responses, it can be seen that the criteria laid down in Table 4.17 are satisfied by the pupils' responses to this test. There are non-original ideas, such as those in category 1, which are used by most pupils, and the original ideas are non-trivial. Most of the pupils understood what was required and only a few failed to satisfy the requirements of the task. Since eight responses were required from each pupil to complete the task this was clearly a case where credit should be given for the number of ideas used rather than the number of individual responses. So the fluency score was obtained by giving one mark for each of the categories or sub-categories in Table 4.30 used by the pupil in his or her responses. This meant that some responses would be given credit for using more than one idea. For example, the response "the fifth prime number squared" uses ideas from categories 7(e), 15(b) and 2(a). Credit for using a particular idea was given only once. Originality bonuses were added for ideas used by less than 10% of pupils (one mark), less than 5% (two marks) and less than 1% (three marks), with the exception of category 5(a) which was given to the pupils as an example and should not therefore be considered original. Incorrect clues were deleted as usual. Table 4.31 gives a summary of these pupils' overall scores on this test.

Table 4.31

Summary of Scores on Test 11 (Cross-Number)

	Number	Mean	Standard deviation
Boys	135	9.1	5.9
Girls	115	7.5	3.7
All	250	8.4	5.1

The fluency scores, that is the number of ideas used by the pupils in their responses, ranged from 0 to 17, with 5.8 being the mean fluency score. Table 4.32 gives a summary of the fluency scores obtained by the 250 pupils taking this test. With originality bonuses added the overall scores ranged from 0 to 32.

Table 4.32

Frequencies of Various Numbers of Mathematical Ideas Used by Pupils (Fluency Scores) in Responses to Test 11 (Cross-Number)

Fluency score	0	1	2	3	4	5	6	7	8
No. of pupils	3	3	8	15	34	48	48	39	25
%age	1.2	1.2	3.2	6.0	13.6	19.2	19.2	15.6	10.0

Fluency score	9	10	11	12	13	14	15	16	17
No. of pupils	11	6	4	4	1	0	0	0	1
%age	4.4	2.4	1.6	1.6	0.4	0	0	0	0.4

Test 12: Scattergram. This was a problem-posing task which involved interpretation of a graph and thus is considered to be essentially in a spatial domain. The pupils had explained to them the construction of a scattergram showing the distribution of boys

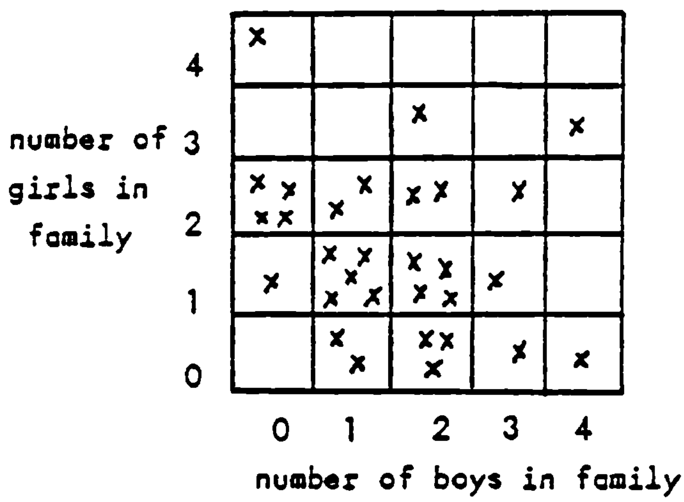


Figure 4.18. Scattergram used in Test 12.

Table 4.33

Acceptable Responses to Test 12 (Scattergram)

	Problems posed	No. of pupils using this idea	%age of pupils
1.	How many families have x boys, y girls? (answer not zero) or equivalent statements, like 'one of each','only 4 boys', 'just 3 girls'	238	99.2
2.	How many families have x boys, y girls? (answer zero)	133	55.4
3.	How many families have only boys (no girls)?	28	11.7
	How many families have only girls (no boys)?	28	11.7
4.	How many families?	31	12.9
5.	How many families have no children?	22	9.2
6.	Questions about the largest family...	20	8.3
7.	Questions about the smallest families...	9	3.8
8.	How many families have equal boys and girls?	17	7.1
9.	How many have not equal boys and girls?	1	0.4
10.	How many have x boys (any no. of girls?)	13	5.4
	How many have y girls (any no. of boys)?	13	5.4
11.	There are n of one type of family, what is it? (n = 3 or 5)	13	5.4
12.	How many families have just 1 child	10	4.2
13.	How many families have n children? (n = 2, 3, 4, 5, 6, 7)	6	2.5
14.	How many families have 8 or 9 children? (answer zero)	3	1.2
15.	Questions about/using coordinates...	9	3.8
16.	How many families have more than n children? (or, n or more children)	8	3.3
	How many have less than n (n or less)?	3	1.2
17.	How many girls altogether? (answer 39)	7	2.9
	How many boys? (answer 46)	6	2.5
	How many children? (answer 85)	6	2.5
18.	Combining questions about the graph with irrelevant calculations (e.g. how many have 4 boys)?	7	2.9
19.	Questions about most common family...	6	2.5
20.	List all combinations with n crosses (families)	7	2.9
21.	Questions about the limitations of the graph (e.g. limits on size of family which could be shown)	5	2.1
22.	How many families have boys (answer 24)? or have girls (answer 23)	1	0.4
23.	How many families have children all same sex?	2	0.8
24.	How many have more boys than girls?	4	1.6
	How many have more girls than boys?	4	1.6
25.	How many have twice as many boys as girls?	4	1.6
26.	How many have x boys, y girls (x or y > 4)? (answer zero)	1	0.4
27.	How many families do not have x boys, y girls?	3	1.2
28.	How many have more than x boys and y girls?	2	0.8
29.	How many have more than x boys (y girls)?	2	0.8
30.	What is the average no. of children per family?	1	0.4
	What is the average no. of boys (girls) per family?	1	0.4
31.	Questions about how many had so many brothers...(sisters)	4	1.6
32.	What is most no. of girls in a family?	3	1.2
	What is most no. of boys in a family?	3	1.2
	Which family will have to pay for most weddings?	1	0.4
33.	Is there a family with x boys, y girls? (or children)... answer yes/no	3	1.2
34.	If certain families were added or removed...	2	0.8
35.	Questions about grid shape, layout axes...	3	1.2
36.	Use of algebraic representation...	1	0.4
37.	How many children altogether in families with equal numbers of boys and girls?	1	0.4
38.	How many girls altogether in families with no boys?	1	0.4
	How many boys altogether in the families with 4 girls?	1	0.4

and girls in the families of a class of schoolchildren. This is shown in Figure 4.18.

They were then instructed to make up as many interesting and different questions as they could which could be answered from the graph. As an example they were given the question, "How many families had 2 boys and 1 girl?". Many pupils just provided a long list of questions of this sort, with some pupils working systematically through all the possibilities. Others showed divergent thinking by producing questions which required consideration of rows, columns, diagonals, and regions of crosses on the graph. Much originality was evident in some of the responses as shown in the analysis below. Again the pupils were required to answer their own questions. The field testing of this task described earlier had indicated that this would help enormously in the interpretation of their questions. For example, the question, "How many families had 4 boys?" might mean "just 4 boys and no girls" or "4 boys regardless of the number of girls". The pupil's own answer to the question would make the intention clear. Table 4.33 gives an analysis of the responses of the 240 pupils who took this test.

This test quite clearly met the criteria for a valid test of divergent production laid down in Table 4.17. Nearly all pupils produced the obvious questions in category 1 above, but apart from these a wide range of mathematical ideas was used and a large number of responses were possible. The less frequently used ideas were on the whole appropriate and non-trivial. Originality bonuses were awarded in the usual way, with three marks for those responses in categories uses by less than 2% of pupils, two marks for those in categories used by less than 5% of pupils and one mark for those in categories used by less than 10% of pupils. The only exceptions were categories 18 and 35 above where some of the responses were of dubious appropriateness and were not given extra credit for originality. Fluency scores were obtained by awarding one mark for each appropriate question posed after excessive repetitions of the same idea had been deleted. On the whole this only applied to category 1 where some pupils, ignoring the encouragement to seek varied and interesting questions, produced a long list of questions of this type. No more than three questions in this category with non-zero answers were accepted, with an extra one with zero answer allowed in addition. Thus pupils using only category 1 could score no more than 4 marks on this test. Some questions had to be ignored simply because it was impossible to decipher the pupil's meaning. However consideration of the answer provided by the pupil usually made the intention clear. If a pupil made an arithmetic slip in answering his or her own question but there was no doubt about what was intended in the question then the response was accepted. The test was designed as a problem-posing item, not a problem-solving task. The answers were required merely to assist in the interpretation of the questions. Table 4.34 gives a summary of the performances of these pupils on this test.

Table 4.34

Summary of Scores on Test 12 (Scattergram)

	Number	Mean score	Standard deviation
Boys	136	6.6	6.2
Girls	104	6.0	3.9
All	240	6.4	5.3

The fluency scores on this test ranged from 0 to 18, with the mean score, dominated by those pupils using only category 1, being 4.8. Table 4.35 gives a summary of the fluency scores obtained by the 240 pupils who took this test. With originality bonuses added the scores ranged from 0 to 34.

Table 4.35

Frequencies of Various Numbers of Acceptable Questions Posed by Pupils (Fluency Scores) in Test 12 (Scattergram)

No. of questions posed	0	1	2	3	4	5	6	7	8	9
No. of pupils	2	1	6	61	86	23	19	15	5	9
%age	0.8	0.4	2.5	25.4	35.8	9.6	7.9	6.3	2.1	3.8

No. of questions posed	10	11	12	13	14	15	16	17	18
No. of pupils	1	4	4	1	1	1	0	0	1
%age	0.4	1.7	1.7	0.4	0.4	0.4	0	0	0.4

Test 3: Subsets. This test presented the pupils with a task involving redefinition in a numerical domain. They were first reminded of the idea of subsets and then given the set {2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16} to consider. They were invited to think of many different ways of making up subsets from this set, in each case stating what their rule was. Subsets with only one member were excluded in the instructions. As an example they were given: {2, 4, 6, 8, 10, 12, 14, 16} (even numbers). In order to come up with different responses to this task the pupils would have to continually redefine the elements making up the set in various ways. They were able to show their ability to think divergently by bringing to bear upon the situation a wide range

of mathematical ideas, such as odd, even, greater than, less than, between, prime, square, factors, multiples and so on, as shown in the analysis of their responses in Table 4.36.

There is no doubt again that the criteria laid down in Table 4.17 are satisfied by this set of responses. There were a number of obvious responses obtained by many pupils, such as using multiples and odd numbers. But there were very many other interesting and creative ideas produced. The original ideas can be recognised immediately as having validity in mathematical terms. Sometimes these would be obtained by using the logical complement of an earlier subset, such as: {4, 6, 8, 9, 10, 12, 14, 15, 16} (not prime). A number of original ideas were obtained by using the intersection of two attributes, such as: {2, 4, 8} (factors of 16 less than 10). It is interesting to note that only one pupil used the logical union of two attributes in a response: {3, 6, 9, 12, 13, 15} (multiples of 3 or contain the digit 3).

There were the expected problems in marking the responses related to appropriateness, accuracy and superfluity. Some pupils listed subsets without stating the rule - this applied particularly to sets of multiples. Also some pupils gave a rule which was sequential rather than global in its description of the subset, such as: "go up in threes" or "missing out every other number". After much deliberation and examination of the pupils' scripts it was decided to delete all responses where the rule was not stated or where the rule was not a description of what the members of the subset had in common. Some indication of the attribute defining membership of the subset was considered necessary for an acceptable response in a test based upon the notion of <u>re</u>-definition. Some of the categories listed above are clearly open to producing many responses all using essentially the same idea. The usual practice

Table 4.36

Acceptable Responses to Test 3 (Subsets)

Subset description	No. of pupils	%age
1. Odd numbers	199	78.9
2. Multiples of 3 (3 goes into/3 times table)	168	66.1
Multiples of 4	175	68.9
Multiples of 5	162	63.8
Multiples of 6	134	52.8
Multiples of 7	121	47.6
Multiples of 8	127	50.0
3. Factors of 4 (number that go into 4)	5	2.0
Factors of 6	14	5.5
Factors of 8	20	7.9
Factors of 9	5	2.0
Factors of 10	26	10.2
Factors of 12	32	12.6
Factors of 14	24	9.5
Factors of 15	18	7.1
Factors of 16	32	12.6
4. Factors of 18	2	0.8
Factors of 20	5	2.0
Factors of 22	1	0.4
Factors of 24	6	2.4
Factors of 25	1	0.4
Factors of 30	4	1.6
Factors of 32	1	0.4
Factors of 36	1	0.4
Factors of 40	3	1.2
Factors of 50	1	0.4
Factors of 100	2	0.8
5. The whole set (numbers from 2 to 16 inclusive)	40	15.7
6. Not multiples of...	2	0.8
7. Prime numbers	23	9.1
8. Not prime numbers	4	1.6
9. Two-digit numbers	22	8.7
One-digit numbers	18	7.1
10. a) Numbers greater than...	20	7.9
b) Numbers less than...	28	11.0
c) Numbers between...and...	15	5.9
11. Squares	10	3.9
12. Triangle numbers	3	1.2

Subset description	No. of pupils	%age
13. ...are square roots of numbers less than...	2	0.8
14. Contain a given digit	24	9.5
15. Do not contain a given digit	2	0.8
16. Have more than one factor (not including 1)	1	0.4
17. Are not factors of 16	1	0.4
18. Powers of 2, 3, or 4	2	0.8
19. The first and last numbers in the given set	3	1.2
20. Numbers related to facts about the pupils	2	0.8
21. Subsets formed by references to the shapes of the digits	3	1.2
22. Have the initial letter T (F, S, etc)	3	1.2
23. Two-syllable (one-syllable) words	1	0.4
24. Numbers which have 3 (4) letters in the word	1	0.4
25. "teens" (i.e 13, 14, 15, 16)	3	1.2
<u>Intersection of two attributes</u>:		
26. Even, greater than	8	3.1
Even, less than	14	5.5
Odd, greater than	10	3.9
Odd, less than	14	5.5
27. Even, between	7	2.8
Odd, between	5	2.0
28. Even, one-digit	6	2.4
Odd,one-digit	3	1.2
Even, two-digit	7	2.8
Odd, two-digit	4	1.6
29. Prime, greater than	1	0.4
30. Factors, less than	1	0.4
31. Multiples, less than	1	0.4
32. Initial letters, even	1	0.4
<u>Union of two attributes</u>:		
33. Multiples of 3 or contain the digit 3	1	0.4

of allowing no more than four in each category was adopted in the marking. Factors are put into two categories because there seemed to be a distinction between listing factors of numbers up to 16, and listing factors of numbers greater than 16, the latter requiring reference outside of the given universe. Where the rule being used to define the subset was quite clear, minor slips in reasoning or recording were overlooked. For example, the subset: {3, 5, 11, 13} (prime number) would be accepted, even though '2' was omitted.

This test provided plenty of opportunity to reward originality, with bonuses being applied as follows: one mark: categories 7, 9, 10(c), 14, 26; two marks: categories 4, 11, 27, 28; three marks: categories 6, 8, 12, 13, 15 - 25, 29 - 33.

The overall performances of the 254 pupils taking this test are summarised in Table 4.37.

Table 4.37

Summary of Scores on Test 3 (Subsets)

	Number	Mean score	standard deviation
Boys	138	8.2	6.5
Girls	116	8.6	5.8
All	254	8.4	6.2

Fluency scores on this test were the number of acceptable responses remaining after superfluous and inappropriate responses had been deleted. The fluency scores ranged from 0 to 20, with the mean fluency score being 7.2. These scores are summarised in Table 4.38. With originality bonuses included the overall scores on Test 3 (Subsets) ranged from 0 to 32.

Table 4.38

Frequencies of Various Numbers of Acceptable Responses

(Fluency Scores) on Test 3 (Subsets)

No. of responses	0	1	2	3	4	5	6	7	8	9	10
No. of pupils	22	27	5	10	12	10	20	41	22	11	12
%age	8.7	10.6	2.0	3.9	4.7	3.9	7.9	16.1	8.7	4.3	4.7

No. of responses	11	12	13	14	15	16	17	18	19	20
No. of pupils	12	7	12	9	7	3	4	3	4	1
%age	4.7	2.8	4.7	3.5	2.8	1.2	1.6	1.2	1.6	0.4

Test 4: Shape-finding. This test of divergent production involved redefinition in a spatial domain. It is similar to some of the tests used by Krutetskii (1976) for the assessment of mathematical ability in which he posed problems with interpenetrating elements. In the present research the pupils were given the following geometric figure shown in Figure 4.19 and asked what shapes they could see within the figure. They were required to draw their answers and indicate how many of each shape they could see. Thus they would need to discern and assess the interpenetrating elements of the given figure from different points of view, and in so doing redefine a given element in terms of its place in a figure being isolated from the background. To get them started the pupils were given the example quoted in Figure 4.20. The 254 pupils taking this test found between them a total of 31 different shapes within the given figure, as shown in the summary given in Figure 4.22. Apart from reference to a "box" or "a cuboid" the responses were entirely

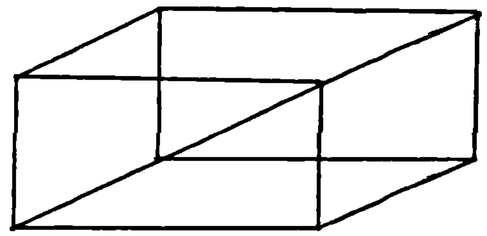


Figure 4.19. Diagram used for shape-finding in Test 4.

"I can see two of these rectangles:"

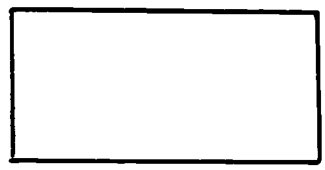
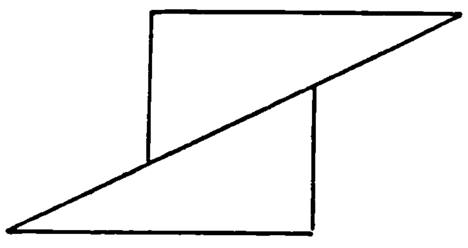


Figure 4.20. Example given to pupils in Test 4.



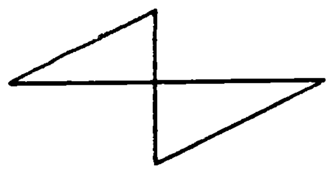
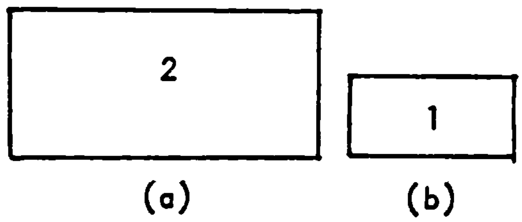
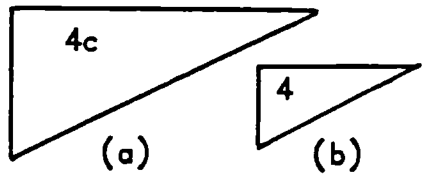


Figure 4.21. Examples of inappropriate responses in Test 4.

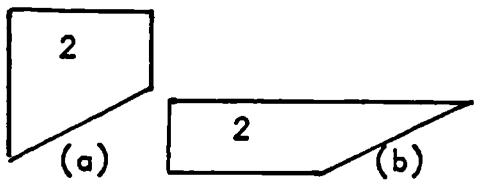
1. Rectangles: 228 (89.8%)



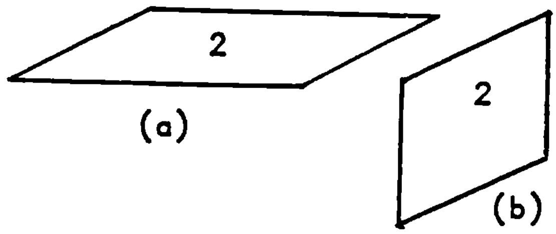
2. Triangles: 250 (98.4%)



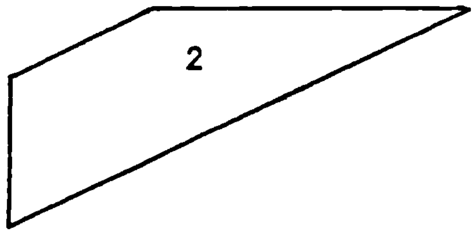
3. Right angled trapeziums: 171 (67.3%)



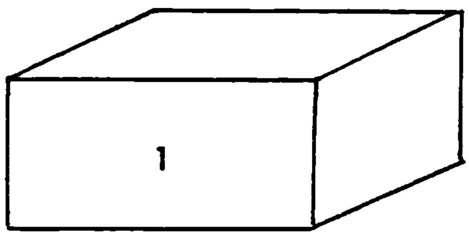
4. Parallelograms: 193 (76.0%)



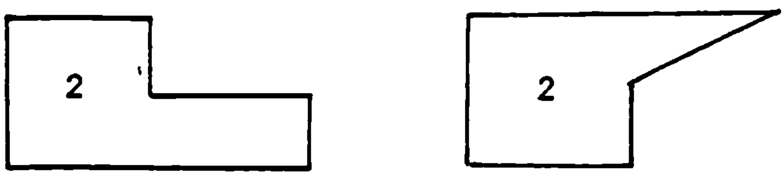
5. Non right angled trapezium: 31 (12.2%)



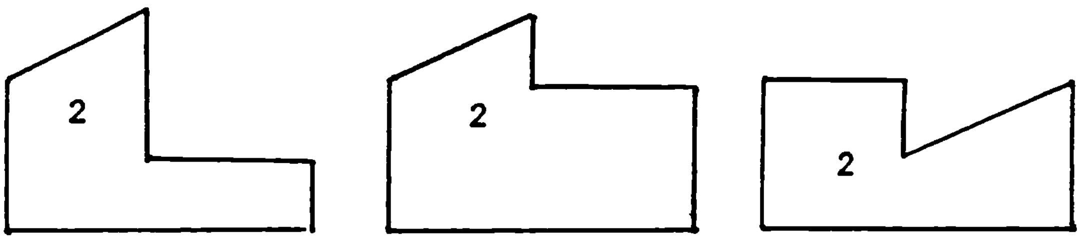
6. A box (cuboid) 12 (4.7%)



7. Nonrectangular shapes containing three or more right angles:



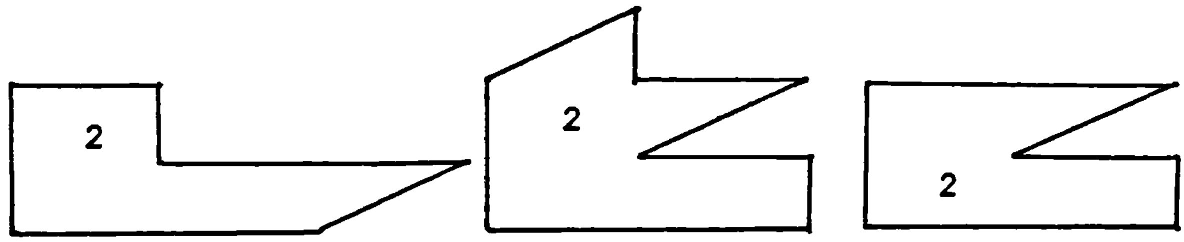
(a) 22 (8.7%) (b) 13 (5.1%) (c) 5 (2.0%)



/cont.

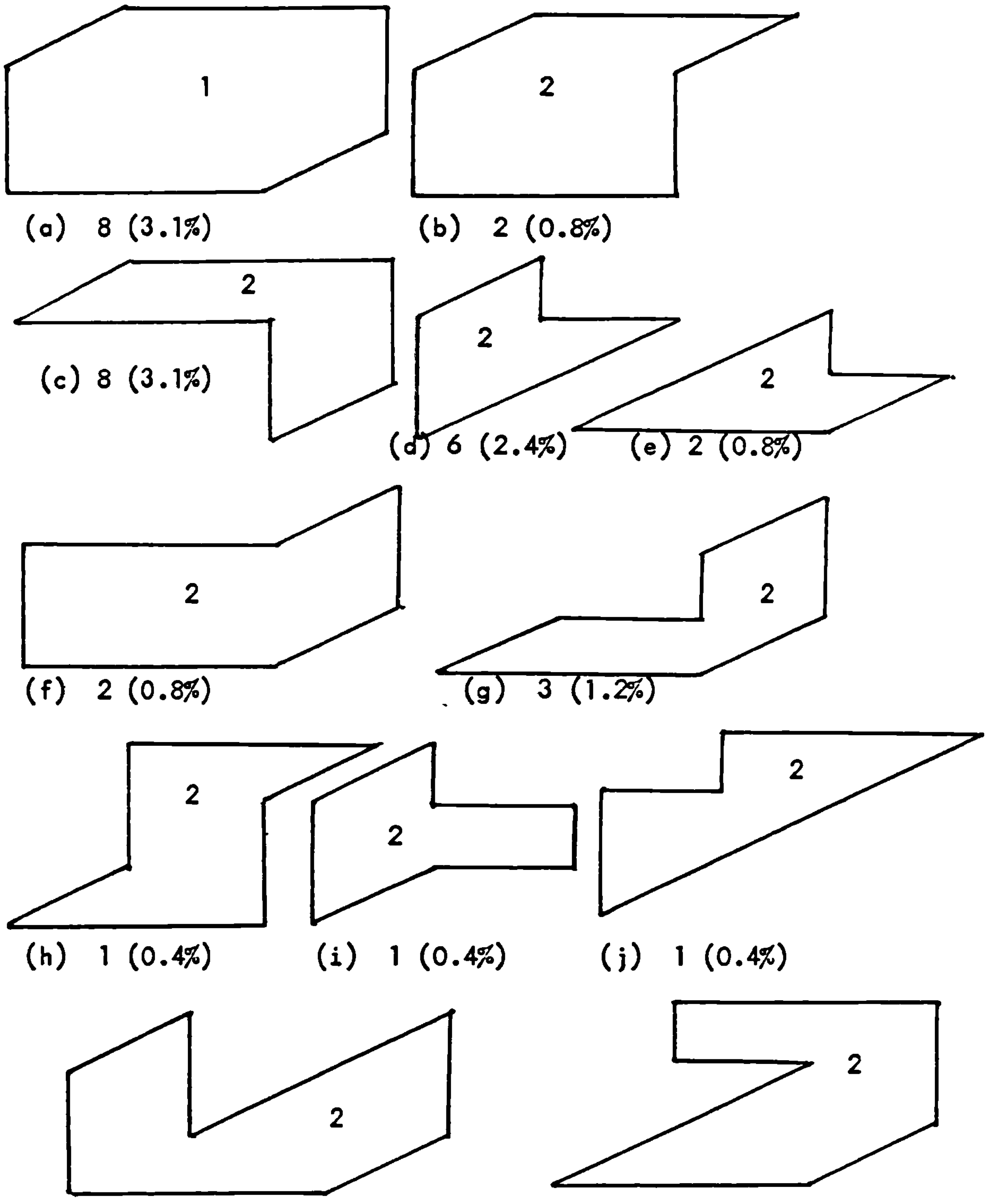
(d) 3 (1.2%) (e) 3 (1.2%) (f) 1 (0.4%)

Figure 4.22. Responses to Test 4 (Shape-finding). (The number of pupils and the corresponding percentage finding each shape are given. The numbers written on each shape indicate how many may be found).



(g) 2 (0.8%) (h) 1 (0.4%) (i) 1 (0.4%)

8. Other shapes:



(a) 8 (3.1%) (b) 2 (0.8%)

(c) 8 (3.1%) (d) 6 (2.4%) (e) 2 (0.8%)

(f) 2 (0.8%) (g) 3 (1.2%)

(h) 1 (0.4%) (i) 1 (0.4%) (j) 1 (0.4%)

(k) 1 (0.4%) (l) 1 (0.4%)

Figure 4.22. continued

restricted to two-dimensional shapes. Since the object of this task was to isolate shapes, composite figures such as those shown in Figure 4.21 were rejected as inappropriate responses. Fluency scores were obtained by giving credit for each shape indentified: half a mark for each triangle, since there were eight of them to be found, and one mark for each other shape. Thus a pupil identifying all four parallelograms would obtain four marks. Originality marks were added as follows: one mark for shapes 6, 7(a), 7(b); two marks for shapes 7(c), 8(a), 8(c); three for all other shapes in categories 7 and 8.

However since the discovery of a large number of shapes in categories 7 and 8 did not appear to involve any mathematical ideas which could be considered more original than the thinking required to discover a few shapes in these categories it was decided to limit the originality bonuses to 15. This meant that the ratio of potential originality to fluency scores was comparable to that in other tests of divergent production used in the research. Table 4.39 summarises the overall scores obtained by the 254 pupils taking this test.

Table 4.39

Summary of Scores on Test 4 (Shape-Finding)

	Number	Mean score	standard deviation
Boys	138	10.7	6.2
Girls	116	10.6	6.5
All	254	10.7	6.3

Fluency scores on this test ranged from 0 to 30, with 10.0 being the mean fluency score. Table 4.40 gives a summary of the fluency scores of the 254 pupils taking this test. With originality bonuses added the overall scores ranged from 0 to 44.

Table 4.40

Frequencies of Various Fluency Scores on Test 4 (Shape-Finding)

Fluency score	0	1	2	3	4	5	6	7	8	9	10
No. of pupils	2	1	1	18	8	13	21	14	27	17	17
%age	0.8	0.4	0.4	7.1	3.1	5.1	8.3	5.5	10.6	6.7	6.7

Fluency score	11	12	13	14	15	16	17	18	19
No. of pupils	23	23	12	19	14	6	5	3	2
%age	9.1	9.1	4.7	7.5	5.5	2.4	2.0	1.2	0.8

Fluency score	20	21	22	23	24	25	26	27	28
No.of pupils	1	0	1	2	0	2	1	0	0
%age	0.4	0	0.4	0.8	0	0.8	0.4	0	0

Fluency score	29	30
No. of pupils	0	1
%age	0	0.4

Test 5a: Similarities (numbers). This test was also constructed on the basis of redefinition. The task given to the pupils was one which is basic to mathematical thinking. The pupils were required to look at two numbers which are different and consider the ways in which they are the same. Looking at things which are different but seeing them as being in some sense the same is an activity

Table 4.41

Acceptable Responses to Test 5a (Similarities: Numbers)

Statements		No. of pupils	%age
1.	They are both multiples of...(or, both can be divided by...)		
a)	4	219	86.2
b)	2	217	85.4
c)	1	38	15.0
d)	0.5, ½	3	1.2
e)	¼	1	0.4
f)	An even number	1	0.4
g)	Themselves	1	0.4
2.	Other statements related to multiples:		
a)	They both have factors/are multiples/are in the tables/are not prime	9	3.5
b)	They are both not multiples of 5, 10 etc	7	2.8
3.a)	They are both factors of 144	1	0.4
b)	They are both factors of 576	1	0.4
c)	They are both not factors of 100	1	0.4
4.a)	They are both even	125	49.2
b)	They are both not odd	6	2.4
5.	They are both squares (have square roots)	27	10.6
6.a)	They are both numbers	27	10.6
b)	They are both whole numbers	3	1.2
7.	Statements about the '6'	89	35.0
a)	They both end in 6, have 6 units		
b)	They both contain a 6	156	61.4
8.	Various statements about digits. They both have:		
a)	An even digit, an odd digit, one odd and one even digit, first digit odd, second digit even	30	11.8
b)	Two digits, tens and units, a tens digit and a units digit	1	0.4
c)	Not a single digit	1	0.4
d)	Two different digits	1	0.4
e)	Last digit divisible by 3, 2, 1 (first digit divisible by 1)	3	1.2
f)	First digit prime (accepted although strictly 1 is not prime).	2	0.8
g)	They are both in the sixties when digits reversed	2	0.8
h)	They both begin with 9 when inverted	13	5.1
9.	Order properties. They are both:		
a)	Greater than some integer	50	19.7
b)	Less than some integer	46	18.1
c)	Between two integers	13	5.1
d)	Greater than 15½	1	0.4
e)	Greater than every number less than 16	1	0.4
10.	Arithmetic Relationships. They both:		
a)	Are 4 less than a multiple of 10 (are rounded up to nearest 10 if 4 added), or 5	7	2.8
b)	Are 6 more than a multiple of 10	1	0.4
c)	Give remainder when divided by 10, give remainder 6 when divided by 10	3	1.2
d)	Give remainder 1 when divided by 5	1	0.4
e)	Will end in 8 when multipled by 3	1	0.4
f)	Will end in 8 when halved	1	0.4
11.	Operations on/relationships between digits.		
a)	They both have the first digit a factor of the second	1	0.4
b)	They both have the sum of their digits less than ten	1	0.4
c)	They both have the sum of their digits odd	4	1.6
12.	Potential applications.		
a)	They both can be percentages	1	0.4
b)	They both can be door numbers.3	1	0.4
13.	Physical appearance. They are both:		
a)	In a box, square	108	42.5
b)	In same size box	5	2.0
c)	Same size, height	6	2.4
d)	In black ink	34	13.4
e)	On white background	9	3.5
f)	Not pink, not on green background	2	0.8
g)	Away from edges of the box	2	0.8
h)	Same way up	1	0.4
i)	Written clearly	1	0.4
j)	On this paper	13	5.1
k)	Part of a question, test	3	1.2
l)	Being looked at by the class	1	0.4
m)	Surrounded by words	1	0.4
n)	They both contain curves, loops	8	3.1

which occurs in all classifications and equivalences in mathematical experience. In this case the pupils were given the two numbers 16 and 36 and instructed to make as many different statements as they could beginning with the phrases, "They are both...", or, "They both...". To get them into the way of making statements like this an introductory exercise carried out by the whole class involved saying what was the same about two cartoon characters, a dog and a cat.

The 254 pupils taking this test once again showed that it was possible for 11 - 12 year old pupils to bring to bear a wide range of mathematical ideas on an open-ended situation such as this one. The summary in Table 4.41 demonstrates the divergent thinking of these pupils.

Analysis of the above responses indicates that on the whole the statements produced by these pupils about the two numbers 16 and 36 are varied, interesting and mathematically non-trivial. There were some statements, such as, "they are both multiples of 4" and "they are both even", which were given by most pupils. But there was a wide range of more original statements, which, by the criteria of Table 4.17, justified the assertion that this test was a valid assessment of divergent production in mathematics. Statements such as, "they are both greater than every number less than 16" and "they both have the sum of their digits less than 10", are immediately recognisable as showing some sort of ability worthy of the description 'creative'.

One obvious problem in marking the responses of these pupils to this test relates to the statements in category 13 in Table 4.41. Statements about the physical appearance of the numerals as they were written on the test paper, their colour, size and so on, are clearly not excluded by the constraints of the task as given to the pupils, so in one sense must be accepted, but in another sense their appropriateness in terms of divergent production in mathematics must be questioned. Fortunately few pupils gave more than two or three of such responses, with the statement about the two numbers both being in a box the only one given by a large proportion. The decision was made to accept up to three of such statements in calculation of the fluency score, but to give no bonuses for originality, because of the dubious appropriateness. This practice is in line with marking procedures adopted in other tests of this sort. It has already been noted that divergent production tasks will almost inevitably produce problems related to the appropriateness criterion, but the occasional element of subjectivity in assessment is a small price to pay for giving the pupils the opportunity to produce responses as original and creative as some of those seen in these tests.

The only category of responses where pupils gave superfluous responses was category 9. Occasionally it was found that a pupil would cover the answer paper with statements all beginning, "they are both greater than...". The usual practice of accepting no more than three responses in each of categories 9(a), 9(b) and 9(c) was adopted. Originality bonuses were awarded in the usual way for categories used by less than 10% (one mark), 5% (two marks) and 2% (three marks), with the exception of category 13. Table 4.42 gives a summary of the overall scores achieved by the pupils taking this test.

The number of acceptable statements produced by the 254 pupils taking this test ranged from 0 to 12, with 5.5 being the mean number of statements. These fluency scores are summarised in Table 4.43. With originality bonuses added the overall scores for Test 5(a) ranged from 0 to 20.

Table 4.42

Summary of Scores on Test 5a (Similarities: numbers)

	Number	Mean score	standard deviation
Boys	138	6.6	3.6
Girls	116	6.3	3.3
All	254	6.5	3.5

Table 4.43

Frequencies of Various Numbers of Statements Produced by Pupils (Fluency Scores) in Test 5a (Similarities: numbers)

No. of statements	0	1	2	3	4	5	6	7
No. of pupils	3	6	9	24	45	53	40	25
%age	1.2	2.4	3.5	9.4	17.7	20.9	15.7	9.8
No. of statements	8	9	10	11	12			
No. of pupils	16	14	9	5	4			
%age	6.3	5.5	3.5	2.0	1.6			

Test 5b: Similarities: (Shapes). The second part of Test 5 was a divergent production task involving redefinition in a spatial domain. The task was identical to the numbers part of this test except that the pupils were given two shapes. Thus they were required to make statements about the ways in which the two shapes given in Figure 4.23 are the same.

The wide range of statements produced by the 252 pupils taking this test is summarised and categorised in Table 4.44. Once again great ingenuity was shown by some pupils in producing many and varied answers.

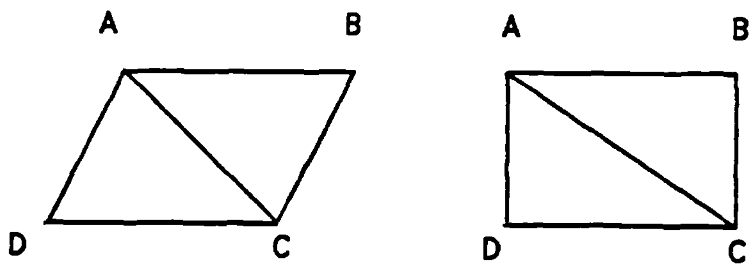


Figure 4.23. Two shapes used in Test 5b. (Pupils were required to state in what ways these two shapes are the same).

Table 4.44

Acceptable Responses to Test 5b (Similarities: shapes)

Statement	No. of pupils	%age
1. Quadrilaterals 'fourness' They both:		
a) are quadrilaterals	37	14.8
b) have four sides	122	48.8
c) have four corners (angles/points)	73	29.2
2. The triangles. They both:		
a) contain two triangles	186	74.4
b) have two triangles same size (area)	6	2.4
c) have two triangles same type, positions	2	0.8
d) have triangles ABC and ADC	2	0.8
e) have triangles with non-equal sides (scalene)	3	1.2
f) would have four triangles if BD were drawn	2	0.8
3. Letters. They both:		
a) have A, B, C, D on the corners	136	54.4
b) have A, B, C, D in same positions (and many similar statements)	34	13.6
c) have a side called AD etc.	4	1.6
4. Parallelism. They both:		
a) are parallelograms	24	9.6
b) have parallel sides	30	11.9
c) have two pairs of parallel sides/have top parallel to bottom have opposite sides parallel/have AB, DC parallel and AD, BC parallel	15	6.0
5. Properties of sides. They both:		
a) have two pairs of equal (opposite) sides	10	4.0
b) have AB = DC (top = bottom) two sides of length 3 cm	12	4.8
c) have AD = BC	1	0.4
d) two long, two short sides/unequal sides	13	5.2
e) contain horizontal lines	4	1.6
f) have both DC and AB horizontal	1	0.4
6. Measurements held in common. They both:		
a) have AB (top) same length (3 cm)/have DC (bases) same length (3 cm) are the same length (3 cm)	43	17.1
b) are same size/area (6 cm^2)	63	25.2
c) occupy the same amount of paper	1	0.4
d) are same height (2 cm)	15	6.0
e) have sides less than 10 cm	1	0.4
7. Mere possession. They both have:		
a) sides, angles, corners, points, area, a base (properties covered by more precise statements elsewhere)	26	10.3
b) a base, height and width	1	0.4
c) a centre	1	0.4
d) a perimeter (not specified)	2	0.8
8. Diagonals. They both:		
a) have a diagonal (line across the middle)	142	56.8
b) have a diagonal AC (line from A to C)	31	12.4
c) are cut in two bits, halved, by AC (a diagonal)	34	13.6
d) do not have a diagonal BD drawn	3	1.2
9. Straightness. They both:		
a) are made up of straight lines	13	5.2
b) contain no curved lines	2	0.8
c) are made by using a ruler	1	0.4
10. Angles. They both contain:		
a) 6 angles	5	2.0
b) 8 angles (better answer)	2	0.8
c) angles which sum to 360°	12	4.8
d) angles greater than 10° etc	1	0.4
e) angles greater than 89.999°	1	0.4
f) angles less than 90° etc	4	1.6
g) angles ABC,ADC, etc	1	0.4
h) equal opposite angles	5	2.0
11. Transformations.		
a) they are both squares seen from an angle	1	0.4
b) they both were the same shape before...	1	0.4
c) they both can be made into the same shape...	1	0.4
d) they both will tessellate	1	0.4
12. they both contain 5 lines	31	12.4
13. they both contain three shapes	2	0.8
14. they are both not squares, circles etc	8	3.2
15. they both can be lids of boxes	1	0.4
16. they are both shapes	32	12.8
17. Physical appearance (non-geometric properties) They both:		
a) are drawn in black ink	24	9.6
b) are in this question	4	1.6
c) are on white background	7	2.8
d) are something to do with maths	2	0.8
e) are on this paper	10	4.0
f) are being looked at, thought about	2	0.8

Once again the opportunity to redefine in different ways the constituent elements in the given situation by means of seeking similarities between two different entities has provided the pupils with a task in which they are able to demonstrate their ability to apply a wide range of mathematical ideas in a divergent and creative way. There were a number of simple similarities which were given by most pupils, such as statements about the shapes both having four sides and both containing two triangles. But amongst the more original responses a number of surprising and imaginative statements were made, such as, "they both contain angles greater than 89.999^{o}" and "they will both tessellate". There were, of course, a number of incorrect statements which had to be deleted, the most common of these assuming that AD was the same length in both shapes. There were only a few instances of superfluous repetitions of the same idea, in particular in categories 3 (b) and (c), 10 (g) and 14. In these cases the usual practice of allowing no more than three of each was adopted. Responses in category 16 dealing with non-geometric properties of the physical appearance of the shapes were treated in the same way as the similar collection of responses about the physical appearance of the numbers used in part (a) of Test 5.

Originality bonuses were then added for those statements made by fewer than 10% (one mark), 5% (two marks) and 2% (three marks). The overall scores of the 252 11 - 12 year old pupils taking this test are summarised in Table 4.45.

The fluency scores for this test, that is the number of acceptable responses remaining after the deletion of incorrect, superfluous and inappropriate responses, ranged from 0 to 12, with 5.5 being the mean fluency score. These figures are, interestingly, identical to those quoted above for Test 5(a) (Similarities: numbers). The fluency scores for the 252 pupils taking Test 5(b) are summarised in Table

4.46. With originality bonuses added the overall scores for this test ranged from 0 to 18.

Table 4.45

Summary of Scores on Test 5b (Similarities: shapes)

	Number	Mean score	Standard deviation
Boys	137	6.7	3.7
Girls	115	6.5	3.3
All	252	6.6	3.5

Table 4.46

Frequencies of Various Fluency Scores for Test 5b

(Similarities: shapes)

No. of statements	0	1	2	3	4	5	6	7	8
No. of pupils	10	7	12	23	31	36	50	34	24
%age	4.0	2.8	4.8	9.1	12.3	14.3	19.8	13.5	9.5

No. of statements	9	10	11	12
No. of pupils	15	6	2	2
%age	6.0	1.2	0.8	0.8

Divergent Production Tests Not Used in the Final Analysis

There were four tests included in the battery administered to the sample of children used for the main piece of research which were designed with a divergent production construct in mind, but which were discarded for not meeting the criteria laid down in Table 4.17.

Test 6 (Nine-dot routes). This test was intended to be a problem-solving task in a spatial domain. It required the pupils to find as many different routes as possible from one point to another on a nine-dot square grid by using straight line paths joining dots to dots. (See the copy of the test in Appendix 1). This was designed as a problem-solving task in a spatial domain, with many possible solutions, but proved to be unsuccessful.

It was expected that certain categories of routes, such as those involving anti-clockwise initial moves (as opposed to the clockwise move in the example given to the pupils), those involving doubling back at angles of 45° or 135°, and those involving paths between non-adjacent dots, would prove to be more difficult to find and hence meet the criterion for originality. In fact this did not occur. There were not sufficient routes falling into the categories of being found by less than 10%, 5% and 2% of the pupils, to give credit for originality. Most pupils found most routes. This test failed to meet criterion (5) in Table 4.17, as well as criterion (1), and hence was not used in the final analysis.

Test 13 (Classroom). This test was intended to be a problem-posing item, in a mainly spatial domain. Pupils were invited to make up questions which they might want to answer if they were doing a mathematics project 'all about your classroom'. This test was not successful as a problem-posing task for assessing divergent production because it was too open-ended. Most pupils gave a large number of different questions concerned with measuring and counting different aspects of the classroom. It proved to be impossible to categorise their responses and to identify those which showed creative mathematical thinking. The less frequently given responses did not have face validity for indicating mathematical creativity. This test did not satisfy criterion 6 in Table 4.17, and hence was not used in the final analysis. It is important in setting open-ended tasks for assessing divergent production to include sufficient constraints within the tasks to demand the application of non-trivial mathematical thinking in order to produce other than the obvious responses to the task.

Test 17 (What can you see?). This was designed as a redefinition task in a spatial domain. The same difficulty as experienced in Test 13 above was apparent here. The pupils were simply asked to write down everything they could think of to say about a given diagram. (See the copy of this test in Appendix 1.) Once again it proved a near impossible task to categorise their responses simply because the situation was too lacking in constraints. There were some interesting and mathematically valid statements made, but too many of the less frequently used statements were not significant enough to satisfy criterion 6.

Test 19 (Factory). This test posed a situation describing wages and overtime rates for employees in a factory, and invited the pupils to make up problems about this factory. The test was derived from the 'Make-up-Problems' test of divergent production used by Getzels and Jackson (1962) and was intended to be a problem-posing task in a numerical domain. The responses of the pupils to this task failed to satisfy criterion 1, in that a wide range of mathematical ideas was not brought to bear upon the situation. The responses were almost entirely limited to calculations involving addition, multiplication, and to some extent subtraction and division involving money and time. Divergent thinking may be shown by this task, but the limitation in the responses to basic arithmetic calculations does not suggest that it could be rightly associated with mathematical creativity.

Table 4.47

Summary of Results on Tests of Divergent Production

Test	Mean score	Standard deviation	Range of scores obtained
	Problem-solving		
7 (Nine-Dot Areas)	6.7	4.1	0 - 39
16 (Results)	6.2	4.5	0 - 27
18 (Three Cards)	8.3	5.9	0 - 31
	Problem-posing		
2 (Counters)	11.4	5.6	1 - 35
11 (Cross-number)	8.4	5.1	0 - 32
12 (Scattergram)	6.4	5.3	0 - 34
	Redefinition		
3 (Subsets)	8.4	6.2	0 - 32
4 (Shape-Finding)	10.7	6.3	0 - 44
5a (Similarities: numbers)	6.5	3.5	0 - 20
5b (Similarities: shapes)	6.6	3.5	0 - 18

Summary of Results for Assessment of Pupils' Ability for Divergent Production in Mathematics

Ten tasks have now been considered as possibly valid tests of divergent production in mathematics. Three of these have been constructed as essentially problem-solving tasks, three involve problem-posing, and four are concerned with the notion of redefinition. Six of the tasks are based on numerical and four on spatial domains. Analysis of the responses of approximately 250 11 - 12 year old pupils to each of these tasks has provided support to the assertion that performance on these tests may be an indication of mathematical creativity.

Table 4.47 summarises the means and standard deviations for these ten tests. It is intended in the subsequent analysis of creative behaviour in mathematics to combine the pupils' scores on these ten tests into an aggregate score for divergent production, giving equal weighting to each of them. This forms part of the quantitative analysis of pupils' scores on the battery of mathematical creativity tests undertaken in the next chapter.

CHAPTER 5

ANALYSIS OF MATHEMATICAL CREATIVITY TEST DATA

This chapter describes how the scores obtained by the 11 - 12 year old pupils on the battery of mathematical creativity tests considered in Chapter 4 were combined to give aggregate scores for overcoming fixation (OF) and divergent production (DP) in mathematics. A further measure used in the analysis is the mathematics attainment score (MA) obtained from the NFER EF test. Correlations between these three measures, OF, DP and MA, and the corresponding scattergrams are discussed in order to determine any relationships between them. Consideration is then given to pupils' performances on numerical and spatial domains separately, and to any discernible group differences between boys and girls on the mathematical creativity measures. Finally the pupils' scores on the problem-solving test are considered in relation to OF, DP and MA scores.

Aggregate Scores for Overcoming Fixation (OF)

Table 4.16 gives a summary of the mean scores obtained on the five tests related to overcoming fixation in mathematics which were found to meet the stated criteria. Each of these tests was scored out of a possible maximum score of ten marks. For the subsequent analysis aggregate scores for overcoming fixation in mathematics were obtained as follows. Each pupil's scores on these five tests were simply added, and then standardised using the formula:

$$x' = 100 + 15\,(x - \bar{x})\,/s$$

(where x' is the standardised score, x the raw score, $\bar{x}$ the mean score and s the standard deviation of the raw scores) to produce a set of scores for overcoming fixation (OF scores) in mathematics with mean 100 and standard deviation 15.

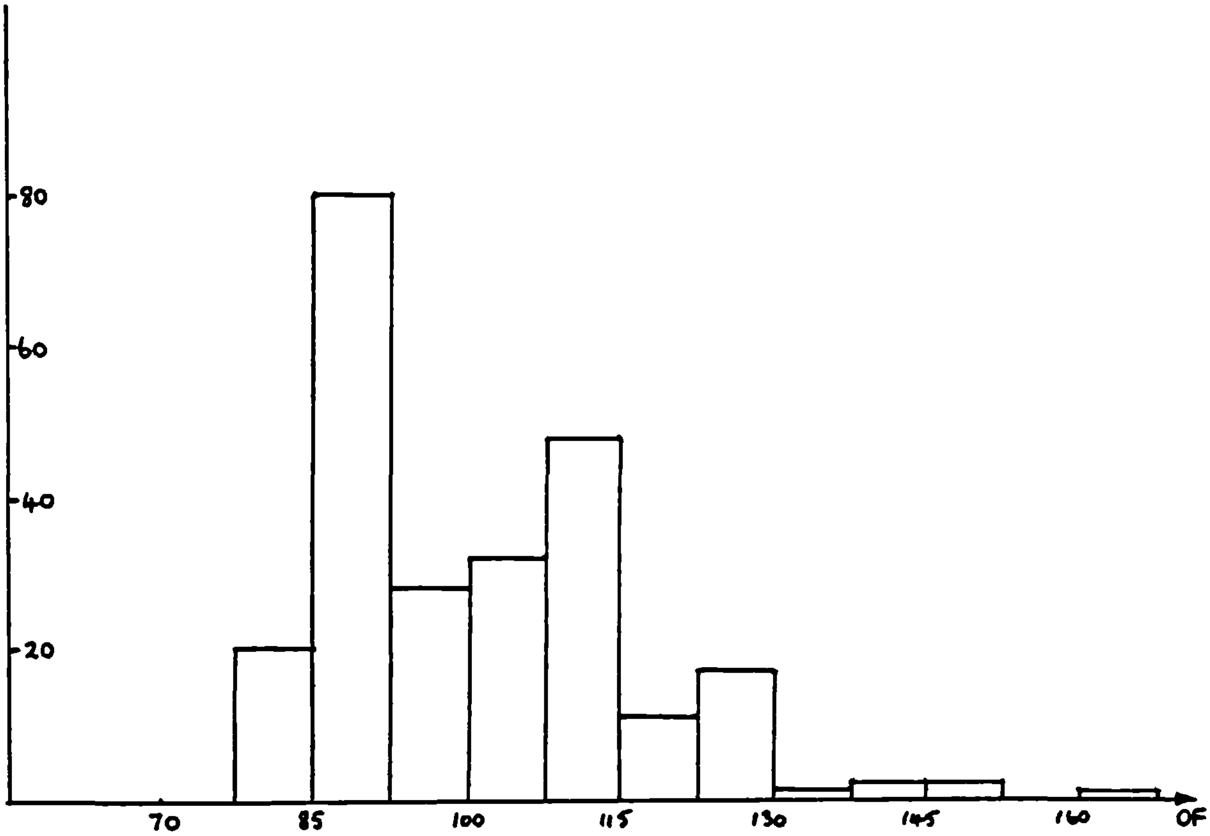


Figure 5.1. Distribution of OF scores.

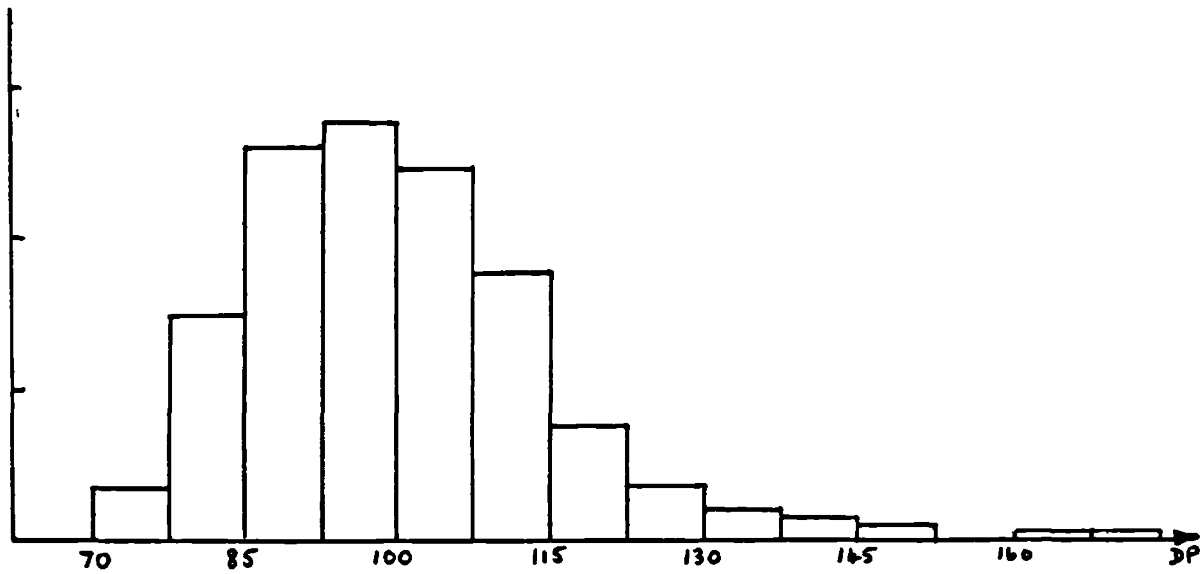


Figure 5.2. Distribution of DP scores.

The resulting distribution of OF scores is shown in Figure 5.1. It is clearly skewed to the right, as a consequence of the wide range of mean scores in the five contributing items (0.08 - 6.2). The large concentration of pupils in the 85 - 92.5 range of OF scores is made up almost entirely of pupils who by virtue of succeeding in only the first two parts of the Areas test and failing in all other OF tests finished up with an OF score of 85.4.

Appendix 4 gives a complete listing of the scores obtained by all pupils on the individual OF tests and the corresponding overall OF scores. (Note: where a pupil was absent for one of the five OF tests an estimated overall OF score was obtained by averaging the scores of those pupils who obtained identical results on the four tests for which the pupil was present. A pupil who was absent for two or more OF tests did not receive an overall OF score.)

Aggregate Scores for Divergent Production (DP)

The ten tests of divergent production in mathematics which were judged to meet the stated criteria, and which are summarised in Table 4.47, were not marked out of a set maximum score. In fact the maximum scores obtained by the pupils on these tests ranged from 18 to 44. Hence simple addition of raw scores would not be an appropriate way of combining them. Consequently a single, overall score for divergent production (DP) was obtained as follows. First the scores on each DP test were standardised using the formula stated above, so that each test had the same mean and the same standard deviation. Then they were added, and finally the aggregate scores were restandardised to give a mean of 100 and a standard deviation of 15. This overall DP score then gave equal weighting to each of the contributing items. The procedure adopted for combining the scores on the individual DP tests made two assumptions: (a) that the distributions of the scores on the tests being combined were similar in shape, and (b) that the contributing tests were all measuring some aspects of the same ability. The first assumption is justified to some extent by examination of the distributions shown in Figure 5.3. All the distributions are clearly skewed to the right, although the concentrations in the lower ranges of scores are somewhat variable. The skewness is a consequence of the marking scheme adopted which gave larger bonuses for the less frequent, more original ideas used. The second assumption is justified both by the qualitative evaluation of the pupils' responses to the DP tests undertaken in Chapter 4 and also to some extent by the consideration of correlations between the ten DP tests discussed below.

The distribution of overall DP scores obtained by the pupils is shown in Figure 5.2. Appendix 5 gives a complete listing of the scores obtained by all pupils on the individual DP tests and the corresponding overall DP scores. (Note: where a pupil was absent for up to four out of the ten DP tests an estimated overall DP score was obtained in the same way by giving equal weighting to each of the scores on the tests for which the pupil was present. No overall DP score was given to a pupil who was absent for more than four of the ten tests of divergent production).

Correlations Between DP Tests

Table 5.1 shows the correlation coefficients obtained for the whole sample of pupils (this number varying between 216 and 255 for the different pairs of tests) between the ten divergent production tests. All these coefficients are significant at the 1% level, a necessary condition for aggregating the scores on these tests into a single DP score. Although a few of these correlation coefficients are disappointingly low, such as 0.20 between Test 5a (Similarities: numbers) and Test 7 (Nine-dot areas), none of the tests correlates

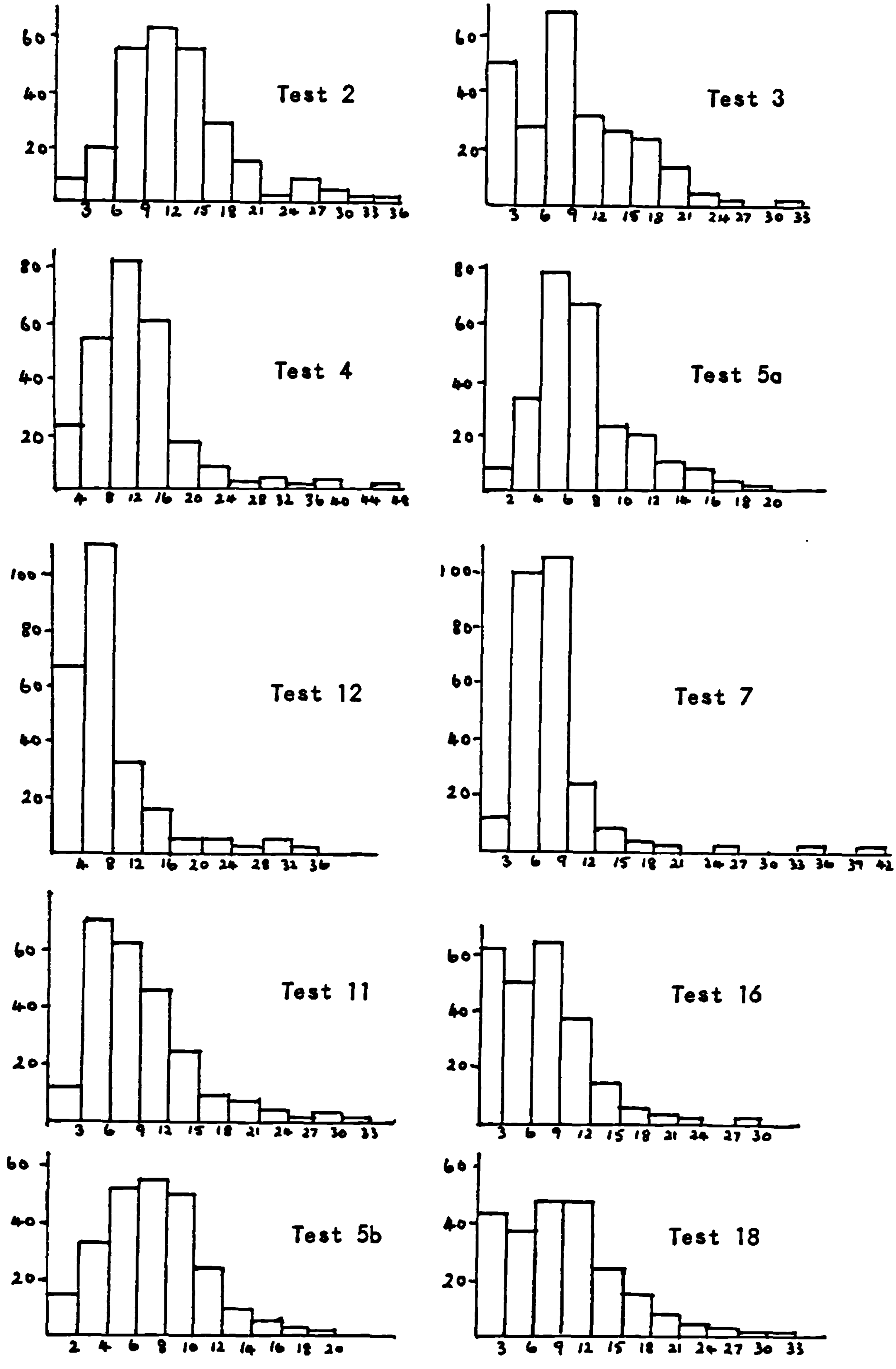


Figure 5.3. Distributions of scores on ten DP tests.

so poorly with the others that it should be withdrawn from the battery. Table 5.2 in fact demonstrates this with the correlations between the individual DP tests and the aggregate DP scores. Divergent production tests in general do not have an outstanding record of reliability as indicated by correlation coefficients between different divergent production tasks within a battery. For example, Getzels and Jackson (1962) obtained correlations varying between 0.17 and 0.37 for the five tests in their battery. Wallach and Kogan (1965) report average correlations of 0.21 for boys and 0.24 for girls in the tests used by previous researchers. They themselves, and likewise Hasan and Butcher (1966), obtained correlation coefficients of around 0.4 by giving attention to the conditions under which the tests were administered, aiming for a 'game-like' atmosphere. Haylock (1977) obtained coefficients of around 0.5 for scores on two divergent production items in mathematics with 128 pupils. Nuttall (1971) obtained correlation coefficients as low as 0.12 between the mathematical divergent production tests used in his research. In view of such previous results, the correlations obtained in Table 5.1 with the 45 coefficients having a mean of 0.39 for about 240 pupils, are satisfactory and sufficiently high to support the assumption implicit in combining the scores. If the ten tests are split into two groups as follows:

2 (Counters)	4 (Shape-Finding)
3 (Subsets)	5a (Similarities: no.)
5b (Similarities: shape)	7 (Nine-Dots areas)
12 (Scattergram)	11 (Crossnumber)
16 (Results)	18 (3 Cards)

so that spatial and numerical tests are shared equally between the two groups, as are, as far as possible, tests dealing with problem-solving, problem-posing and redefinition, then a split-half coefficient of reliability of 0.89 is obtained for the battery of DP tests.

Table 5.1

Correlation Coefficients Between Ten Divergent Production Tests

DP Test number	2	3	5a	11	16	18	4	5b	7	12
2	.	35	37	37	33	33	43	41	38	42
3	35	.	51	44	46	48	35	49	33	44
5a	37	51	.	37	33	42	32	56	20	31
11	37	44	37	.	40	25	38	44	42	47
16	33	46	33	40	.	46	32	40	39	48
18	33	48	42	25	46	.	35	40	24	26
4	43	35	32	38	32	35	.	43	50	41
5b	41	49	56	44	40	40	43	.	38	37
7	38	33	20	42	39	24	50	38	.	57
12	42	44	31	47	48	26	41	37	57	.

numerical domains (columns 2–18; rows 2–18) — spatial domains (columns 4–12; rows 4–12)

(decimal points omitted)

Mathematics Attainment of the Sample

Prior to the testing programme all the schools had recently administered to their pupils the NFER EF test of Mathematics Attainment, as part of a local authority policy of assessment in the final year of the Middle School. A copy of this test is reproduced in Appendix 1. The NFER EF test is a norm-referenced, standardised test of mathematical attainment. It covers five areas of content: number (concepts), number (operations), space, tabular/geographical representation, and geometry, of which the two number categories contain half the questions. The test is also analysed into behaviour categories: knowledge, technique and skill, comprehension (translation, interpretation, extrapolation), and application, of which knowledge, technique and skill amount to 60% of the test. In any analysis of categories of mathematical behaviour, such as that of Hollands (1972) or Wood (1968), the behaviours assessed by this test would constitute aspects of mathematics which have a lower hierarchical position than creativity or inventiveness. Hence it would be expected that a pupil's

Table 5.2

Correlation Coefficient Between Scores on Individual DP Tests and Overall DP Scores

Test	Correlation coefficient
2 (Counters)	0.66
3 (Subsets)	0.72
5a (Similarities:number)	0.66
11 (Crossnumber)	0.67
16 (Results)	0.68
18 (3 Cards)	0.62
4 (Shape-Finding)	0.66
5b (Similarities:shape)	0.72
7 (Nine-dots areas)	0.65
12 (Scattergram)	0.69

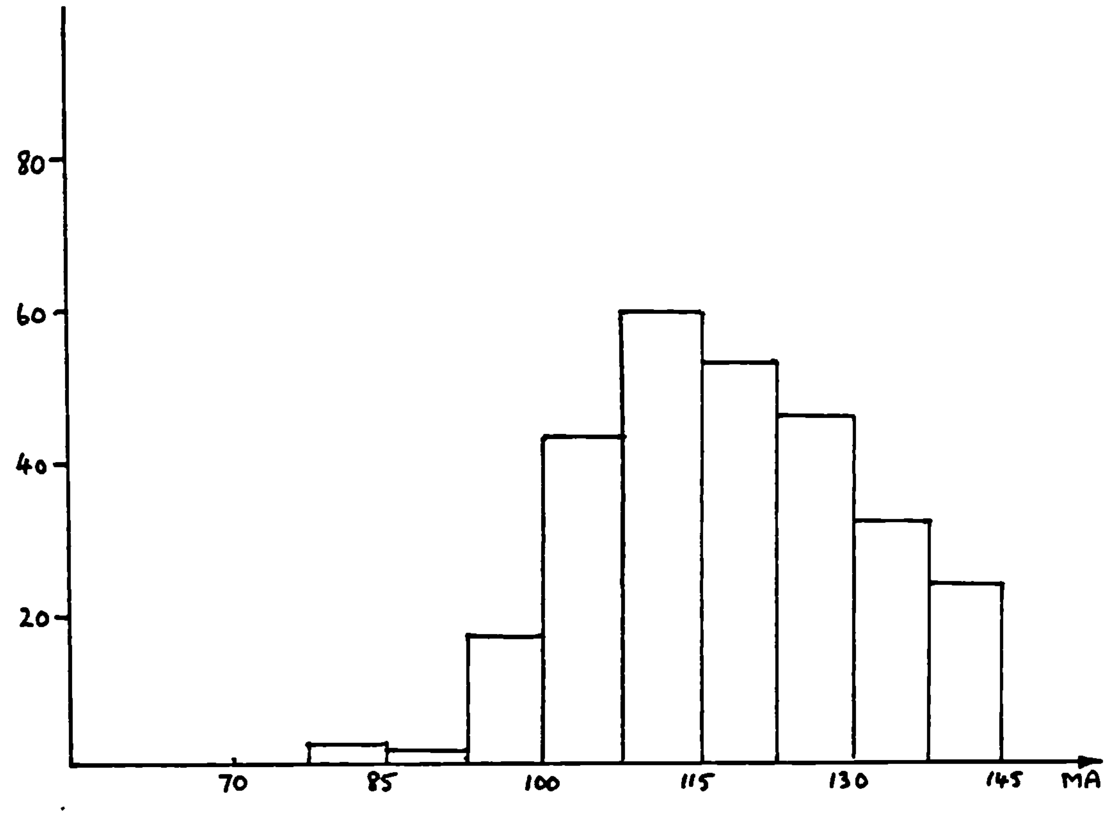


Figure 5.4. Distribution of MA scores.

level of mathematics attainment as measured by this NFER EF test would be a limiting factor on performance in the mathematical creativity tests. (This suggestion is considered further in the discussion of correlation coefficients and scattergrams below). Consequently the standardised scores (MA scores) on this test are used in the subsequent analysis to sort the pupils into bands of similar levels of mathematics attainment. In particular the performances on mathematical creativity tests of the four groups of pupils given in Table 5.3 will be considered.

Table 5.3

Four Mathematics Attainment Groups

A.	very high attainers	$MA \geq 130$	56 pupils
B.	above average attainers	$115 \leq MA < 130$	99 pupils
C.	average attainers	$100 \leq MA < 115$	103 pupils
D.	below average attainers	$70 \leq MA < 100$	22 pupils

The distribution of MA scores, shown in full in Figure 5.4, confirms that in terms of national norms the sample was top-heavy in mathematics attainment.

Relationships Between OF, DP and MA Scores

Table 5.4 gives the correlation coefficients obtained between the three measures, OF, DP and MA. Figures 5.5, 5.6 and 5.7 are scattergrams showing the relationships between these measures.

Figures 5.6 and 5.7 are markedly similar. Horizontal lines have been drawn on these two scattergrams to divide the sample into the four mathematics attainment groups defined in Table 5.3. These show clearly the way in which the level of mathematics attainment has some sort of limiting effect upon the performance in the mathematical creativity tests, as was suggested might be the case earlier. Pupils with higher levels of mathematical attainment would be expected to have a more secure basis of mathematical skills and knowledge with which to think creatively in the OF and DP tests. Consequently the correlation coefficients of 0.65 between OF and MA, and 0.69 between DP and MA, are not surprising. The scattergrams show that the higher levels of mathematics attainment have larger ranges of OF and DP scores. This is indicated also by Table 5.5 which shows that as the level of mathematics attainment decreases, not only do the mean scores for OF and DP decrease, but also the standard deviations within each band of MA scores. Thus the largest variation in scores for OF and

Table 5.4

Correlation Coefficients Between OF, DP and MA

	Correlation coefficient	Number of pupils
OF/DP	0.59	238
OF/MA	0.65	241
DP/MA	0.69	259

DP is in each case with the very high mathematics attainment group (group A), and the smallest variation is in the below average mathematics attainment group (group D). This is not to suggest that mathematics attainment determines the performance of the pupil on the mathematical creativity tests, nor that these OF and DP tests are only of value for discriminating between pupils in the highest levels of mathematics attainment. On all four bands of attainment

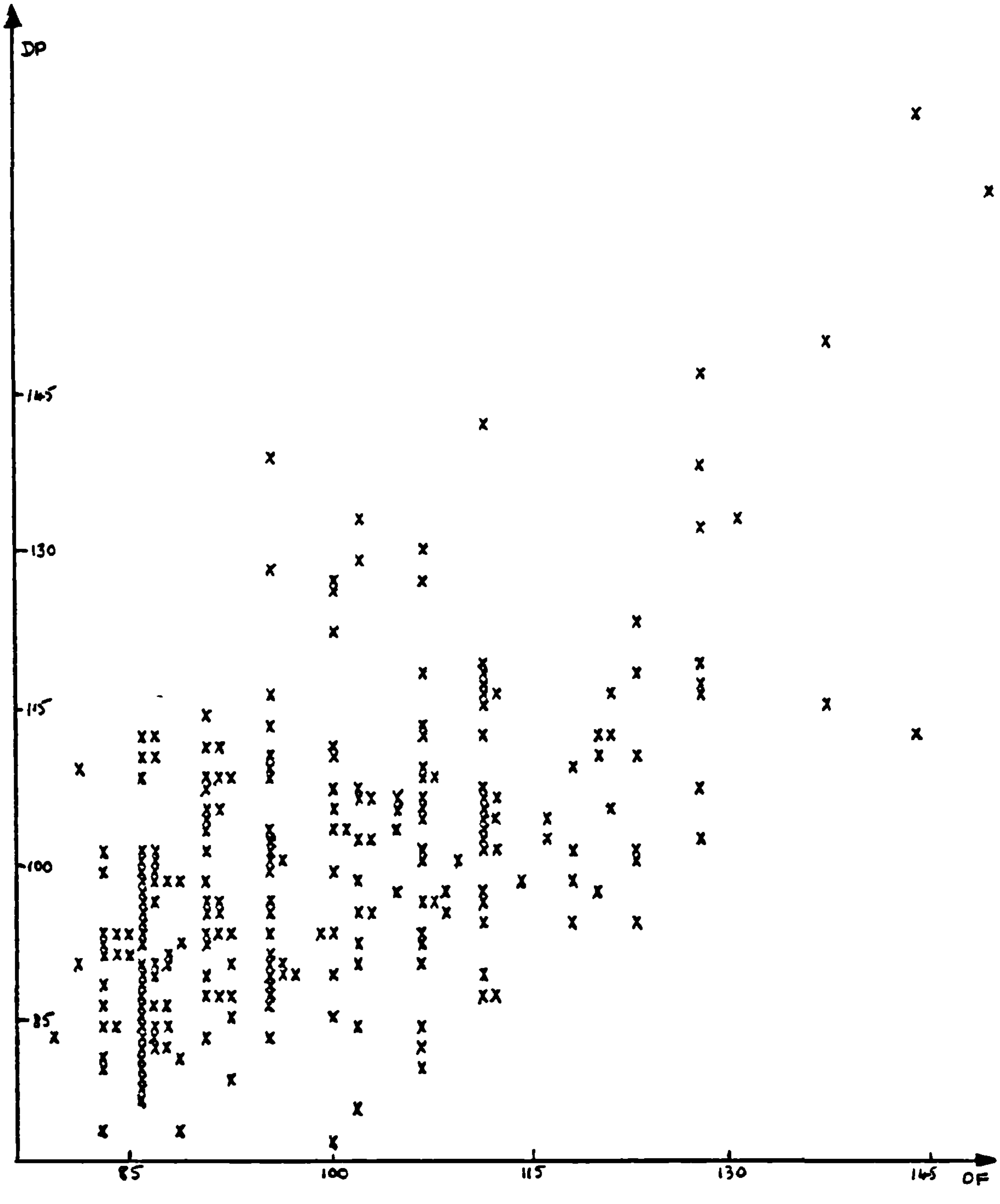


Figure 5.5 Scattergram showing relationship between OF and DP scores. (N.B. when points have coincided they have been plotted next to each other horizontally).

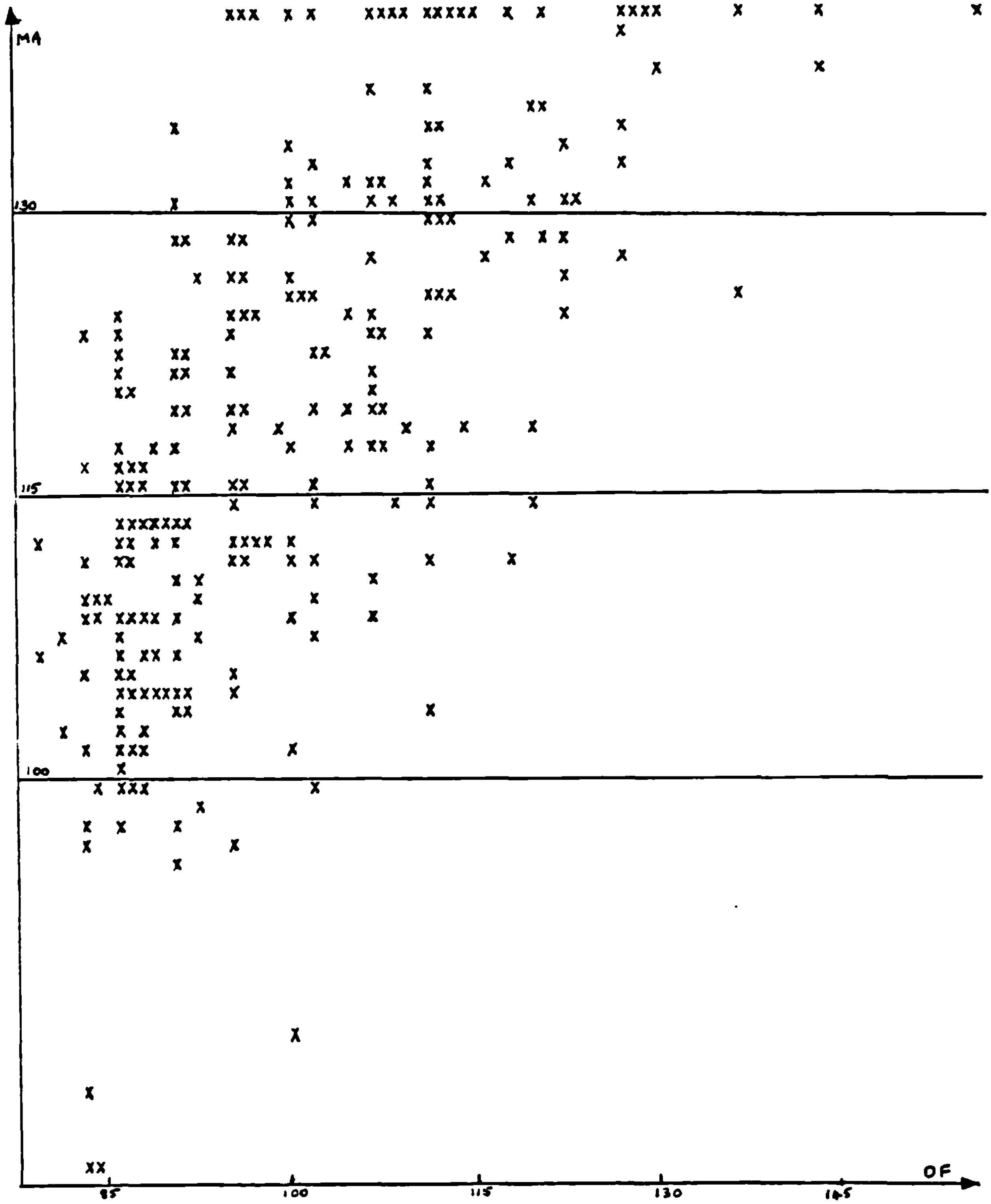


Figure 5.6. Scattergram showing relationship between OF and MA scores. (N.B. when points have coincided they have been plotted next to each other horizontally).

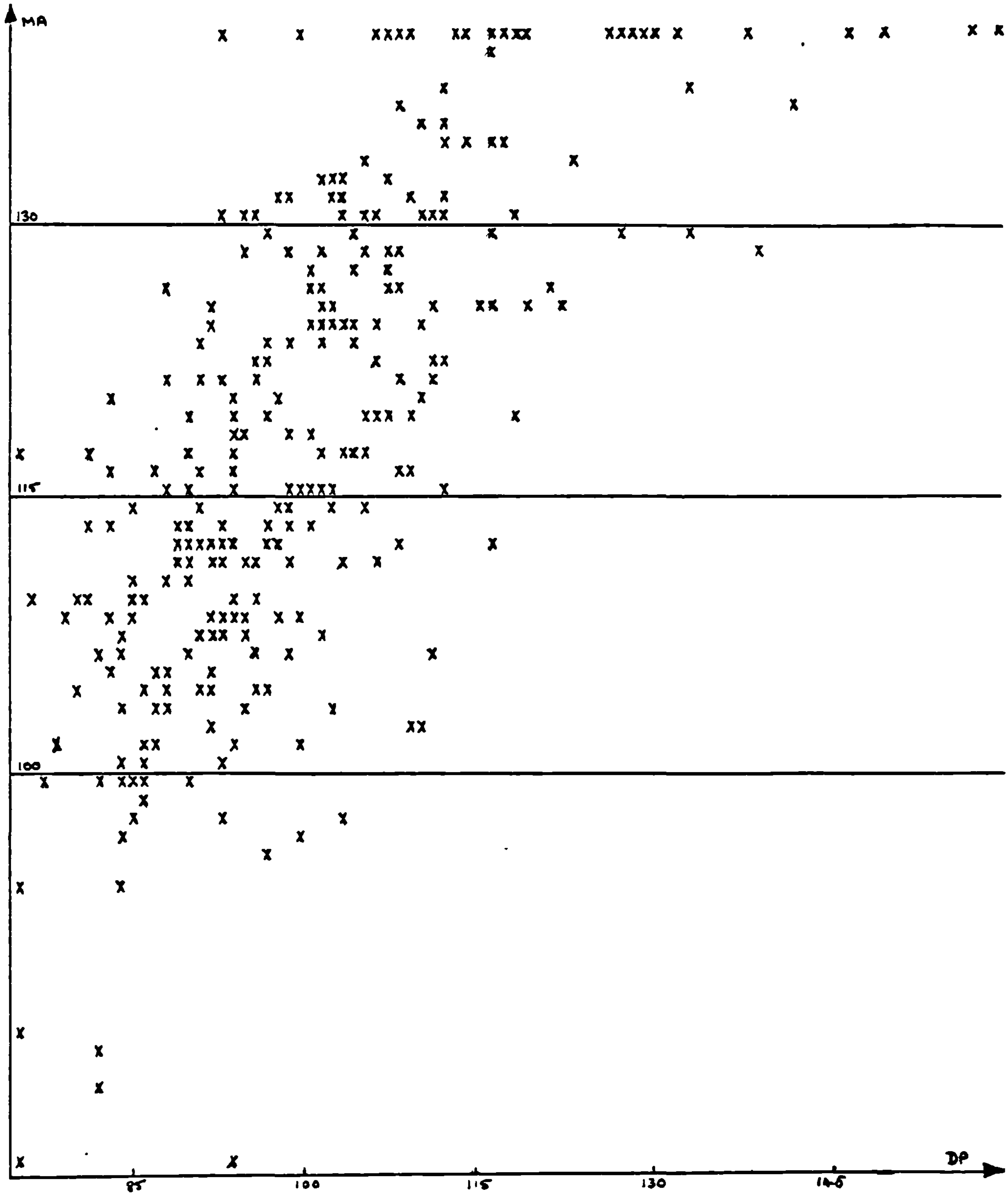


Figure 5.7. Scattergram showing relationship between DP and MA scores. (N.B. when points have coincided they have been plotted next to each other horizontally).

there are still considerable variations in OF and DP scores, as can be seen in the four sections of the scattergrams in Figure 5.6 and 5.7. But it is probably fair to conclude, as has been suggested in the conceptual analysis of the relationship between mathematical skills and knowledge and mathematical creativity, that the correlations and scattergrams obtained support the view that mathematical attainment has a limiting effect on the performance in the mathematical creativity tests. The higher the level of attainment the more possible it becomes to discriminate between pupils on the basis of their abilities to overcome fixation and for divergent production in mathematics. A similar relationship has been noted by many previous researchers (e.g. Haddon and Lytton, 1968): that creativity in general (as measured by general divergent production tests) and intelligence become progressively more distinct as one moves up the scale of general ability.

Table 5.5

Means and Standard Deviations of OF and DP Scores for Four Mathematics Attainment Groups

Mathematics attainment:	OF Mean	OF St. d.	OF (No.)	DP Mean	DP St. d.	DP (No.)
A. very high	115.3	14.3	(56)	115.7	16.7	(56)
B. above average	100.7	12.3	(87)	101.8	11.0	(92)
C. average	91.1	9.4	(82)	91.7	8.1	(91)
D. below average	88.1	6.5	(16)	85.2	8.1	(20)

Numerical/Spatial Aspects

There is no strong support from the results obtained in the testing programme for considering the numerical and spatial tests as two separate groups. For example the mean of the six correlation coefficients in Table 5.1 between pairs of divergent production tests with spatial domains is 0.44, which is slightly higher than the overall mean of 0.39. The mean of the fifteen correlation coefficients for pairs of tests with numerical domains is 0.39 also. If the scores on the six numerical divergent production tests are combined into a single score (NDP score), and likewise those on the four spatial divergent production tests (SDP score), then the correlation between these NDP and SDP scores is found to be 0.71. The scattergram in Figure 5.8 shows the relationship between these NDP and SDP scores for the pupils who took all ten tests. The scattergram indicates that pupils who perform poorly in numerical divergent production items tend to perform poorly in spatial ones also. There is a small number of pupils who excel in both. In view of the correlations obtained above between numerical and spatial tests, for subsequent purposes the ten tests will be considered as a single battery of tests of divergent production in mathematics. The reason for including both numerical and spatial domains in these tests, as with the tests of overcoming fixation, was to obtain composite measures representing performances on a wide range of creativity tasks across the mathematics curriculum of 11 - 12 year old pupils, but within the general constructs of overcoming fixation and divergent production.

Boy/Girl Differences

In terms of mathematics attainment, as measured on the NFER EF Test, there was no significant difference in the level of the boys and girls in the sample. The boys had a mean MA score of 117.5 with a standard deviation of 13.5. The girls had a mean MA score of 116.8 with a standard deviation of 12.3.

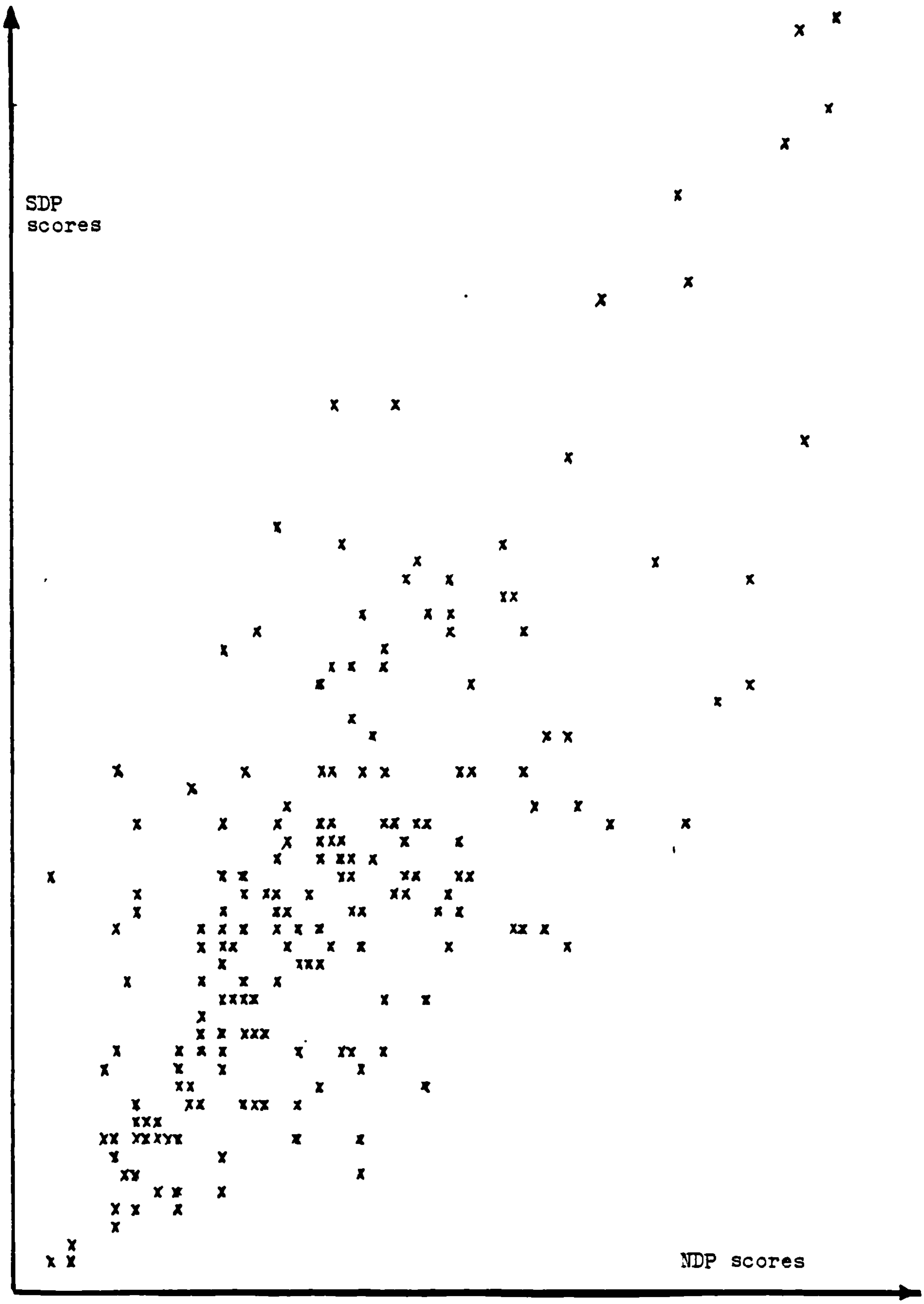


Figure 5.8. Scattergram showing relationship between overall scores on spatial divergent production tests and numerical divergent production tests. (N.B. where points have coincided they have been plotted next to each other horizontally).

Table 5.6

Means and Standard Deviations of Boys and Girls:

Overall OF and DP Scores

	OF scores		DP scores	
	Boys	Girls	Boys	Girls
Mean score	101.6	98.1	100.6	99.3
St. dev.	16.1	13.3	16.6	12.8
Number	133	109	142	119

On the mathematical creativity tests there was a slight tendency for boys as a whole to perform better than girls. This difference is analysed below.

In four out of the five OF tests and eight out of the ten DP tests the mean score for boys exceeded that of girls. The comparison between overall scores for boys and girls on the OF and DP tests is given in Table 5.6.

For the OF scores the difference in the means between boys and girls is equivalent to 1.84 standard error units, which is significant at the 10% level only. For DP scores the figure is 0.72 standard error units, which is not significant. However Table 5.7 indicates that in both OF and DP tests there is only any marked difference in performance between boys and girls in the higher group of mathematical attainment. In group A, the very high attainers, the performance of the 33 boys is better overall than that of the 23 girls in both OF and DP. The difference in the means for OF is significant at a 6% level and that for DP at a 1% level. However in the mathematical attainment groups B, C and D the performances of boys and girls as groups are very similar. Figures 5.9 and 5.10 show the distributions

Table 5.7

Means and Standard Deviations of OF and DP Scores for Groups of Boys and Girls of Varying Levels of Mathematical Attainment

	OF scores		DP scores	
	Boys	Girls	Boys	Girls
	A. very high attainers:			
Mean	118.2	111.1	119.8	109.8
St. dev.	14.2	13.5	18.8	10.6
(number)	(33)	(23)	(33)	(23)
	B. above average attainers:			
Mean	102.1	99.0	100.1	103.6
St. dev.	12.8	11.5	9.9	11.8
(number)	(47)	(40)	(48)	(44)
	C. average attainers			
Mean	91.1	91.1	91.1	92.0
St. dev.	9.5	9.2	7.8	8.4
(number)	(48)	(34)	(53)	(38)
	D. below average attainers			
Mean	87.4	88.3	84.4	85.8
St. dev.	7.4	6.0	10.9	5.4
(number)	(5)	(11)	(8)	(12)

for boys and girls separately for OF and DP scores respectively. It is clear from these graphs that the distributions of scores for OF and DP for boys and girls within the lower and middle ranges of

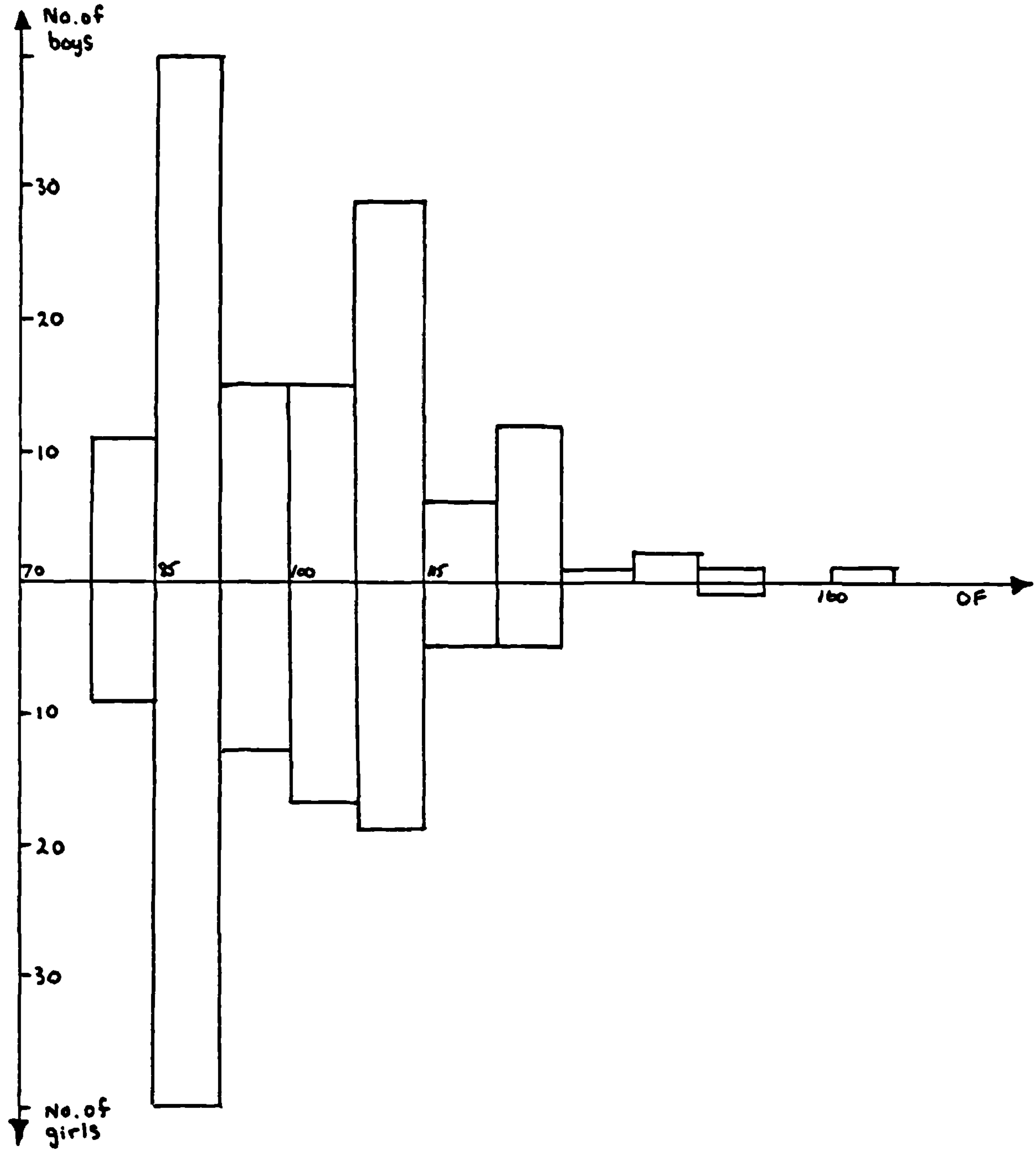


Figure 5.9. Distribution of OF scores for boys/girls.

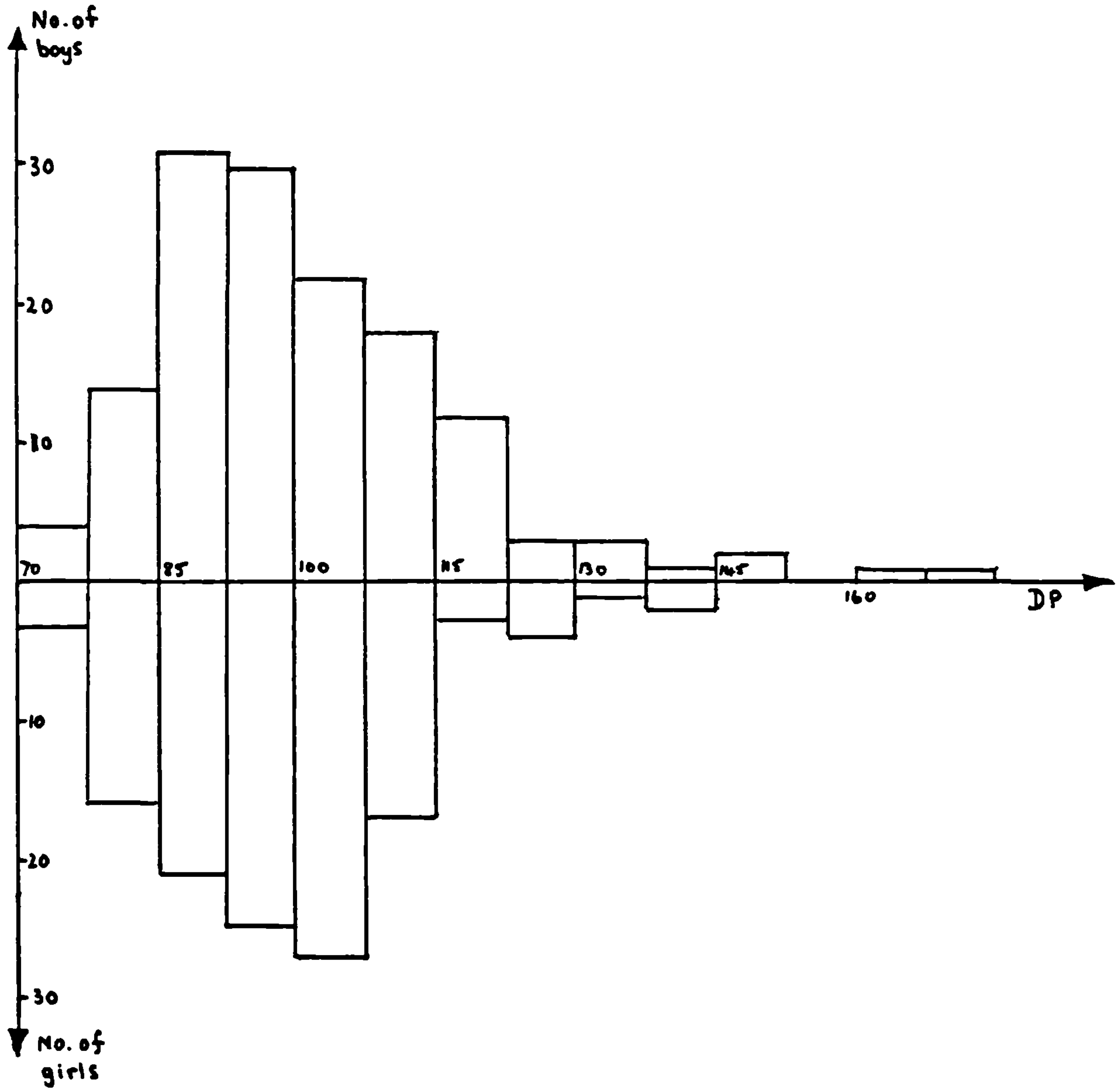


Figure 5.10. Distribution of DP scores for boys/girls.

mathematical creativity measures are very similar. Any difference which can be detected is near the top end of each graph. This is shown by consideration of proportions of pupils who achieved more than one standard deviation above the mean (i.e. more than 115) for their OF or DP scores. In fact, for OF scores, 17% of the boys achieved this level, compared with 10% of the girls. For DP scores, 16% of the boys compared with 8% of the girls came into this category.

So, to summarise, this analysis of group data suggests that only amongst the high attainers is there any discernible boy/girl difference in performance in the mathematical creativity tests. Then it appears that boys as a group score significantly more highly than girls. Furthermore the proportion of the boys in the most mathematically creative group in terms of both OF and DP scores is greater than the proportion of the girls.

Problem-Solving (PS)

Implicit in this investigation into two particular aspects of what has been termed mathematical creativity - namely, the ability to overcome fixation and the ability for divergent production - has been the assumption that these abilities might play an important role in creative problem-solving in mathematics. It would be expected that the more mathematically creative pupils as indicated by the battery of tests used would actually be more successful as problem-solvers.

Hence, to lend some support to this assumption and to give some degree of validity to the tests devised in this research programme, the pupils were given a problem-solving test. This appears as Test 20 in Appendix 1. (Questions 1 - 5: question 6 was used as part of the battery of items dealing with fixation). The five problems were adapted from some used by Krutetskii (1976), and chosen

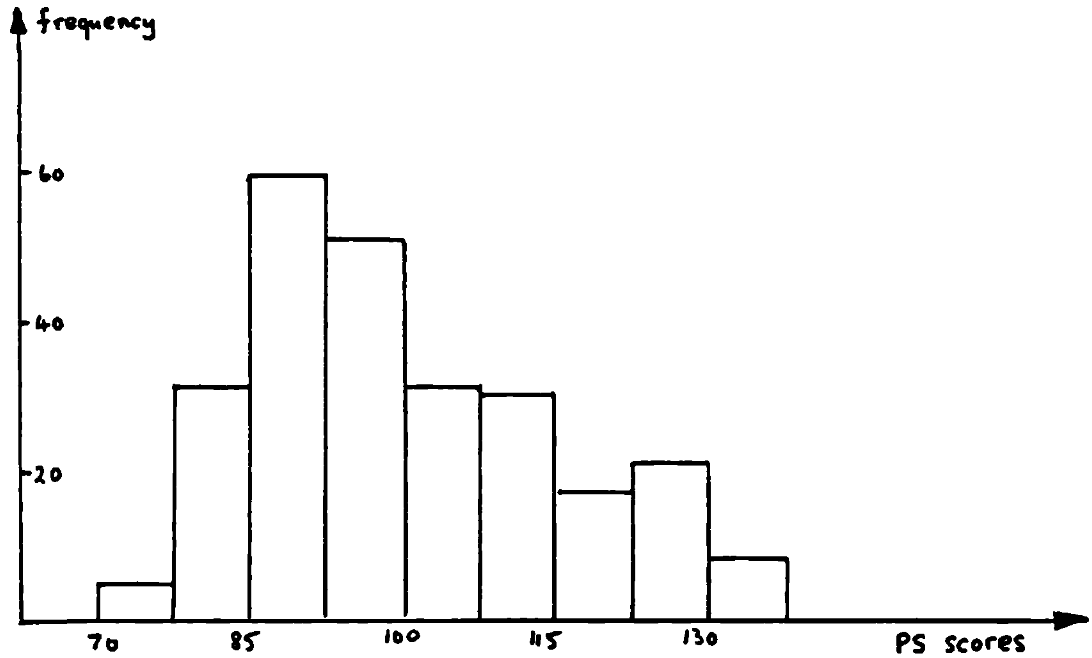


Figure 5.11. Distribution of (standardised) PS scores.

for one particular feature: all of them require nothing more than elementary arithmetic skills, but these skills have to be applied in unusual ways in order to solve the problems. Thus each problem requires flexibility of mental processes. By awarding six marks per question, with part marks where appropriate, a problem-solving score (PS score) was obtained for each pupil. Of course, problem-solving is a complex process to analyse; and breaking from mental sets and thinking divergently may contribute in only small ways to the total process for any given problem. But it would be surprising - and disappointing - if there was not some indication that the more mathematically creative pupils as defined by success in the OF and DP tests were not on the whole good problem-solvers.

In fact, correlation coefficients of 0.63 (231 pupils) between OF and PS scores, and 0.55 (235 pupils) between DP and PS scores, were obtained. These are, of course, highly significant, but no more so than the correlation coefficient of 0.67 obtained between mathematics attainment scores and PS scores.

It would be expected that all three scores, OF, DP and PS, would be related to the level of mathematics attainment, so the last coefficient quoted is not surprising. Higher coefficients between OF and PS, and between DP and PS, may have been expected from a theoretical standpoint, but in fact the distribution of the PS scores, shown in Figure 5.11, being dissimilar in shape to those of OF and DP (Figure 5.1 and 5.2), militates against this. The lack of homoscedasticity in these distributions makes the interpretation of a product-moment correlation coefficient a little doubtful.

However the problem-solving performances of the pupils in the sample will be additional information in the profile analyses of individual pupils to be undertaken in Chapter 7.

CHAPTER 6

THE INVESTIGATION INTO HYPOTHESES CONCERNING CHARACTERISTICS OF THE MATHEMATICALLY CREATIVE PUPIL

It has already been seen (Figures 5.6, 5.7 and Table 5.5) that within the various bands of mathematical attainment there were considerable spreads of mathematical creativity performance as indicated by OF and DP scores, though these were more marked in the higher attaining groups. The basic question to be considered in this chapter is the extent to which such a spread in mathematical creativity performance might be related to variability in such aspects of the pupils' personality traits as their willingness to take risks in mathematics, their nonconformity in mathematics, their tendency to narrow or broad categorisation, and such attitudinal factors as self-concept in mathematics, test anxiety and anxiety towards mathematics. For any analysis of such questions it will be necessary, because of the relationship between mathematical creativity and mathematical attainment, discussed in Chapter 5 to consider the behaviours of pupils within various bands of attainment.

Hypotheses

The following six hypotheses therefore were formulated in Chapter 3 and are to be examined in this chapter.

Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to:

1. be more inclined to take risks in mathematics;
2. be less conformist in their behaviour in mathematics;
3. be broader categorizers;
4. have higher self-concepts in mathematics;
5. have lower levels of anxiety towards mathematics;
6. have lower levels of test anxiety.

The pupils used for examining these hypotheses are the same sample of 11 - 12 year olds used for the main part of the research programme, described in Chapter 4. The level of mathematical creativity will be determined by performances on OF and DP tests. "Similar levels of mathematical attainment" will be taken to mean pupils within the same band of mathematics attainment as defined in Table 5.3. The various behaviours related to personality aspects included in the hypotheses were assessed in the research programme by means of a number of instruments, some newly devised and some already in existence, which are described and the results of which are analysed in this chapter. The instruments were included in the testing programme as Tests 21, 22, 23, 26 and 27. They are reproduced in Appendix 1, with full details of administration in Appendix 2.

In this chapter the investigation will be considered in two parts. The first will be concerned with hypotheses 1 and 2. No existing instruments for assessing willingness to take risks or nonconformity specifically in the context of doing mathematics could be found. These two hypotheses therefore were investigated by means of four somewhat arbitrary investigator-devised instruments. This part of the investigation is to be regarded therefore as only a preliminary and exploratory examination of the hypotheses. The second part of the chapter will consider hypotheses 3 - 6. These were investigated by means of (slightly modified) existing and well-proven instruments, and the conclusions drawn from these are less tentative.

The method used to examine the hypotheses in this chapter will be essentially to consider correlation coefficients within the various bands of mathematics attainment between scores on various tests. This leads to a tentative profile of a hypothetical mathematically creative pupil, based on group data. Chapter 7 then looks at individual profiles of high mathematically creative and low mathematically creative pupils arising from their performances on the various instruments described in this chapter.

Willingness to Take Risks and Nonconformity (Hypotheses 1 and 2)

Background

A summary of some of the relevant background to the present research, which was discussed in more detail in Chapter 3, is provided below.

Risk-taking. Pankove and Kogan (1968) have equated creativity with cognitive risk-taking. This is clearly a rich metaphor. To be creative, either in terms of overcoming fixation or in thinking divergently, it could be argued that a pupil must free himself from the inhibiting effect of customary modes of thought, thus risking the uncertainty of the unknown. Getzels and Jackson (1962) refer to this characteristic as typical of creative adolescents. Bruner (1960) talks of the child's need to free himself from the fear of making an error in order to make intuitive leaps in his thinking. Creativity literature abounds with such metaphorical images: leaping into the unknown, willingness to pursue unexplored territory, calculated risk-taking (e.g. McCleeland 1963), fearlessness of error and failure (Pankove and Kogan, 1968), personal impulsivity and daring (Barron, 1963), and so on. As has been noted in Chapter 3, Anderson and Cropley (1966) found creative 13 year olds to be more willing to take intellectual risks, and have suggested that the clearest barrier to creative thinking is an internalized stop-rule "don't take risks". The metaphor becomes even more convincing when considered in terms of mathematics. Fear of error and failure must be inevitably more acute in a subject in which a pupil is continually required to produce large numbers of answers to exercises, which are then subjected to the black and white, incontestable judgment of the teacher: "right" or "wrong". Pupils learn procedures and algorithms which bring them success on a range of exercises: to be prepared to depart from such safe procedures in favour of something more adventurous or unusual could be said to involve an element of cognitive risk-taking. Problem-solving in mathematics may involve the investment of time and effort into one particular approach or strategy which may not be leading to a solution: the decision to abandon this in favour of an alternative approach can be seen potentially to involve a willingness to take a risk. Such considerations as these led the present researcher to attempt to devise ways of assessing the willingness of 11 - 12 year old pupils to take risks - not risks in general, but specifically risks in the context of their doing mathematics. The tasks to be given to the pupils would therefore need to be perceived by them as mathematical exercises. Three instruments related to risk-taking were devised and included in the testing programme. (Tests 23, 27, 22).

Nonconformity. An operational definition of conformity is provided by Crutchfield (1962). A subject must make a judgment in which the subject's private conviction is clearly at variance with the group consensus. Then to express that judgment shows nonconformity, to agree with the consensus, conformity. Crutchfield provides evidence that creative adults tend to be more nonconformist than their peers. Barron (1963, chapter 14) also describes experiments linking creativity with nonconformity in this sense of showing independence of group judgments. MacKinnon (1962) found creative architects to be more ready to admit unconventional views. Yamamoto and Genovese (1965) have equated creativity in children with lack of conformity to group pressure. It is not surprising that many authors and researchers have linked creativity with nonconformity. It is to be expected that a person must be prepared to resist pressure to conform if he is to demonstrate original thought and to think divergently. Allen and Levine (1968) investigated the effect of a creativity training programme on 10 - 11 year old children's readiness to conform. In that investigation, which has been described more fully in Chapter 3, pupils were asked a series of questions requiring them to make judgments about both attitudes and facts. Later they were informed (falsely, on critical items) which were the most popular answers, and then asked to answer the questions again. Their subjective conformity (i.e. that related to the attitude items) and objective conformity (i.e. that related to the items of fact with one correct answer) were then assessed by the changes made in favour of the purported most popular answers. Allen and Levine found that a creative training programme reduced the objective conformity of the pupils, but not the subjective conformity. This study again links creativity with preparedness to resist pressure to conform in one's judgments, particularly objective, cognitive judgments.

The present research sought to investigate any such link in terms of mathematical creativity and nonconformity in mathematics. A similar assessment procedure to that used by Allen and Levine was devised but using mathematical questions. This appears as Test 26 (Multi-choice Test B) in Appendix 1.

<u>Instruments Used for Hypotheses 1 and 2</u>

To assess the pupil's willingness to take risks in mathematics it was considered necessary to devise situations in mathematics which contained the following elements. The pupil would be able to choose whether or not to take a particular action. The consequences of that action might be some desirable reward, but there is also the possibility of not achieving the reward or even obtaining some undesirable penalty. The pupil needs to be aware of both possibilities and of the risk involved in choosing action rather than inaction.

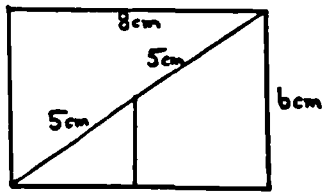
This description clearly could cover a very wide range of behaviours, and it should not be assumed that "willingness to take risks in mathematics" is in any sense a narrowly-defined tendency in pupils. The types of risk which pupils might take, even when restricted to the context of their doing mathematics, may be very different from each other. This is because of the potential variability in the elements in the description of a risk-taking situation given above. For example, the natures of the reward and penalty are variables. They may be essentially extrinsic, such as winning or losing a prize, or gaining teacher's approval or disapproval. On the other hand, they may be more intrinsic, such as the satisfaction of getting some mathematics questions correct or the disappointment of failure. The desirability of the rewards or undesirability of the penalties are also variables from one situation to another, and from one pupil to another. The probability of the action leading to the reward rather than the penalty is yet another variable.

In Chapter 3 it was seen that the approaches adopted to assessing risk-taking in previous research have involved very diverse types of behaviour and consequently the hypothesised relationship between risk-taking and creativity has not always been apparent.

Hence it was considered desirable to devise more than one risk-taking situation with a mathematical context for the pupils in the present research programme. Three instruments were devised (Tests 23, 27 and 22), all probing the pupil's willingness to take risks in mathematics. The programme also included one instrument based upon the notion of nonconformity in mathematics (Test 26). These four instruments, as has been stated above, were newly devised for this present investigation. They are described below.

Test 23. This test of willingness to take risks in mathematics had the appearance of a conventional multi-choice test paper, with which all the pupils were familiar. For each of the 20 questions there were five options provided, A, B, C, D being possible answers, and option E being "don't know". Some non-critical items, distributed throughout the test, were straightforward mathematical questions. Other items were deliberately designed to put the pupils in a situation where they might have to take some sort of risk in choosing anything other than the "don't know" option. Some examples of such items are shown in Figure 6.1 with, in each case, percentages of pupils choosing various options given in parentheses underneath. Some of these items used bogus mathematical terms, but in a context where a possible, plausible meaning could be guessed, such as in questions 2 and 5. The rationale of such items is as follows. In question 2, for example, it is assumed that the 30.2% of pupils who chose option A presumably 'risked' the idea that the word 'conjugal' must refer to the only line in the diagram for which they do not know a name already, and the 33.9% who took option D could be said to be taking a risk that the word 'conjugal' is just another word for 'diagonal'. The safe thing to do, however, is to take option E, which 16.9% of the pupils did. Similarly, in question 5, large numbers of pupils took a chance on option A, guessing that the principal factors must be 5 and 12, and option D (30 + 30?), whereas 23.6% chose the safe "don't know" option.

Other critical questions such as question 6 in Figure 6.1, used correct mathematical terms, such as median, vector, zero matrix, which the pupils, because of the syllabus they had followed, would not have met. But the questions were set up so that reasonable guesses could be made, with a certain element of doubt. In question 6, for example, it could be argued that the 47.5% choosing option A are



(2) What is the length of the conjugal in this rectangle?

A. 3 B. 4 C. 5 D. 10 E. don't know

(30.2%) (11.2%) (7.9%) (33.9%) (16.9%)

(5) What is the sum of the two principal factors of 60?

A. 17 B. 8 C. 50 D. 60 E. don't know

(21.9%) (13.2%) (10.3%) (31.8%) (23.6%)

(6) The median of the numbers: 3, 4, 8, 9, 12, 16, 25, 26, 30, is 12.

What is the median of the numbers: 4, 6, 9, 15, 21, 34, 35?

A. 15 B. 35 C. 4 D. 12 E. don't know

(47.5%) (7.0%) (11.2%) (5.4%) (20.7%)

(11) Which of the following is not equivalent to $\frac{1}{2}$?

A. $\frac{2}{3}$ B. $\frac{2}{4}$ C. $\frac{6}{12}$ D. $\frac{4}{8}$ E. don't know

(81.4%) (7.9%) (5.4%) (2.5%) (2.9%)

(12) Which of the following is equivalent to $\frac{6}{7}$?

A. $\frac{1}{7}$ B. $\frac{3}{7}$ C. $\frac{2}{3}$ D. $\frac{29\ 46}{34.37}$ E. don't know

(1.7%) (8.3%) (26.4%) (47.9%) (15.7%)

Figure 6.1 Examples of items in Test 23. Questions 2, 5, 6, 12 are crticial items, question 11 non-critical. Percentages of pupils choosing various options are shown in parentheses.

taking a calculated risk, trusting their judgment, based on the internal evidence, as to the meaning of the unfamiliar term 'median', compared with the 20.7% choosing the safe option E.

Other questions, such as question 12, gave the pupils potentially difficult, time-consuming calculations, which could be by-passed by a bit of judicious guesswork. Would the pupils be prepared to risk the guess, or opt for safety with option E? Since 81.4% chose the correct option in question 11, it is not unreasonable to assume that most of the pupils understood what was meant by equivalent, and could presumably deduce fairly easily that options A, B and C in question 12 were <u>not</u> equivalent to 6/7. So option D becomes a reasonable guess, with a certain element of risk involved.

The theoretical basis of this instrument is that the pupil's action in choosing other than the "don't know" option involves the possibility of an intrinsic reward (the pupil's self-satisfaction in getting the question right) and the corresponding penalty (failure). Inaction (choosing E) avoids the risk. The possibility of informing pupils that negative marks would be awarded for any incorrect answers was considered, in order to increase the undesirability of the penalty for failure. But this would, of course, involve a subtle shift in emphasis away from the intrinsic reward of self-satisfaction towards the more extrinsic reward of marks. (Test 22 is an instrument using a greater emphasis upon marks).

To obtain an index for willingness to take risks in doing mathematics questions from this test, first each item was classified for the degree of risk-taking involved. (The complete list of percentages of pupils choosing each option on each of the twenty questions is given in Appendix 3 Table A3.1). This was done on the basis of the percentage of pupils choosing the "don't know" option, E. The more pupils choosing this option, it was reasoned, the more risk involved for the others in choosing an alternative option. Thus 'E points' were allocated to questions as follows:

E chosen by less than 10%..........no risk..........0 points

E chosen by 10 - 14.9%............low risk..........1 point

E chosen by 15 - 19.9%........medium risk...........2 points

E chosen by more than 20%........high risk..........3 points.

On this basis the twenty items were classified as follows:

no risk:	questions 1, 3, 7 - 11, 15, 18, 19
low risk:	questions 4, 13, 14, 16, 17, 20
medium risk:	questions 2, 12
high risk:	questions 5, 6

Some items which had been designed as critical items in the event did not turn out to contain a risk element. By choosing option E on all the critical items above the maximum E-score that could be obtained is 16. An index of risk-taking willingness (RT_{23}) therefore was calculated using the formula:

$$RT_{23} \text{ index} = \frac{(16 - \text{E-score}) \times 100}{16}$$

So, for example, a pupil scoring maximum E-points, the ultimate safety-first option, would be given an index of zero for risk-taking, but a pupil obtaining zero E-points would be given the maximum risk-taking index of 100.

<u>Test 27</u>. The second instrument which was devised and used for investigating risk-taking appears as Test 27 in Appendix 1, with full details of administration in Appendix 2.

This was the final task in the main programme of testing, and was presented in a more game-like atmosphere. It was an adaptation of the procedure used by Kogan and Wallach (1964) for assessing risk-taking in adults. Pankove and Kogan (1968) modified this for use with 10 - 11 year old children. Here the procedure was modified further and used with mathematical entities. It consisted of four rounds. The pupils were required in each round to guess the identity of an element known to the teacher. In rounds 1 and 3 this was a number, and in rounds 2 and 4 this was a shape selected from a large sheet of shapes on display. A series of eleven clues to the identity of the element was given in each round, with each clue reducing the number of possibilities available. At any stage in each round pupils could hand in their guesses and the number of clues received was recorded. The fewer clues taken, therefore, before making a guess, the lower is the probability of accurate identification. Hence it is assumed that the number of clues received will be determined by a pupil's willingness to take risks. Piaget and Inhelder (1951) have shown that it is reasonable to assume that children of this age have a sufficient intuitive grasp of probability to appreciate the element of risk in this game. Points were available for correct identification according to a scheme displayed and explained to the pupils beforehand:

No. of clues:	1	2	3	4	5	6	7	8	9	10	11
Points:	60	59	57	54	50	45	39	32	24	15	5

It can be seen that the difference in the number of points scored increases as each extra clue is given, thus putting greater pressure on the pupils to take a risk. A prize of sweets was promised (and given) to the pupil in each class scoring the most points over the four rounds.

It is clear from the above description of the game, that the notion of risk-taking involved here is very different from that involved in Test 23. In that test the pupils were competing against a mathematics test paper, and were motivated solely by whatever usually motivates them to do well in mathematics tests. The reward offered there was essentially intrinsic. But here they are competing against their peers for a prize, an extrinsic reward, in an open, highly competitive task. Inevitably the timing of a pupil's guess, for example, will be influenced by that of others in the group. This is a public rather than a private risk-taking behaviour.

A risk-taking index for this test (RT_{27}) was obtained. Only rounds two and three were used for this purpose. The first round was excluded to allow the pupils one round to get used to how the game worked. Round four was excluded because by then a pupil's chances of winning the prize may have become non-existent and so the probability of gaining the reward would be perceived as being near zero. If n_2 and n_3 are numbers of guesses received in rounds 2 and 3 respectively, then the risk-taking index for this test was calculated using the formula:

$$RT_{27} = (22 - n_2 - n_3) \times 5.$$

Thus a pupil guessing in each case immediately after the first clue was given would obtain an RT_{27} index of 100, and a pupil waiting to receive all clues would obtain an index of zero.

<u>Test 22</u>. This was the third instrument devised to probe the willingness of the pupils to take risks. Test 23 and 27 approached the notion of risk-taking from two very different points of view: first the idea of risking an answer to a mathematical question when there is an element of doubt, but sufficient internal evidence to make the guess reasonable; and secondly, the idea of risking a guess on the basis of very little evidence in a competition with peers, rather than waiting for further evidence. The reward and penalty on offer were in the first case essentially intrinsic and in the second extrinsic. Test 22 introduced another approach, with the reward of 'marks' assumed to be somewhere in the middle of the extrinsic/intrinsic dimension. In this test risk-taking was related to the pupil's self-confidence. MacKinnon (1961) and others have

shown that high creatives tend to be more self-confident than their peers, even egotistical. In the context of school mathematics it could be argued that a pupil's willingness to risk entering an unknown mathematical situation may be related to his self-confidence, the extent to which he expects to succeed. Test 22 was devised to put pupils in a situation where they could show that they were willing to enter an unknown mathematical situation. They were presented with a series of twenty conventional mathematics exercises, getting progressively more difficult. The pupils were told that this would be the case and that consequently the marks for the questions would increase: 1 for question 1, 2 for question 2, and so on. Then it was explained, with an illustration, that if they got a question wrong or failed to do it the marks for that question would be deducted from their score. Finally they could stop at any time before a question was read to them and hand their papers in - otherwise they would have to take the question and risk either gaining or losing that number of marks. Hence it was presumed that the number of questions a pupil received before deciding to stop would indicate the risk being taken. This would presumably be related to their assessment of their own ability to cope with an unknown, but harder question. The risk-taking index for this test (RT_{22}) was simply the number of questions taken multipled by five. This RT_{22} index was considered therefore to be an indicator of the pupil's willingness to risk losing the marks gained so far by taking more and more difficult questions.

It could not be claimed that this is a very refined instrument for measuring willingness to take risks. Again there are many factors affecting the pupil's behaviour. For example, it must be assumed that the pupil wants to do well in the test and gain as many marks as possible. But there was no particular incentive given to the pupils to make them want strongly to score as many marks as possible. Furthermore it is difficult to assess and interpret the influence of other pupils' behaviour, particularly their decisions to stop receiving questions.

However, even though interpretation of the behaviour involved in this test is difficult, the index obtained from it is still presumed to be an indication of willingness to take some sort of risks, along with the indices obtained from Tests 23 and 27.

Test 26. This instrument was devised to probe the pupil's level of nonconformity in mathematics. Because the procedure for administering this test involved an element of deception it was placed near the end of the programme in order not to affect adversely the pupil's attitude to the research.

First the pupils were given a multi-choice test of 20 conventional questions (see Appendix 1). Two days later their papers were handed back to them, unmarked. They were told that because a number of silly mistakes had been spotted they were to get another chance to answer the questions. To help them they would be told which was the most popular answer to each question given when the test was taken by children a year older than them in the High School. A procedure for changing their answers was explained. In the non-critical questions (1, 2, 4, 6, 7, 9, 12, 13, 14, 16, 18, 19) the purported most popular answer given was in fact the correct answer. A large proportion of non-critical items was included to help convince pupils that the procedure was genuine. There were eight critical items (questions 3, 5,8, 10, 11, 15, 17, 20) in which incorrect answers were suggested. (See Appendix 2 for full details of the procedure used). If x represents a pupil's original option, and y the final option, then (x,y) can be used to represent the behaviour on any given item. Both x and y can take three different values on the critical items:

p, the correct option

q, an incorrect, but unsuggested option

r, the suggested, incorrect option.

Hence there are nine different behaviours possible: (p,p), (p,q), (p,r), (q,p), (q,q), (q,r), (r,p), (r,q), (r,r). It was considered that only behaviours (q,r) and (p,r) indicated conformity. Of these, since moving from a correct option to an incorrect suggestion seems to involve more conformity, the behaviour (p,r) scored two points, and behaviour (q,r), one point for conformity.

This method of scoring for conformity, of course, means that a pupil getting all the critical questions wrong first time ($x = q$) can only score up to eight points, whereas the maximum for a pupil getting all the critical items correct first time ($x = r$) is 16. And even fewer points are available if a pupil chooses some of the options which are later to be suggested ($x = r$). This problem is overcome by considering the total number of conformity points scored (T) as a percentage of the maximum number (M) which could be obtained by the pupil, given the original options made. Then, since it is nonconformity, rather than conformity, which is expected to be linked with mathematical creativity, an index of nonconformity (NC) is calculated by means of the formula:

$$\text{NC index} = (1 - T/M) \times 100$$

Using this formula, any pupil who changes all original options to the suggested options ($y = r$ each time) obtains an NC index of zero; and an NC index of 100 is obtained by a pupil making no changes in favour of the suggested options.

Data from Tests 23, 27, 22 and 26

Table 6.1 gives the distribution of RT_{23} indices obtained from Test 23, the first of the instruments used for assessing willingness to take risks in mathematics. It is clear from this table that a large proportion (46.3%) of the pupils in the sample chose the E option on no occasions and consequently emerge as high risk-takers as measured by RT_{23}.

Table 6.1

Distribution of RT_{23} Indices (Risk-Taking Willingness) from Test 23

E-score	RT_{23} index	Frequency	%age
0	100	112	46.3
1	94	11	4.5
2	87	11	4.5
3	81	30	12.4
4	75	16	6.6
5	69	13	3.7
6	63	9	5.4
7	56	9	3.7
8	50	15	6.2
9	44	7	2.9
10	37	3	1.2
11	31	1	0.4
12	25	1	0.4
13	19	3	1.2
14	13	0	0
15	6	1	0.4
16	0	0	0

However, in spite of the concentration of pupils in this maximum RT_{23} group the instrument still succeeded to some extent in discriminating between pupils in terms of how they behaved on this multi-choice test, with RT_{23} indices ranging from 6 to 100. The odd distribution obtained may well be related to a particular problem suggested by analysis of the pupil's responses. A number of pupils were clearly just guessing wildly on this test, since some of their choices could not be explained on any reasonable basis. For example, four pupils chose option A in question 12, and 27 pupils chose option C for question 6. Clearly these 'wild guesses', particularly for the low attaining pupils, constitute a very different type of behaviour from the 'calculated' risk which the test was intended to probe. It can be speculated that, for example, for a pupil who is accustomed to getting answers wrong in mathematics tests, wild guessing on a multi-choice test is as good a strategy as any. There is therefore for such a pupil a different quality of risk in guessing than there would be for a pupil more accustomed to being able to work out the answers to mathematics questions correctly. The mean RT_{23} index was 83.0 with a standard deviation of 21.0.

The indices of willingness to take risks obtained from the second instrument related to this construct, Test 27, were distributed as shown in Table 6.2. It is clear that this instrument was more successful than Test 23 in producing a spread of results. The mean RT_{27} index was 30.4 with a standard deviation of 19.0.

The distribution of RT_{22} indices obtained from the third instrument used for probing willingness to take risks, Test 22, is shown in Table 6.3. In this test the risk element was involved in the pupil's decision when to stop taking questions in a progressively more dificult sequence of 20 mathematics problems. The mean number of questions taken by the pupils was 15.5. A fairly large proportion of pupils , 19.4%, took all 20 questions, and therefore gained the maximum RT_{22} index. But in spite of this the instrument succeeded in getting a fair spread of results for discriminating between pupils in terms of their behaviour on this test.

Table 6.2

Distribution of RT_{27} Indices (Risk-Taking Willingness) from Test 27

RT_{27}	Frequency	%age
85	2	1.0
80	0	0
75	6	3.0
70	2	1.0
65	2	1.0
60	4	2.0
55	17	8.5
50	5	2.5
45	11	5.5
40	13	6.5
35	16	8.0
30	21	10.6
25	21	10.6
20	21	10.6
15	26	13.1
10	14	7.0
5	10	5.0
0	8	4.0

The distribution of scores obtained from the instrument used for assessing nonconformity, Test 26, is given in Table 6.4. The procedure used in this instrument involved suggesting to the pupils what were the most popular answers given by older pupils to a multi-choice test they had themselves taken, and giving them the opportunity to conform by changing their options. Clearly a major problem in using this procedure for assessing nonconformity is that

Table 6.3

Distribution of Risk-Taking Indices (RT_{22})

Arising from Test 22

No. of questions taken	RT_{22} index	Frequency	%age
6	30	1	0.5
7	35	1	0.5
8	40	2	0.9
9	45	6	2.8
10	50	6	2.8
11	55	9	4.1
12	60	21	9.7
13	65	11	5.1
14	70	26	12.0
15	75	20	9.2
16	80	25	11.5
17	85	28	12.9
18	90	10	4.6
19	95	9	4.1
20	100	42	19.4

there is no guarantee that the pupils were taken in by the deception. The distribution of NC indices shown in Table 6.4 indicates that, although a large proportion of the sample resisted the pressure to conform to the suggested choices (40% obtaining NC indices of 100), at least it can be claimed that 60% were persuaded to make some changes. Two pupils demonstrated the totally conformist behaviour indicated by an NC index of zero. The mean NC index obtained was 80.2, with standard deviation of 23.4 (232 pupils).

Table 6.4

Distribution of Nonconformity (NC) Indices Obtained from Test 26

NC Indices	Frequency	%age
100	94	40.3
90 - 99	15	6.5
80 - 89	37	15.9
70 - 79	30	12.9
60 - 69	16	6.9
50 - 59	19	8.2
40 - 49	4	1.7
30 - 39	5	2.1
20 - 29	8	3.4
10 - 19	3	1.3
0 - 9	2	0.9

Results and Discussion of Hypotheses 1 and 2

In spite of the odd distribution obtained for the RT_{23} indices, there were significant (at the 5% level), though low correlation coefficients between RT_{23} and OF scores, and RT_{23} and DP scores. These were 0.14 in both cases (221 and 227 pupils respectively).

But since the hypothesis being investigated here is concerned with "pupils of similar levels of mathematical attainment", and also because of the possible difference in interpretation of the risk-taking behaviour of low and high attainers in connection with this instrument, it is necessary to look for relationships within the different bands of MA scores. Table 6.5 shows the correlation coefficients obtained.

Table 6.5

Coefficients of Correlation Between Mathematical Creativity Scores (OF/DP) and Risk-Taking Indices (RT_{23}) from Test 23

Level of maths attainment:	Correlation coefficients between: OF and RT_{23}	DP and RT_{23}
A. very high	0.24* (51 pupils)	0.11 (51 pupils)
B. above average	-0.06 (81 pupils)	-0.14 (85 pupils)
C. average	-0.11 (72 pupils)	0.11 (73 pupils)
D. below average	0.31 (16 pupils)	0.16 (16 pupils)

* significant at 10% level

It is clear from these results that there is no support for the hypothesis amongst groups B and C. The number of pupils in group D is too small to draw any conclusions, and it has already been noted that there is a difficulty in interpreting the behaviour of the low attainers on this test. For the high attainers in group A there is slight support, but of dubious signifcance, for the hypothesis that those who performed better on the tests of overcoming fixation tend to have higher indices of this type of risk-taking willingness. This is strengthened a little by considering a slightly smaller group of very high attainers, the 41 pupils scoring 131 or over for mathematics attainment. In this case correlation coefficients of 0.30 and 0.18 respectively are obtained between RT_{23} and OF, and RT_{23} and DP scores. The first of these is significant at the 5% level.

Other data relating to the scores on this instrument are given in Appendix 3, including tables relating to boy/girl performances.

No significant difference in behaviour between boys and girls was found on this test.

Analysis of the results of Test 27 shows that the type of risk-taking behaviour involved here is not positively related to performance in mathematical creativity tasks. In fact, the correlation coefficients obtained would support the opposite relationship to that hypothesised. Between OF scores and RT_{27} indices a correlation coefficient of -0.23 (174 pupils) was obtained, and between DP scores and RT_{27} indices one of -0.39 (182 pupils). Both these figures are significant negative correlations at the 1% level. In terms of the whole sample it appears that this type of risk-taking behaviour is inversely related to mathematical creativity. Table 6.6 gives the correlation coefficients obtained for the various groups of similar levels of mathematical attainment. Where risk-taking is determined by the type of behaviour associated with Test 27 it can be concluded that there is no support for the hypothesis.

Table 6.6

Correlation Coefficients Between Mathematical Creativity (OF/DP) Scores and Willingness to Take Risks (RT_{27}) in Test 27

Maths attainment:	Correlation coefficients between:	
	RT_{27} and OF	RT_{27} and DP
A. very high	0.10 (43 pupils)	-0.10 (43 pupils)
B. above average	-0.04 (57 pupils)	-0.35** (59 pupils)
C. average	-0.17 (64 pupils)	-0.24* (68 pupils)
D. below average	0.04 (10 pupils)	-0.26 (12 pupils)

* significant at 5% level ** significant at 1% level

In fact for the average and above average pupils it is found that the more mathematically creative in terms of divergent production, tend to be lower takers of this kind of risk.

Additional data arising from this test may be found in Appendix 3. Table A3.5 shows clearly that the higher mathematics attainment groups show less willingness to take this sort of risk. This can be understood in terms of their preference to rely on their ability to work out the answer rather than take an unjustified gamble. No particular difference in behaviour between boys and girls was discerned, as can be seen from Tables A3.6 and A3.7.

The third measure of willingness to take risks was that obtained from Test 22. Highly significant (at the 1% level) correlation coefficients between RT_{22} indices and mathematical creativity scores were obtained: 0.46 between RT_{22} and OF scores (197 pupils), and 0.45 between RT_{22} and DP scores (202 pupils). But the hypothesis being investigated requires consideration of groups of pupils with similar mathematical attainment. This is clearly important here because it would be expected that the higher attaining pupils would be more prepared to risk exposure to harder questions, and that, in fact, there is less risk involved for them in doing this. (This is confirmed by a correlation coefficient of 0.53 between MA scores and RT_{22} indices, for 216 pupils. See also Table A3.8 in Appendix 3). The correlation coefficients obtained for the four mathematics attainment groups are shown in Table 6.7. The two most significant correlations are between RT_{22} and OF for the average attaining group and between RT_{22} and DP for the above average attaining group. The coefficients for the very high attainers are in the right direction, but are not sufficiently high to be significant. There is therefore some support for the hypothesis that pupils who perform better in mathematical creativity tasks than those with similar levels of mathematics attainment, particularly amongst the average and above average groups, are more willing to take risks, where taking risks is identified by the behaviour shown on Test 22.

Table 6.7

Correlation Coefficients Between Mathematical Creativity Scores (OF/DP) and Willingness to Take Risks (RT_{22}) Indices from Test 22

Maths attainment:	Correlation coefficients between:	
	RT_{22} and OF	RT_{22} and DP
A. very high	0.16 (50 pupils)	0.15 (50 pupils)
B. above average	0.27*(65 pupils)	0.09 (66 pupils)
C. average	0.20 (66 pupils)	0.33** (70 pupils)
D. below average	-0.23 (16 pupils)	0.15 (16 pupils)

*significant at 5% level **significant at 1% level

Additional data arising from this test may be found in Appendix 3. In particular it may be seen that no significant differences in behaviour between boys and girls is apparent.

The results on the test devised for assessing nonconformity in mathematics, as shown in Table 6.4, indicate that, in view of the large proportion of maximum indices obtained, Test 26 was not a particularly sensitive instrument. However, the hypothesised relationship with mathematical creativity will be considered.

Significant (at the 1% level) correlations of 0.29 in each case (207 pupils and 217 pupils respectively) were obtained between the NC indices and the two mathematical creativity scores (OF/DP). But these were again not as high as the correlation with the mathematics attainment scores, which was found to be 0.43 (230 pupils). These figures indicate that the behaviour which has been labelled 'nonconformity' in Test 26 is most strongly related to the level of mathematical attainment. This would be expected, of course. The higher attainers would be more likely to be less influenced by the purported opinions of other's about answers to mathematics questions. But hypothesis 2 suggests that, for groups of pupils of similar levels of mathematical attainment, variation in their mathematical creativity performances may be related to their nonconformity. Table 6.8 shows that there is no evidence to support this hypothesis from the results of Test 26. None of the correlations obtained within the attainment groups is significant. The correlation coefficients obtained for the group of very high attainers are in the right direction but too low to lend even slight support to the hypothesis for this group in the case of overcoming fixation. The figure of 0.19 for 59 pupils is only significant at a 20% level of probability.

Table 6.8

Correlation Coefficients Between Mathematical Creativity Scores (OF/DP) and Nonconformity (NC) Indices from Test 26

Maths attainment:	Correlation coefficients between:	
	NC and OF	NC and DP
A. very high	0.19 (49 pupils)	0.11 (49 pupils)
B. above average	-0.07 (77 pupils)	0.09 (81 pupils)
C. average	0.09 (66 pupils)	0.04 (70 pupils)
D. below average	0.25 (14 pupils)	-0.31 (15 pupils)

Other data arising from this test may be found in Appendix 3. No significant differences in behaviour were found between boys and girls.

Summary of Findings With Respect to Hypothesis 1 and 2

It is concluded from the above analyses that the notion of risk-taking in mathematics is not to be considered as an easily identifiable single type of behaviour. Three different, albeit rather crude, instruments have been described, in which pupils were put into situations requiring different kinds of risk-taking. Three possible different interpretations of "willingness to take risks in mathematics" might therefore be as stated below.

Test 23 risk-taking. What might be called 'the reasonable guess'. The pupil shows willingness to hazard a reasonable guess for a solution to a mathematics problem, where there is an element of doubt or uncertainty, but sufficient internal evidence to make the guess reasonable.

Conclusion. There is a weak suggestion that this type of risk-taking is associated with mathematical creativity performance amongst the high attaining pupils. The hypothesis is supported only to a small extent, and only with respect to high attainers and overcoming fixation.

Test 27 risk-taking. What might be termed 'the sporting chance'. The pupil is prepared to make a guess on the basis of very little mathematical evidence, rather than wait for more conclusive evidence to become available. There is a preference for risking a guess to win a prize rather than waiting to draw a reasoned conclusion.

Conclusion. There is evidence that this type of risk-taking is typical of low attaining pupils. The hypothesis is certainly not supported. In fact there is some support for the opposite hypothesis, particularly with respect to divergent thinking amongst average and above average pupils: that the more mathematically creative tend to show less of this type of risk-taking behaviour.

Test 22 risk-taking. Confidence to take on the unknown. The pupil is prepared to risk entering an unknown mathematical situation. This willingness to risk exposure to an unknown question is presumed to be an indication of the pupil's self-confidence in mathematics.

Conclusion. This type of risk-taking is associated with the higher levels of mathematical attainment. There is some support for the hypothesis, particularly amongst average and above average attaining groups.

That these are different aspects of risk-taking behaviour is indicated by the correlations obtained between the three indices shown in Table 6.9. The 'sporting chance' type of risk-taking is not related at all to the 'reasonable guess' type, and is inversely related to the 'confidence to take on the unknown' behaviour. Only the measures of willingness to take risks indicated by the indices RT_{22} and RT_{23} are significantly and positively correlated.

Table 6.9

Correlations Between Three Risk-Taking Indices

Indices	Correlation coefficients	No. of pupils
RT_{22} / RT_{23}	0.21**	202
RT_{22} / RT_{27}	-0.24**	171
RT_{23} / RT_{27}	0.01	179

(**significant at 1% level)

The results from the instrument used for assessing nonconformity in mathematics (Test 26) give no support to hypothesis 2.

Category Width, Self-Concept and Anxiety (Hypotheses 3 - 6)

This second part of the investigation into personality aspects related to mathematical creativity was based upon the use of existing instruments.

Background

A summary of some of the relevant background to the present investigation is provided below. A more detailed consideration was given in Chapter 3.

Category width. Bruner (1957, 1963) describes a model of learning in which the essential intellectual activity consists in reorganising data input from the external world into categories. New information and experiences are made meaningful by the learner by connecting them with past data which they resemble. This intellectual activity is referred to as 'coding' or 'categorising'. As has been seen in Chapter 3, Cropley (1967) has identified the tendency of some individuals to make novel or unusual codings as a basis for creative thinking. Individuals vary between narrow coders on the one hand and broad coders on the other. Some tend to make narrow, fine discriminations between data, concentrating on differences rather than similarities. Others tend to group together in broad categories items which are roughly equivalent, seeing similarities rather than differences. It is clear that theoretically creative thinking is likely to be more typical of broad rather than narrow categorisers. Anderson and Cropley (1966), in their study of 320 Canadian 12 year olds, found that the top ten per cent of high creatives showed a marked tendency for broader coding than the bottom ten per cent of low creatives.

This finding supports the results of the work in this area carried out by Wallach and Kogan (1965). They found a significant tendency for the higher creative pupils in their sample of 151, 10 - 11 year olds to score higher on a test of category width. The test they used was an adaptation of the Pettigrew (1958) Category Width Test. It was constructed to assess the subject's tendency to think in narrow or broad categories. Presented with a central tendency value for a category - such as, "most roads are about 18 feet wide" - the subject is invited to estimate the most deviant members of the category - such as the width of the narrowest/widest road. The theoretical basis of this instrument is that narrow coders will tend to give estimates for the deviants very close to the given value, because of their inclination to think in narrow categories - for example, to interpret 'road' in a very restricted and narrow sense. Broad coders will behave in a contrasting way, opting for estimates deviating considerably from the central value.

As has been noted above, it is clearly appealing to link category-width with creativity. Narrow coding would hardly seem a likely basis for divergent production, for example, and would seem potentially to be a contributing factor in content universe fixation. These considerations led to the formation of the hypothesis above, and the decision to investigate whether for 11 - 12 year old pupils their mathematical creativity performance, as measured by divergent production in mathematics and their ability to overcome fixation, is in fact linked with any tendency for narrow or broad coding.

Furthermore, the relationship of category-width to mathematical thinking is similarly appealing. Mathematics often involves ignoring differences between elements in order to group them togehher according to some equivalence relation. This would seem to involve the formation of broad categories. For example, Biggs (1967) argues that the tendency to narrow or broad coding is a significant factor in the pupil's reaction to a multi-base approach to learning arithmetic. Those who tend to make narrow discriminations between experiences find initial difficulties in seeing the underlying common structure in activities with a variety of number bases. Biggs suggests that the observed slowness and initial anxiety of girls in particular in adjusting to a multi-model method of teaching arithmetic is a consequence of the oft-reported tendency of girls to be narrower coders than boys. (See,for example, Wallach and Caron, 1959;

Wallach and Kogan, 1965,pp 124 - 126).

Attitudes: self-concept and anxiety. Hypotheses 4, 5 and 6 are concerned with pupils' self-concept in mathematics.

Self-concept in mathematics refers to the pupils' images of themselves with regard to performance in mathematics. So pupils with high self-concepts, for example, will indicate that they think they do well in the subject, they find it easy, and they expect to understand and succeed. Such an attitude is clearly related to the behaviour discussed in relation to Test 22, that described as self-confidence, which is associated there with willingness to risk exposure to unknown mathematics questions. The same theoretical considerations as outlined in the discussion of Test 22 led to the decision to include Hypothesis 4 in the present investigation. It would be expected that in order to engage in original, divergent thinking in mathematics a pupil would need a high self-concept. Expectation of success would seem theoretically to be a prerequisite attitude for embarking on unknown or unfamiliar territory in mathematics. Conversely, a pupil with low self-concept is likely to be inhibited and will tend to stay on familiar and therefore less original ground.

Torrance (1965, pp 24 - 25) has underlined the importance of self-concept in developing creative thinking in schoolchildren. He includes self-confidence and self-sufficiency in a list of characteristics which differentiate highly creative from less creative individuals. Hypothesis 4 in the present investigation seeks to relate all this to mathematics in particular. The question to be considered then is whether in fact a high self-concept in mathematics impinges significantly on mathematical creativity performance for pupils of similar levels of mathematics attainment.

Fennema and Sherman (1977) have shown that although self-confidence and anxiety are usually defined as separate traits it appears that in relation to mathematics learning they are very closely related. In fact they may be considered as two poles of one dimension. So the considerations above about self-concept in mathematics led to further consideration of the influence of anxiety towards mathematics on mathematical creativity performance, as stated in Hypothesis 5. Callahan and Glennon (1975) conclude that anxiety and mathematics achievement are clearly related, and that in general there is evidence that high anxiety is associated with low achievement. But the question of interest in the present investigation is whether within groups of similar levels of mathematical attainment high anxiety is associated with poor performance in the creativity tasks, and vice versa.

It was seen in Chapter 3 that the evidence about the relationship between anxiety and performance in creativity tests is somewhat equivocal. Klein, Frederikson and Evans (1969) found that an intermediate level of anxiety was associated with poor performance in divergent production amongst first year undergraduates. However, working with 10 - 11 year olds, Wallach and Kogan (1965) found that high creative boys displayed intermediate levels of anxiety, both in general and towards tests. Low creativity and high IQ in boys was associated with low test anxiety, presumably because their competences matched conventional classroom assessment expectations. But high creativity and low IQ in boys was associated with high test anxiety. In common with other findings in this area, they found that girls were consistently more anxious in general than boys, but no clear relationship between creativity and anxiety in girls emerges. White (1968) reports that students with low anxiety performed significantly better on two of the Minnesota Tests of Creative Thinking, and he identifies the typical divergent thinker as being a non-anxious extrovert. A number of studies have indicated that induced stress and anxiety positively inhibits divergent production and increases rigidity in thinking (Beier, 1951; Cowen, 1952; Krop et al, 1969; Hadley, 1965). In relation to performance on difficult arithmetic questions, Biggs (1962) suggests that in differing conditions, anxiety - general anxiety, test anxiety, number anxiety - can be sometimes inhibiting, but sometimes motivating and facilitating.

It is clear, therefore, with this background, that in terms of performance in tasks related to both mathematics and creative thinking, the levels of a pupil's anxiety both towards mathematics and towards tests in general could be significant factors. Hypotheses 5 and 6 were formulated as the basis for the investigation into this area in the present research.

Instruments Used for Investigating Hypotheses 3 - 6

For the purpose of investigating hypothesis 3 in the present research the Category Width Test used by Wallach and Kogan was modified further - anglicising the language and metricating most of the units - to make it appropriate for the 11 - 12 year old pupils concerned. The instrument appears as Test 25 in Appendix 1. Being concerned with numerical estimates of quantity and measurements it has the appearance of a mathematical exercise, though it was presented to the pupils as a guessing game.

An example of the ten items included in the instrument is:

"Most roads are about 9 metres wide.

(a) How wide is the widest road?

52 metres

27 metres

12 metres

36 metres

(b) How wide is the narrowest road? 8 metres

3 metres

1 metre

6 metres"

In scoring the pupil's responses to such items, the options were assigned 0, 1, 2, 3 marks respectively according to their closeness to the stated central value. Thus a pupil choosing 27 metres and 6 metres in the above example would receive 2 marks for part (a) and 3 marks for part (b). Hence a large score indicates a preference for broad band widths, and a small score a preference for narrow band widths.

For investigating hypotheses 4 - 6, a questionnaire was used to assess pupils' self-concept in mathematics, anxiety towards mathematics and test anxiety. This appears as Test 21 in Appendix 1, with instructions for administration in Appendix 2. Thirty items appear in the test, classified as follows:

A. Items 2, 5, 8, 13, 16, 19, 21, 26: self-concept in mathematics.

B. Items 12, 22, 25, 27, 28, 30: anxiety towards mathematics.

C. Items 3, 6, 7, 10, 14, 15, 17, 20, 24, 29: test anxiety.

D. Items 1, 4, 9, 11, 18, 23: fillers.

Categories A and B items were adapted from the Mathematics Attitude Inventory developed by the National Science Research Project at the University of Minnesota. Category C items were based on items dealing with Test Anxiety adapted by Wallach and Kogan (1965) from the work of Sarason and his associates (Davidson and Sarason 1961) for use with 10 - 11 year olds. Each item took the form of a first-person statement, with which the pupil would indicate the extent of his aggreement or disagreement on a five-point scale. Some trial statements were used by the teachers administering the questionnaire with the pupils to ensure that they understood how to respond. The teachers reported that the pupils found the instructions clear and unambiguous. The scoring procedure was to award four points for a response showing very strong agreement with a statement indicating high self-concept or anxiety, then three, two, one, and finally zero points for strong disagreement. (Two items, 5 and 16, for self-concept were reverse-scored). The scores for the items in each category were aggregated, and, for convenience in interpretation expressed as percentages of the maximum possible score.

Data from Tests 25 and 21

The minimum score which could be obtained on the Category Width test (test 25) was zero, with the maximum score being 60. In fact the scores ranged from 4 to 59, with a mean score of 30.6 and a standard deviation of 10.4 marks. A split-half coefficient of reliability of 0.88 was obtained for this test indicating highly consistent behaviour of pupils in their responses to these items. The full distribution of scores for category width (CW scores) is given in Table 6.10.

The scores arising from Test 21, the attitude questionnaire, will be referred to as the SCM index (Self-Concept in Mathematics), the ATM index (Anxiety Towards Mathematics) and the TA index (Test Anxiety). Table 6.11 gives the overall distributions of these indices. Table 6.11 shows that the central tendency for the sample of 11 - 12 year olds used was to have a just positive self-concept in mathematics (59.3%), a fairly low level of anxiety towards mathematics (36.4%), and to be neutral in terms of test anxiety (46.9%).

Results and Discussion of Hypotheses 3 - 6

Correlation coefficients of 0.15 (218 pupils) between Category Width (CW) scores and OF scores, and 0.20 (226 pupils) between CW scores and DP scores were obtained. The first of these is significant at the 5% level, and the second at the 1% level. Interestingly there was a less significant correlation between mathematics attainment and category width, a coefficient of 0.13 (238 pupils) being obtained. These figures suggest that CW scores may be related to mathematical creativity.

Table 6.10

Distribution of Category Width (CW) Scores from Test 25

CW scores	Frequency	%age
0 - 4	1	0.4
5 - 9	3	1.2
10 - 14	9	3.7
15 - 19	20	8.3
20 - 24	33	13.7
25 - 29	45	18.7
30 - 34	50	20.7
35 - 39	35	14.5
40 - 44	24	10.0
45 - 49	11	4.6
50 - 54	4	1.7
55 - 59	6	2.5

But hypothesis 3 requires consideration of pupils in their mathematics attainment groups. The correlation coefficients obtained for the four groups are given in Table 6.12. The only significant correlation coefficients are for the group of high attainers. We may conclude therefore that there is some support for the hypothesis amongst the very high mathematics attainment group. Apart from this group, category width appears to be unrelated to mathematical creativity scores.

Table 6.11

Distribution of Attitude Indices Obtained from Test 21

Indices range	Frequencies SCM	ATM	TA
0 - 9	1	15	6
10 - 19	1	20	10
20 - 29	10	77	27
30 - 39	15	42	41
40 - 49	25	34	51
50 - 59	78	29	50
60 - 69	53	13	30
70 - 79	39	12	17
80 - 89	12	1	8
90 - 99	8	1	3
100	2	0	1
Mean Index:	59.3	36.4	46.9
St. Dev.	16.6	17.7	19.0
Number of pupils	244	244	244

Table 6.12

Correlation Coefficients Between Mathematical Creativity Scores (OF/DP) and Category-Width (CW) Scores from Test 25

Maths attainment:	Correlation coefficients between: CW and OF	CW and DP
A. very high	0.34* (49 pupils)	0.32* (49 pupils)
B. above average	-0.08 (79 pupils)	0.15 (81 pupils)
C. average	0.07 (74 pupils)	0.01 (77 pupils)
D. below average	-0.36 (15 pupils)	-0.24 (17 pupils)

* significant at 5% level

Other data related to this instrument may be found in Tables A3.16-18 in Appendix 3. Highly significant differences in behaviour between boys and girls in terms of category width emerge from these tables. The mean score for boys was 32.9, compared with 28.0 for girls. This difference between the means represents 3.7 standard error units, which is extremely significant. The results confirm the findings of previous researchers that boys tend to be broader categorisers than girls. More difficult to interpret is the differences obtained in correlation coefficients when boys and girls are considered separately. For the whole sample, the correlations with OF and DP for girls were -0.01 and 0.11 respectively. Neither of these is significant at the 5% level. For boys the figures are 0.22 (significant at the 5% level) and 0.25 (significant at the 1% level). This difference is even more clearly marked if just the boys and girls in the very high attaining group are considered. For the 19 girls concerned the correlation coefficients between CW and OF, and CW and DP are near zero (-0.06 in each case). For the 30 boys in this group, the figures are 0.43 and 0.28 respectively.

To summarise these findings, it appears that only in the case of higher attaining boys is category-width significantly linked with mathematical creativity performance. Girls tend to be narrower categorisers than boys overall, and it seems to be the case that for the girls variation in category width is unrelated to mathematical creativity performance.

It is particularly interesting to note that Wallach and Kogan found that the hypothesis that children exhibiting broad band width on the Pettigrew Test would rank higher on the creativity index was confirmed for both boys and girls, but much more conclusively in the case of girls. The difference between their finding and the finding in the present research supports the view that the behaviour being discussed under the heading 'mathematical creativity', although conceptually related to general notions of creativity, is a distinct, domain-specific ability or set of abilities. It could be suggested for example that those girls who do show a greater tendency to think in broad categories in general, and will tolerate wide deviations from the norm in the examples used in Test 25, are less prepared than boys to think in a similar way when operating in a mathematical domain.

Highly significant correlations, in the expected directions, were obtained for the whole sample between each of the three indices obtained from the attitude questionnaire and similar and slightly stronger correlations were obtained with mathematics attainment scores. These are shown in Table 6.13.

Table 6.13

Correlations Between Attitude Indices from Test 21 and Both Mathematical Creativity (OF/DP) and Mathematics Attainment (MA) Scores

	OF	DP	MA
SCM	33	43	45
ATM	-32	-34	-39
TA	-26	-23	-33
No. of pupils:	221	227	243

(Decimal points omitted. All correlations significant at 1% level).

So since there is evidence here that high self-concept in mathematics and low levels of anxiety towards mathematics and tests are linked with the level of mathematical attainment, it will be necessary to pursue any link with mathematical creativity by separate consideration of the four mathematics attainment groups, as required in the hypotheses.

Table 6.14 shows the correlation coefficients obtained within the four groups of pupils of various levels of mathematics attainment.

Table 6.14

Correlation Coefficients Between Attitude Indices from Test 21 and Mathematical Creativity Scores (OF/DP) for Various Levels of Mathematics Attainment

	Correlation coefficients with:					
	SCM		ATM		TA	
Maths attainment	OF	DP	OF	DP	OF	DP
A. very high	25	45**	-30*	-41**	-16	-32*
	(54)	(54)	(54)	(54)	(54)	(54)
B. above average	00	-05	-08	16	-03	31**!
	(80)	(82)	(80)	(82)	(80)	(82)
C. average	07	20	-14	-21	-18	-08
	(72)	(76)	(72)	(76)	(72)	(76)
D. below average	-18	60*	25	-31	14	-18
	(15)	(15)	(15)	(15)	(15)	(15)

Notes: Decimal points omitted. Numbers of pupils in parentheses.
* significant at 5% level, ** significant at 1% level
(!) this correlation, although highly significant, is in the opposite direction to that predicted by the hypothesis.

The most significant results are in relation to self-concept in mathematics and anxiety towards mathematics within the group of very high attainers. It can be inferred that within this group self-concept in mathematics is clearly linked with performance in mathematical creativity tests, there being a highly significant correlation of 0.45 with divergent production in mathematics, and a near significant correlation of 0.25 with overcoming fixation in mathematics. Apart from the coefficient obtained with the small number of below average pupils for divergent production, no other correlations in this SCM column are significant. So for the very high attaining group hypothesis 4 is confirmed. That is to say, that amongst the very high mathematics attainment pupils, those who show higher levels of mathematical creativity, particularly in terms of divergent production, tend to have a higher self-concept in mathematics. For other groups of pupils self-concept in mathematics is not significantly related to mathematical creativity performance. Similar conclusions may be drawn with respect to hypothesis 5. The only significant correlations in the ATM column are for the very high mathematics attainment group. For these pupils the hypothesis is confirmed, again slightly more strongly for divergent production than for overcoming fixation. Amongst the very high mathematics attainment pupils, those who show higher levels of mathematical creativity, particularly in terms of divergent production, tend to have lower levels of anxiety towards mathematics. For other groups of pupils the level of mathematics anxiety does not seem to be significantly related to mathematical creativity. Slightly less strong are the relationships for the group of very high mathematics attainers between Test Anxiety and the mathematical creativity measures. There is for these pupils some support for Hypothesis 6, but only in terms of divergent production, with a correlation of -0.32. For overcoming fixation the correlation coefficient obtained is -0.16, which is in the right direction but not high enough to be significant. However the hypothesis is turned on its head by the figure obtained for the correlation between DP and TA for the above average group. This is highly significant (at the 1% level), but in the wrong direction. In line with the earlier discussion about previous results in the area, it is found that in terms of test anxiety the conclusions are equivocal and difficult to interpret. But the question must be asked: why for the very high mathematics attainers is the hypothesis supported that low levels of test anxiety are associated with high levels of mathematical creativity, but for pupils of slightly lower, but still above average mathematics attainment, the reverse is found to be the case? A possible interpretation of this phenomenon is provided by considering four hypothetical pupils:

Pupil P: High mathematics attainment, high test anxiety

Pupil Q: High mathematics attainment, low test anxiety

Pupil R: Moderate mathematics attainment, high test anxiety

Pupil S: Moderate mathematics attainment, low test anxiety

Pupils P and Q would be in Group A, R and S in Group B. The following is a hypothetical discussion of their behaviour. Pupil P, although very good at conventional mathematics assessment, still worries about tests and consequently performs badly in the unusual tests of divergent production. Pupil P is keen to do well, and usually does so in mathematics test, but is put off by the unfamiliar style of the DP tasks. Pupil Q on the other hand, not worrying about tests is able to bring all available mathematical skills and knowledge to bear in an uninhibited way upon the divergent production tasks . Pupil Q is not disturbed by their unfamiliarity. Pupil R is actually a highly creative thinker, but in conventional assessments this pupil's performance is only moderate and results in test anxiety. However this pupil does not recognise the divergent production tasks as being the sort of tests on which R normally does only moderately well, so the normal test anxiety does not function and R is able to perform well. Pupil S is actually a low creative person, but moderately competent at conventional classroom assessments in mathematics. Pupil S's style and competences match fairly closely what is normally required in tests, so consequently this pupil has a low level of test anxiety.

If such tendencies amongst pupils as these exist in the groups under consideration then the correlations obtained can be explained. The ambiguous nature of the results may therefore be due to the possibility that sometimes low or high test anxiety may be the cause of good or poor performance in mathematical creativity tasks, facilitating or inhibiting the pupil in the approach to unfamiliar tasks, and sometimes high or low test anxiety may be the result of a mismatch or a close match, as the case may be, between the pupil's competences (including creativity) and the conventional assessments used in schools.

Other data relating to the results of the Questionnaire (Test 21) may be found in Appendix 3. Of particular interest are the differences which may be observed between the boys and the girls on the attitude indices, as shown in Tables A3.19 and A3.20. Consistent with the findings of other researchers it is seen that the girls tend to have a lower self-concept in mathematics, the difference between the boys' and girls' means being equivalent to 3.03 standard error units, which is significant at the 1% level. Similarly, it is found that the mean level of anxiety towards mathematics for girls is higher than that of boys, the difference being equivalent to 2.48 standard error units, significant at the 1% level. Furthermore the girls show higher levels of test anxiety, the difference in the means in this case being equivalent to 2.20 standard error units, also significant at the 5% level. It is also interesting to note that only for the girls are there any non-significant correlations with mathematical creativity scores in Table A3.20. In particular the relationship between Test anxiety and Divergent

Production for girls is weaker than any of the other relations in that table. This weak result is consistent with the inconclusive findings of Wallach and Kogan in connection with girls mentioned earlier.

Summary of Findings with Respect to Hypotheses 3 - 6

Hypothesis 3: Category-width. The hypothesis is supported only for high attaining boys. For these pupils the tendency for broad coding is significantly linked with higher levels of mathematical creativity. Girls tend to be narrower coders than boys overall, and it seems that for girls variation in category width is unrelated to mathematical creativity performance.

Hypothesis 4: Self-concept in mathematics. The hypothesis is supported, most strongly in the case of divergent production, but again only amongst the group of very high attainers. For these pupils it seems to be the case that a higher self-concept is associated with higher mathematical creativity.

Hypothesis 5: Anxiety towards mathematics. The hypothesis is supported, most strongly in the case of divergent production, but again only amongst the group of very high attainers. For these pupils it seems to be the case that lower levels of anxiety towards mathematics are associated with higher mathematical creativity.

Hypothesis 6: Test anxiety. The results are difficult to interpret. The hypothesis linking lower levels of test anxiety with higher levels of mathematical creativity is supported for the group of very high attainers, particularly in terms of divergent production. But for the above average attainers the opposite conclusion may be drawn: higher test anxiety is associated with higher mathematical creativity, in terms of divergent production.

The Mathematically Creative Pupil: a Picture Drawn from Group Data

The examination of group data undertaken so far suggests that a tentative picture may be drawn of the high mathematically creative 11 - 12 year old pupil, bringing together the results from the consideration of hypotheses 1 - 6. The clearest results were obtained in the part of the investigation which used existing instruments (hypotheses 3 - 6: category-width and attitudes), rather than that which employed newly devised instruments (hypotheses 1, 2: risk-taking and nonconformity). Most of the significant correlations obtained in the consideration of hypotheses 3 - 6 referred to the group of very high attainers (Table 6.12 and 6.14). Also in Chapter 5 it was seen that it is within this group that most of the high mathematically creatives are found and also that there is the greatest deviation in terms of mathematical creativity measures (Figures 5.6 and 5.7, Table 5.5). Consequently the investigation from this point on will concentrate mainly on this group of pupils. The description below is merely a tentative profile of a hypothetical high mathematically creative pupil, an attempt to draw together the various threads which have emerged so far in this investigation. The question as to whether such pupils, and their less creative counterparts, actually exist will be considered in Chapter 7 by examination of the profiles of individual pupils from the group of high attainers.

A tentative profile. The high mathematically creative, high attaining pupil almost certainly has a high self-concept in mathematics (Table 6.14 SCM column). That is to say, this pupil expects to do well in mathematics and to be able to solve the problems which are given to the class. It can be imagined that such a pupil would therefore embark upon the unfamiliar divergent production tests with confidence. The pupil probably has a low level of anxiety towards mathematics (Table 6.14 ATM column) and possibly towards tests in general (Table 6.14 TA column), and is consequently likely to be uninhibited in approaching the open-ended style of question. It can be imagined that the pupil lets the mind run freely over the whole range of available mathematical skills and knowledge previously learnt, confident that these can be called upon without difficulty, and without anxiety. The pupil would thus be able to demonstrate high levels of fluency and originality in responses to divergent production tests. The pupil may well be willing to take risks in answering mathematics questions, provided these are reasoned, calculated risks (Table 6.5). Such a pupil would not guess wildly (Test 27) but may be more prepared to trust a personal judgment in a mathematical situation in which there is some degree of uncertainty (Test 23). It can be imagined that this might enable the pupil to move away from the safe, familiar ground adhered to by less creative, high attaining contemporaries. Consequently greater ability to break from mental sets and overcome fixation might be shown. The pupil is more likely to be a boy than a girl (Table 5.7, Figures 5.9, 5.10). Girls show tendencies to have lower self-concepts in mathematics and higher levels of anxiety than boys (Table A3.18) and these factors could well inhibit them in situations requiring creative thinking. Boys also tend to be broader coders than girls (Table A3.15). The mathematically creative boy will show a predisposition to think in broad categories (Table A3.16), thus concentrating on similarities rather than differences. It can be imagined that this feature of the pupil's cognitive behaviour would enable him not to restrict his thinking in mathematical problem-solving to narrow domains, and hence he will show greater abilities for breaking from mental sets and for divergent production in mathematics.

The characteristics suggested in the above profile are derived from group data. It would be unsound to conclude from the analysis of group data that certain behaviours associated with personality traits and attitudes must necessarily be demonstrated by high mathematically creative pupils. It is therefore very likely that individual pupils will show variations from this overall description. Consequently it will be helpful to conclude the investigation of the hypotheses about personality aspects by locking at the profiles of individual pupils across the range of instruments used in the research. In particular, Chapter 7 includes a comparison of the profiles of individual high mathematically creative, high attaining pupils and low mathematically creative, high attaining pupils.

CHAPTER 7

PUPIL PROFILES

There were 56 pupils in the sample of 11 - 12 year olds who came into the highest group for mathematical attainment with MA scores greater than or equal to 130. In this chapter it is intended to examine further, particularly for this highest attaining group, the relationships between the personality aspects investigated in the research programme and mathematical creativity as indicated by OF and DP scores. Complete listings of all pupils' scores on the OF tests and DP tests are given in Appendices 4 and 5 respectively. Appendix 6 contains listings of the scores obtained on all the tests and measures used in Chapter 6. These will be the basis of the pupil profiles used in this present chapter. Particular groups of individual pupils will be isolated for comparison. First, the 12 pupils with the highest OF scores and the 12 pupils with the lowest OF scores will be separated out from the group of high attainers. Profiles of the pupils in these two subsets will be compared. Then the same will be done for DP scores. Any pupil appearing in the highest subset for both OF and DP will then be considered as a high mathematically creative pupil. Likewise any pupil appearing in both the lowest subsets will be considered as a low mathematically creative pupil. These high and low mathematically creative pupils will be considered in more detail. Some consideration will also be given to any pupils who appear as high on one creativity measure and low on the other. Then two pupils both with MA scores of 140 will be contrasted: one of them the most mathematically creative pupil and the other the least mathematically creative pupil in the group of high attainers. Finally some consideration is given to relatively high and low mathematically creative pupils in other than the highest attaining group.

Highest and Lowest OF Pupils

Table 7.1 gives profiles of the 12 pupils gaining highest OF scores and the 12 pupils scoring lowest OF scores in the group of very high attainers. The data in this table have been taken from Appendix 6 and relate to risk-taking in mathematics (RT23, RT22), nonconformity (NC), category width (CW), self-concept in mathematics (SCM), anxiety towards mathematics (ATM) and test anxiety (TA).

All scores (except MA) have been standardised for the whole sample to a mean of 100 and standard deviation of 15. MA scores are standardised according to national norms. This, of course, is a procedure which is conventionally applied to normally distributed data. But it is used here even with some of the very non-normal distributions obtained, in particular for the RT23, RT22 and NC indices. The relative failures of Test 23, 22 and 26 to discriminate finely at the top end of the risk-taking and nonconformity constructs have resulted in the maximum standardised scores for RT23, RT22 and NC being only 112, 120 and 113 respectively. However the standardisation procedure has been used here simply to aid in quick interpretation of the figures in the table (and in Appendix 6), since any score around 100 can be immediately recognised as about average for the whole sample, and likewise above average and below average performances can be picked out easily. (Data from Test 27, Clues, have not been included in these profiles: in Chapter 6 it was determined that the risk-taking demonstrated on this test was not the type of risk-taking which could be associated with mathematical creativity and was essentially different behaviour from that shown in Test 22 and 23).

Observations

The following observations arise from examination of Table 7.1.

1. There is a marked sex difference in terms of ability to

Table 7.1

Profiles of the Twelve Highest and Twelve Lowest OF Pupils in the Group of Very High Attainers

Highest OF

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problems
198	M	163	165	140	112	120	113	134	137	69	63	134
199	M	146	172	140	112	120	113	132	126	76	85	134
29	F	146	112	137	112	111	113	102	100	101	109	122
174	M	140	151	140	112	120	103	102	134	76	85	126
66	M	133	134	137	112	116	97	89	100	87	89	89
53	M	130	148	140	112	-1	113	121	117	80	87	126
68	M	130	103	132	112	116	113	108	92	105	126	97
84	M	130	132	140	112	120	113	112	97	94	105	118
137	F	130	119	140	108	120	113	-1	120	87	95	134
204	M	130	117	139	112	120	113	112	117	90	81	124
232	F	130	139	140	94	107	-1	-1	114	94	91	132
241	M	130	118	134	-1	102	113	112	129	87	99	128

Lowest OF

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problems
54	M	102	129	140	72	116	102	109	126	76	69	124
73	M	102	108	132	94	111	113	114	126	101	91	112
112	F	102	92	130	112	102	106	85	103	87	77	112
63	M	101	106	133	90	107	103	105	109	97	101	118
218	F	101	127	140	99	111	-1	112	117	84	73	114
219	F	101	103	131	103	111	113	98	106	101	101	105
239	F	101	110	130	108	116	104	83	103	108	113	124
10	F	96	113	140	99	107	113	101	112	94	95	103
20	F	96	128	140	112	107	108	92	120	87	101	128
172	M	96	99	140	99	120	108	88	100	94	87	107
124	F	91	106	130	112	120	107	116	103	108	97	101
224	F	91	114	134	99	107	97	111	92	118	102	124

overcome fixation in mathematics in this high attaining group. The ratio of boys to girls in the highest OF group is 9:3, whereas the ratio in the lowest OF group is 4:8. This indicates the general trend, but, of course, there are exceptions. One girl, pupil number 29, stands out as being the second highest pupil overall in terms of OF scores.

2. There is a significant difference in willingness to take risks as indicated by the RT23 indices. The highest OF pupils show a markedly stronger preference for this type of calculated, reasoned risk-taking. In the highest OF group nine of the eleven who took Test 23 registered the maximum score of 112, compared with three of the lowest OF pupils. Such a score is understood to mean that on each occasion where there was not a clear, certain correct answer in the multi-choice test the pupil opted for a reasoned guess rather than for the don't know option. One pupil, number 54, in the lowest OF group stands out as being particularly cautious in terms of his behaviour on Test 23. The mean RT23 score for the highest OF group is 109.7, compared with 99.9 for the lowest OF group.

3. There are similar, but less marked, tendencies for the highest OF pupils to register higher RT22 risk-taking indices and also greater levels of nonconformity (NC) than the lowest OF group.

4. There is a significant difference between the two groups in terms of category width. Only one pupil, number 66, in the highest OF group comes out as a narrow coder with a standardised CW score of less than 100. This is compared with five in the lowest OF group. The mean CW scores are 112.4 and 101.2 for the highest and lowest OF groups respectively.

5. From the three scores obtained from the attitude questionnaire (SCM, ATM, TA) the most noticeable feature in Table 7.1 is the clear tendency for the highest OF pupils, with one or two exceptions, to

register very low scores for anxiety towards mathematics (ATM). All but two of the highest OF's are below average in anxiety towards mathematics. Five of the lowest OF's score over 100 for ATM, with one pupil, number 224, registering 118, a surprisingly high ATM score for a high attainer. This pupil is not only the most anxious towards mathematics but also has the lowest OF score (91) for the group of high attainers. It is interesting to note also that the highest OF score (163) is achieved by the pupil (number 198) with the lowest level of anxiety towards mathematics (69). The mean ATM scores for the highest OF and lowest OF groups are 87.2 and 96.3 respectively. There are no marked differences between the two groups in terms of SCM and TA scores.

6. Apart from two pupils (66 and 68), the highest OF group are on the whole better problem-solvers than the lowest OF pupils, based on their performances in Test 20. Given such unusual problems to solve, the pupils who show less reliance upon fixed methods and procedures, as evidenced by their OF scores, are more likely to succeed.

Discussion

From the above observations it seems, therefore, that in terms of the ability to overcome fixations in mathematics and for the group of very high attainers, there is some support from the analysis of the highest OF and the lowest OF pupils for hypotheses 1, 3 and 5, which associated mathematical creativity with willingness to take risks in mathematics, broad categorising and low levels of anxiety towards mathematics. Most of the pupils who show highest ability for overcoming fixation show an attitude of willingness to make judgments in mathematics involving a degree of calculated, reasoned risk. This could be understood in terms of a risk-taking attitude which manifests itself in the pupil maintaining only a loose commitment to safe, proven algorithms or to safe, familiar domains, and therefore always being prepared to consider alternatives. This would no doubt be facilitated by the low level of anxiety towards mathematics which seems to be typical of the pupils who show greatest ability to overcome fixation.

Furthermore it is clear that thinking in broad categories could be an important factor in overcoming fixation in mathematics. Faced with a question about numbers, the broad coding 11 - 12 year old is more likely to think in terms of other than natural numbers. Given a problem requiring the construction of a quadrilateral to fit certain conditions, it can be envisaged that the narrow coder would be more inclined to restrict thinking to particular subsets such as rectangles or convex quadrilaterals. It has been noted earlier that girls tend to be narrower coders than boys in general, and this might well be the basis of the sex difference noted in observation 1 above, particularly if broad coding, as has been suggested by this analysis, has a strong relationship with overcoming fixation.

Highest and Lowest DP Pupils

Table 7.2 gives profiles of the 12 pupils gaining highest DP scores and the 12 pupils scoring lowest DP scores in the group of very high mathematics attainers. The data in this table will be used to discuss any differences which may emerge between pupils who did well in divergent production tasks and pupils who did relatively poorly.

Observations.

The following observations arise from examination of Table 7.2.

1. Once again there is a marked sex difference, this time in terms of ability for divergent production in mathematics, but again in favour of the boys. In the highest DP group the ratio of boys to girls is 10:2, compared with a ratio of 5:7 in the lowest DP

Table 7.2

Profiles of the Twelve Highest and Twelve Lowest DP Pupils in the Group of Very High Attainers

Pupils no.	Sex	CF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problems
Highest DP												
199	M	146	172	140	112	120	113	132	126	76	85	134
198	M	163	165	140	112	120	113	134	137	69	63	134
174	M	140	151	140	112	120	103	102	134	76	85	126
53	M	130	148	140	112	-1	113	121	117	80	87	126
97	M	113	143	136	112	120	113	79	131	76	83	112
232	F	130	139	140	94	107	-1	-1	114	94	91	132
66	M	133	134	137	112	116	97	89	100	87	89	89
84	M	130	132	140	112	120	113	112	97	94	105	118
56	M	108	130	140	90	120	113	114	117	76	83	122
54	M	102	129	140	72	116	102	109	126	76	69	124
20	F	96	128	140	112	107	108	92	120	87	101	128
52	M	108	127	140	81	80	69	116	114	87	71	112
Lowest DP												
68	M	130	103	132	112	116	113	108	92	105	126	97
219	F	101	103	131	103	111	113	98	106	101	101	105
4	F	118	102	131	112	107	113	104	112	94	97	120
139	F	119	102	132	99	93	101	92	117	84	87	112
152	F	113	102	132	99	93	105	122	112	76	79	105
172	M	96	99	140	99	120	108	88	100	94	87	107
188	M	113	98	131	-1	120	-1	109	117	101	113	109
110	F	106	97	131	112	-1	113	101	-1	-1	-1	111
93	M	109	96	130	-1	-1	88	-1	114	80	85	89
125	F	113	94	130	112	120	108	105	109	94	101	112
112	F	102	92	130	112	102	106	85	103	87	77	112
161	M	113	88	140	103	120	85	118	109	84	81	111

Notes:

(1) -1 indicates that the pupil was absent for the test

(2) all scores (except MA) standardised for the whole sample to give mean 100 and standard deviation 15

group. This is the general trend, but again there are exceptions. One girl, pupil number 232, scores particularly highly for divergent production, and the lowest DP score overall in the group of high attainers is registered by a boy, pupil number 161.

2. There are no significant differences between the two groups of pupils in Table 7.2 in terms of either of the risk-taking indices (RT23, RT22). Although eight out of the 12 highest DP's register the maximum RT23 score, compared with five out of 12 of the lowest DP's the four lowest risk-takers using this index are in the highest DP group.

3. The NC indices are distributed across the two groups in very similar ways. No differences in terms of nonconformity emerge.

4. There is a slight tendency for broader coding, indicated by the CW scores, amongst the pupils in the higher DP group, but this observation is influenced by the presence of two very broad coders, pupils 198 and 199, in the group. Overall, category width does not seem to be so significant here as it was for the OF analysis.

5. The most noticeable difference between the two groups are related to their responses to the attitude questionnaire. The highest DP pupils show a markedly higher self-concept in mathematics (SCM). Six of the 12 highest DP pupils register higher SCM scores than the highest SCM score obtained by a pupil in the lowest DP group. On the whole, with one or two exceptions, such as pupils 66 and 84, a very high self-concept in mathematics seems typical of the highest divergent producers. The mean SCM scores for the two groups are 119.4 for the highest DP group and 108.3 for the lowest DP group. Looking down the ATM column in Table 7.2, it is clear that compared to the whole sample, these high attainers do not show particularly high levels of anxiety towards mathematics. But it can be seen that the very lowest ATM scores (e.g. 69 and 76) occur more frequently

in the group of highest DP pupils. Low levels of anxiety towards mathematics are associated again with higher levels of divergent production. No general conclusions may be drawn regarding Test Anxiety (TA). There are two pupils, 68 and 188, in the group of lowest DP pupils, who show particularly high levels of test anxiety, and it may be speculated that this may have affected their performance on the unusual style of test associated with divergent production.

Discussion

From the above observations it seems, therefore, that in terms of the ability for divergent production in mathematics, and for the group of very high mathematics attainers, there is most support from the analysis of the highest and lowest DP groups for hypotheses 4 and 5 which associated mathematical creativity with a high self-concept in mathematics and a low level of anxiety towards mathematics. The most marked characteristics of the pupils in the highest DP group are indeed their high self-concepts in mathematics and their low levels of anxiety towards mathematics. Because they expect to succeed when faced with a problem in mathematics, it may be supposed that they are in a better position than their contemporaries with similar levels of mathematical knowledge and skills when faced with an open-ended divergent production test. They are able more confidently to call upon a wide range of mathematical ideas and are not fearful of trying unusual possibilities. Moreover, being typically less anxious about mathematics they are presumably not worried by the unusual nature of the tests.

Six Pupils Highly Creative in Mathematics and Six Pupils of Low Creativity in Mathematics

Six pupils, numbers 53, 66, 174, 198, 199 and 232, appear in both the highest OF group and the highest DP group in Tables 7.1 and

7.2. These pupils will now be considered as the pupils showing overall the highest levels of mathematical creativity within the group of high attainers. They all have OF scores of 130 or above and DP scores of 134 or above. There were no pupils in other than the highest MA group achieving both these levels of mathematical creativity performance.

Only three pupils in the high attaining group, numbers 112, 172 and 219, appear in both the lowest OF and the lowest DP subset. Three further pupils are in the lowest subset for one measure and only just out of the lowest subset for the other. These are pupils 63, 110 and 124. These six pupils will be considered as the group of lowest creativity in mathematics amongst the high attainers. In this way two groups of six pupils, one a high mathematically creative group, and the other a low mathematically creative group, may be compared. Profiles of the pupils in these two groups are given in Table 7.3.

Observations

Four of the pupils in the high mathematically creative set fit very clearly the tentative description of a hypothetical pupil given at the end of the previous chapter. These are pupils 53, 174, 198 and 199. All four score highly for overcoming fixation and very highly on the divergent production tests. They also have the maximum mathematics attainment score of 140. They are all boys. All emerge as high risk-takers as indicated by their RT23 and RT22 indices, although pupil 53 was absent for Test 22. Apart from pupil 174, who is a relatively average coder according to his responses to the category width test, the other three are very broad coders with CW scores of 121, 134 and 132. All four have high self-concepts in mathematics (SCM scores of 117, 134, 137 and 126) and very low levels of anxiety towards mathematics (ATM scores of 80, 76, 69 and 76). They also indicate low levels of anxiety towards tests in general (with TA

Table 7.3

Profiles of Six High Mathematically Creative and Six Low Mathematically Creative Pupils in the Very High Attaining Group

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problems
High												
198	M	163	165	140	112	120	113	134	137	69	63	134
199	M	146	172	140	112	120	113	132	126	76	85	134
174	M	140	151	140	112	120	103	102	134	76	85	126
53	M	130	148	140	112	-1	113	121	117	80	87	126
232	F	130	139	140	94	107	-1	-1	114	94	91	132
66	M	133	134	137	112	116	97	89	100	87	89	89
Low												
63	M	101	106	133	90	107	103	105	109	97	101	118
110	F	106	97	131	112	-1	113	101	-1	-1	-1	111
219	F	101	103	131	103	111	113	98	106	101	101	105
124	F	91	106	130	112	120	107	116	103	108	97	101
112	F	102	92	130	112	102	106	85	103	87	77	112
172	M	96	99	140	99	120	108	88	100	94	87	107

scores of 87, 85, 63 and 85). All four are good solvers of problems of the type given in Test 20.

The other two pupils in this high mathematically creative group show some deviations from this general description. Pupil number 66, who incidentially has the lowest DP score of the six, is, on the basis of the CW Test, a very narrow coder. He has only an average self-concept in mathematics for the whole sample of pupils. Although he has done well on the mathematical creativity tests he is in fact a poor solver of the unusual type of problems in Test 20. Pupil number 232 is the only girl in this group of high mathematically creatives. She scores rather low on risk-taking and was unfortunately absent for the category width test.

Discussion

This consideration of the six most mathematically creative pupils in the whole sample lends some support to the picture of the mathematically creative 11 to 12 year old put forward in Chapter 6. There may be some individual deviations from this pattern, but the picture of a mathematically creative pupil, who is willing to make judgments based upon a calculated, reasoned risk, who is self-confident and expects to succeed in mathematics, who is not at all anxious about mathematics or tests, and who shows a tendency to think in broad rather than narrow categories, is to some extent confirmed by the analysis in Table 7.3.

Further Observations

Some contrasts with the low mathematically creatives are evident from Table 7.3. The five of them who were present for the attitude questionnaire all emerge with only average self-concepts in mathematics compared to the whole sample (SCM scores of 109, 103, 103, 100 and 106). It should be noted that these are all pupils who presumably do very well in conventional assessments of mathematics attainment. Only one of the six in this low group has the maximum score for mathematics attainment of 140, but this pupil has the lowest self-concept. As a group these six pupils were slightly more anxious towards mathematics than the six high creatives, with pupil 124 showing an above average level of anxiety with an ATM score of 108. None of the six is an exceptionally broad coder, though given the tendency of girls to score lower than boys on the CW test pupil number 124 scores relatively high for CW with 116. The other five pupils in the low creative group are average and narrow coders, with pupils 112 and 172 scoring particularly low for category width (CW scores of 85 and 88). Pupil number 63 is a relatively low risk-taker. Apart from pupil number 66 it is clear that the high creatives are better problem-solvers than the low creatives, based on their scores on Test 20. This last observation lends some degree of validity to the mathematical creativity tests.

Discussion

Each of the low creatives deviates in one or more ways from the description put forward as appropriate for a high mathematically creative pupil. Pupil 63 is relatively unwilling to take risks of the type involved in Test 23. Pupil 112 is a particularly narrow coder. Pupil 124 has an above average level of anxiety towards mathematics. Pupil 172 is a narrow coder, and with a mathematics attainment score of 140 has a surprisingly low self-concept in mathematics. Pupil 219 is just below average for category width and shows average levels of anxiety towards mathematics and towards tests in general. Four out of the six low mathematically creatives are girls.

High OF/Low DP and Low OF/High DP Pupils

Observations

One pupil, number 68, appears in both the group of highest OF pupils in Table 7.1 and the group of lowest DP pupils in Table 7.2. This boy therefore warrants some consideration. Scanning his scores for the various measures reveals that of the 12 highest OF pupils he shows the greatest level of anxiety towards mathematics (ATM = 105) and the lowest self-concept in mathematics (SCM = 92). But even more significant perhaps is his very high level of test anxiety (TA = 126). This is in fact the third highest TA score of all 56 pupils in the high attaining group.

Discussion

His high anxiety level can be interpreted as a possible contributing factor to his poor performance in the divergent production tests. This is presumably a pupil who usually does well in conventional mathematics assessments, but nevertheless worries excessively about tests. Consequently when faced with the unusual style of the divergent production tests his anxiety may inhibit him in calling upon a wide range of mathematical ideas and he shows little flexibility or originality. His low score in the test of unusual problems would confirm this analysis. He is, in fact, a real example of the hypothetical pupil P considered in the discussion about Test Anxiety results in Chapter 6. There are in fact three other high attaining pupils with similar very high levels of test anxiety. These are pupils numbered 86, 101 and 212 - see Appendix 6 for their profiles. All three of these also score relatively low marks for divergent production (109, 113 and 106 respectively) compared to the mean score of 118 for the high attaining group.

<u>Observations</u>

Two pupils, numbers 20 and 54, appear in both the lowest OF group in Table 7.1 and the highest DP group in Table 7.2.

Pupil number 20 is a very high attaining pupil with the maximum score of 140. Her fairly high DP score indicates that she is able to call upon a range of mathematical ideas successfully and show some degree of flexibility and originality. However she shows high levels of fixation. The feature of her profile which stands out is her very low CW score.

<u>Discussion</u>

It has already been suggested that thinking in broad categories is generally associated with overcoming fixation. If this girl (pupil number 20) tends to think in narrow categories when faced with mathematical problems requiring the breaking of mental sets then her failure in such problems is easily understood.

<u>Observations</u>

Pupil number 54 is another very high attainer who shows little ability to overcome fixation. This boy's RT_{23} index stands out as being exceptionally low.

Discussion

If this correctly indicates that this is an over-cautious pupil in his approach to mathematics then this fact could have contributed to the high level of content universe fixation shown by him in the OF tests. The risk-taking willingness indicated by RT_{23} seems to be more associated with overcoming fixation than with divergent production.

Comparisons of Two High Attaining Pupils: one a High Creative, the Other a Low Creative in Mathematics

To clarify further the picture of the high mathematically creative pupil which emerges both from the consideration of correlations in Chapter 6 and the pupil profiles undertaken in the present chapter, it is proposed now to compare two individual pupils in detail. These are two pupils with the same high score for mathematics attainment, but one a very high mathematically creative, the other relatively low. A comparison will be made of their behaviours on the personality-based tests. Then their actual responses on a number of OF and DP tests will be compared. This comparison will (a) give the reader some indication of what the responses of a high mathematically creative pupil look like when set alongside those of a low creative, and (b) allow consideration of the extent to which these responses are consistent with the observed differences in personality characteristics between the two pupils.

The two pupils are both boys, numbers 198 and 172 (see Table 7.3). It is helpful to note that these two boys were at the same school, in the same (top) mathematics set, and they both registered the maximum score of 140 on the NFER EF test of mathematics attainment. It cannot be assumed, of course, that they are equal in terms of mastery of mathematical skills and knowledge, since the standardised scores on the NFER EF test do not allow for anything greater than 140. However it can be assumed, both in terms of national norms, and in terms of the sample used for this research, that these two boys are very high attainers by conventional standards of school mathematics. The reason for comparing these two boys is that they show markedly different abilities in terms of their performances on the mathematical creativity measures. Pupil 198 scores 163 for OF and 165 for DP, whereas pupil 172 scores only 96 for OF and 99 for DP. Using these measures it would appear that in the group of high attainers these two boys are the most and the least mathematically creative respectively. Pupil 198 is rivalled for high mathematical creativity only by pupil 199 (see postscript at the end of this chapter). And pupil 172 gains below average OF and DP scores compared to the whole sample of 283 pupils, even though he is one of the very highest attainers in mathematics.

Responses on Personality-Related Tests

First a comparison will be made of their responses on the personality-related tests.

Small differences are apparent in their responses to Tests 23 and 26. In Test 23, which was concerned with risk-taking, the high mathematically creative pupil 198 was willing to take a risk on every item in the multi-choice test. He chose no "don't know" options at all. Pupil 172, the low mathematically creative boy, opted for the "don't know" option in question 6, rather than make a reasoned, calculated guess as to the meaning of the unfamiliar term 'median'. In the other multi-choice test number 26, concerned with nonconformity, pupil 198 made no changes at all in the direction of the suggested incorrect options. Pupil 172 made one change. In question 3, he had originally given the answer 0.1 for the calculation $(0.1)^2$, but changed this to the suggested most popular answer, 0.2. Nothing of any great significance can be read into these comparisons. All that can be said is that if Test 23 is at all an indication of willingness to take risks of the sort suggested then the high mathematically creative pupil showed the maximum possible willingness, and the low creative did not and if Test 26 is at all an indication of nonconformity, then the high mathematically creative pupil showed the maximum possible level of nonconformity, and again the low creative did not.

However there are some significant comparisons to be made between these two pupils in terms of category-width and their responses to the attitude questionnaire.

Pupil 198 emerges as a very broad coder, whereas pupil 172 is a low coder compared to the whole sample. (Their standardised CW scores are 134 and 88 respectively). In fact pupil 198 registered a raw score of 54 out of a maximum possible of 60, compared with 22 for pupil 172, on Test 25. So for example, given the question:

> Most roads are about 9 metres wide.
>
> (a) How wide is the widest road?
>
> (options: 52m, 27m, 12m, 36m)
>
> (b) How wide is the narrowest road?
>
> (options: 8m, 3m, 1m, 6m),

pupil 198's answers were (a) 52m (b) 1m, whereas pupil 172's answers were (a) 12m (b) 6m. Consistently in questions of this sort the high creative pupil chose options a long way away from the given central tendency, and pupil 172 tended to choose options fairly close to the given central tendency. The theoretical construct of the test is that such behaviours indicate tendencies to code information in broad and narrow categories respectively. So the most mathematically creative pupil is on this basis a very broad coder, and the least mathematically creative pupil in the group of high attainers is a very narrow coder.

On the attitude questionnaire (Test 21) the high creative pupil 198 emerges as being a pupil of very high self-concept in mathematics, with a standardised SCM score of 137, of very low anxiety towards mathematics, with a standardised ATM score of 69, and very low test anxiety, with a standardised TA score of 63. In fact this pupil responded with SA (strongly agree) or SD (strongly disagree) to every item in the questionnaire, except the fillers, and always in the direction of high self-concept or low anxiety. Hence he registered the absolute maximum score for self-concept, and the absolute minimum scores for anxiety toward mathematics and towards tests. By contrast, pupil 172 registers only an average standardised score for self-concept in mathematics (100 exactly). This is a pupil of very high mathematical attainment, yet one who indicates that he is "not sure" about, for example, the following statements related to self-concept.

> I feel at ease in a maths lesson.
>
> I don't do very well in maths.
>
> Maths is easy for me.
>
> I am good at doing maths problems.
>
> I remember most of the things we learn in maths.

His standardised scores for anxiety towards mathematics and test anxiety are 94 and 87, both below average for the whole sample. But some of his responses here are revealing. For example, he agrees with the following statements.

> I worry more about maths tests than most other things at school.
>
> I worry a lot while I am taking a test.

<u>Discussion</u>. These then are some indications of the personality

types being compared. Two pupils, very high in mathematics attainment, but one a high mathematically creative, the other a low mathematically creative. The high creative is, in particular, a very broad coder, has a very high self-concept in mathematics and very low levels of anxiety towards mathematics and tests. He is perhaps a high risk-taker, of the type indicated by RT23, and not easily persuaded away from his own judgments in mathematics. The low creative is a narrow coder and shows a surprisingly modest self-concept in mathematics for a high attainer. There are indications of less confidence in his behaviours in Test 23 and 26, and he admits to worrying about mathematics and while doing tests. It will be illuminating to look now at their actual responses to the mathematical creativity tests.

Responses on OF Tests

Table 7.4 summarises the responses of these two pupils on the tests related to overcoming fixation.

It can be seen from Table 7.4 that pupil 198 is completely successful in all the tasks requiring breaking from mental sets. He does not show any fixations whether in terms of content universe or algorithms. Pupil 172 appears to be particularly inclined to algorithmic fixation. This is a type of behaviour which, relying on safe, learnt procedures, is probably appropriate for most conventional assessments in mathematics. But here his fixation on the algorithms which he finds to work in the early parts means that he fails to find the most efficient ways of solving the final parts in Tests 10 and 24. This type of thinking could well explain his failure in Test 14 and 20 (qu. 6). In the magic square question he could fail to consider the use of fractions because he relies upon a trial-and-error procedure, which works in the other cases, which involves trying each whole number in turn. Also if his response to a mathematical question is to categorise it in terms of which learnt algorithm

Table 7.4

Comparison of High and Low Mathematically Creative Pupils' Performance in OF Tests

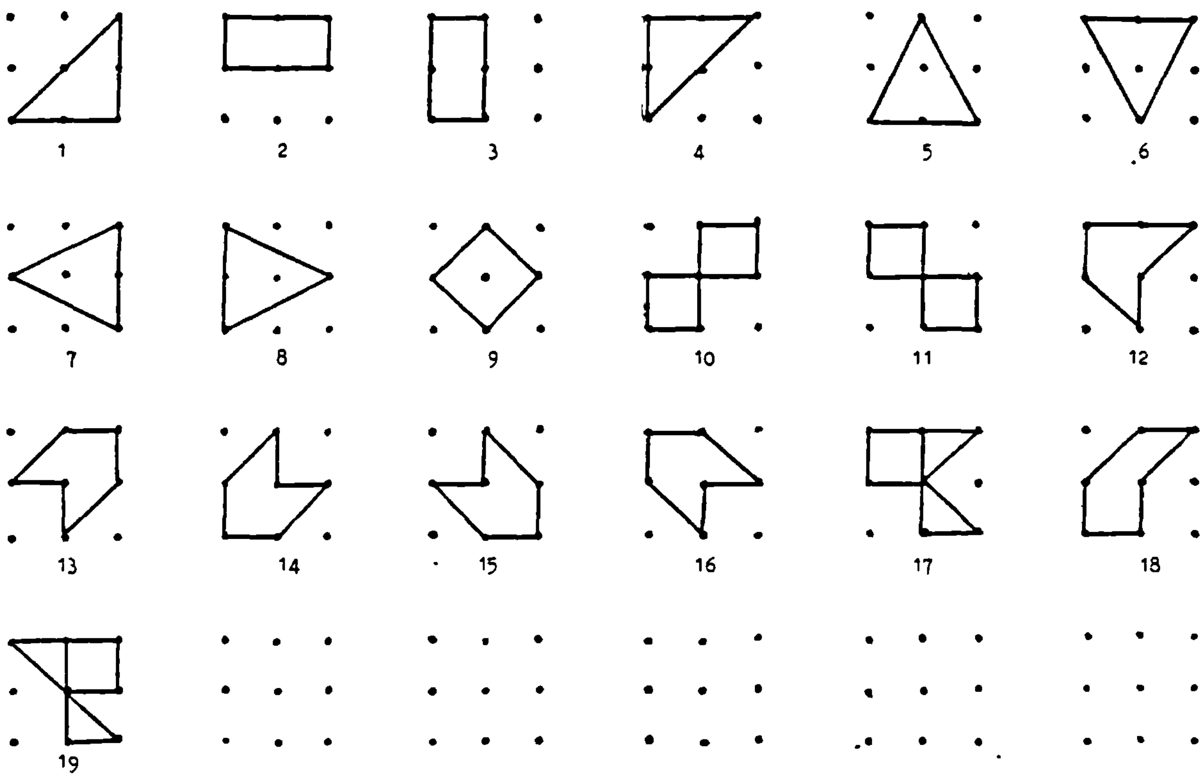
Content universe fixation:	Pupil 198 (high)	Pupil 172 (low)
Test 1: Areas	All four parts solved correctly. No fixation on right angles, concavity etc. shown	All four parts solved correctly. No fixation on right angles, concavity etc. shown
Test 14: Sum and Difference	Solved all parts including (x) correctly: "Find two numbers whose sum is 9 and whose difference is 2. Answers: $5\frac{1}{2}$, $3\frac{1}{2}$"	All ten parts solved by the same incorrect procedure, e.g. "Find two numbers whose sum is 10 and whose difference is 4. Answers: 14, 6"
Test 20 (qu. 6): Magic Square	Solutions all correct, including last part: $2\frac{1}{2}$ 6 $\frac{1}{2}$ 1 3 5 $5\frac{1}{2}$ 0 $3\frac{1}{2}$	Failed to break from fixation on whole numbers in last part: 3 6 0 1 3 5 5 0 4
Algorithmic fixation:		
Test 10: Jugs	Gave solutions breaking from mental set: 1. B - A - C - C 2. B - A - C - C 3. B - A - C - C 4. B - A - C - C 5. A - C 6. C	Demonstrated the anticipated behaviour of the fixation construct: 1. B - A - C - C 2. B - A - C - C 3. B - A - C - C 4. B - A - C - C 5. B - A - C - C 6. B - A - C - C
Test 24: Cuts	Gave solutions breaking from mental set: 3 parts, 2 lines 5 parts, 4 lines 7 parts, 6 lines 9 parts, 4 lines	Gave solutions showing algorithmic fixation: 3 parts, 2 lines 5 parts, 4 lines 7 parts, 6 lines 9 parts, 8 lines

should he apply to it, then the stimulus of the words "sum"and "difference" in Test 14 would produce the algorithmic response of simply finding the sum and the difference of the two given numbers. It could be suggested that a form of narrow coding of mathematics questions is involved here. It has been noted earlier that pupil 172 is a narrow coder. It is tempting also to associate his modest self-concept in mathematics with his responses to these tests. Having found a method which works, the pupil with less confidence in his own abilities in mathematics is more likely than his more self-confident contemporary to stick to that method rather than to seek more efficient alternatives.

Responses on DP Tests

Similar comparisons can be made from their responses to the divergent production tests. Figure 7.1 shows their responses to Test 7 (Nine Dot Areas), a divergent production, problem-solving task in a spatial domain. Pupil 172, the low creative, shows much repetition of the same ideas. In fact, responses numbers 3, 4, 6, 7, 8, 11, 14, 15, 16 are all discounted because they are duplicates of other responses in different positions. Responses 10, 11, 17, 19 are not considered appropriate (see discussion of this test in Chapter 4), leaving only seven acceptable responses. None of these is original. All except number 5 are composed of combinations of unit squares and half-unit square triangles. Pupil 198, by contrast immediately starts off with an original response, number 1. He shows great flexibility, jumping around from one idea to another in seeking solutions. He makes no duplicates, and produces several highly original shapes, such as numbers 6, 12, 14, 16, and 17. Both pupils have provided 19 responses in total, but one shows much narrowness in his thinking and preference for safe, simple solutions, whereas the other shows flexibility and willingness to encounter

PUPIL 172



PUPIL 198

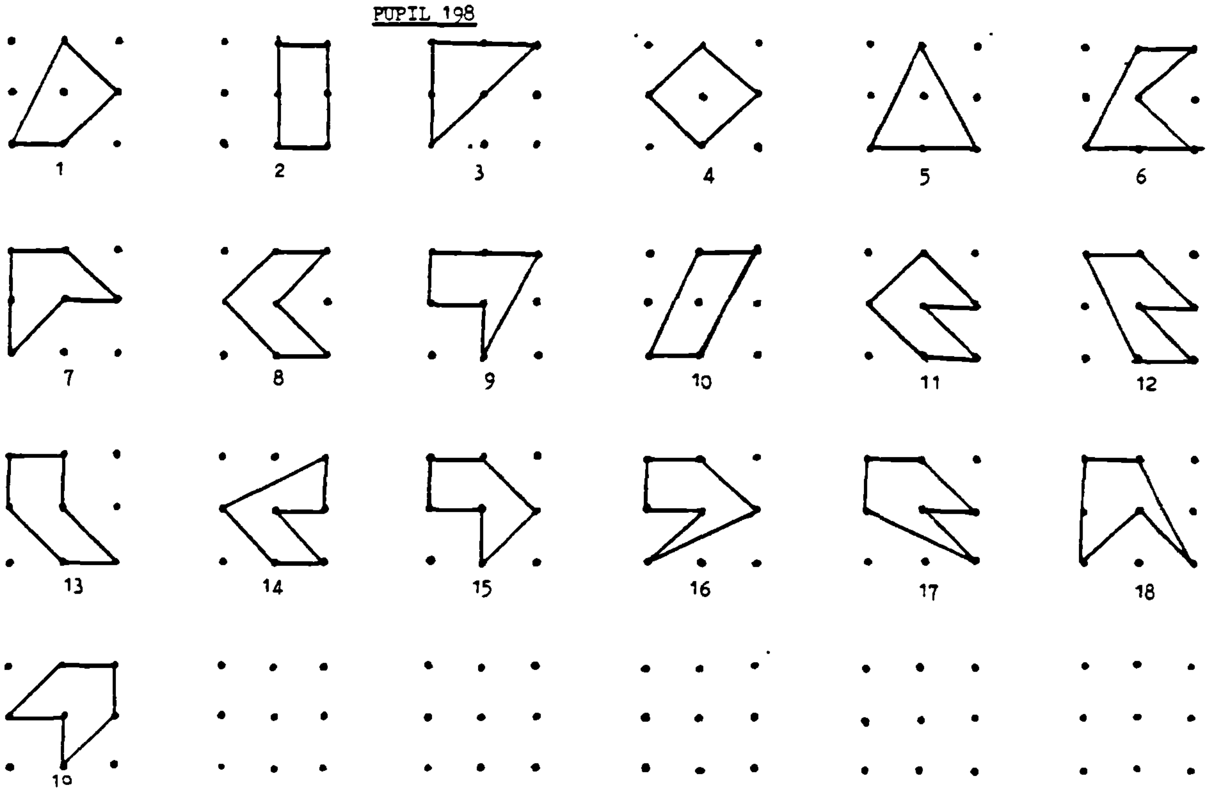


Figure 7.1: Responses of pupils 172 and 198 to Test 7 (Nine Dot Areas)

complexity in seeking solutions.

Again, similar behaviour is demonstrated in Test 16 (Results), another problem-solving task, but in a numerical domain. Table 7.5 gives the complete set of their responses to this test. Pupil 172

Table 7.5

Responses of Pupils 172 and 198 to Test 16 (Results)

Pupil 172

1. 23 x 350 = 8050
2. 230 x 350 = 80500
3. 23 x 3500 = 80500
4. 230 x 3500 = 805000
5. 2300 x 350 = 805000
6. 23 x 35000 = 805000
7. 230 x 35000 = 8050000
8. 2300 x 35000 = 80500000
9. 230 x 350000 = 80500000
10. 2300000 x 3500 = 8050000000
11. 23 x 3500000000000 = 80500000000000
12. 230000000000 x 3500000000000 = 805000000000000000000000
13. 2300 x 350 = 805000
14. 23 x 350000000 = 8050000000
15. 230 x 3500000000 = 8050000000
16. 230000 x 350 = 80500000
17. 23000000 x 35000 = 805000000000
18. 23000 x 35 = 805000

Pupil 198

1. 23 x 250 = 8050
2. 230 x 35 = 8050
3. 230 x 350 = 80500
4. 805 ÷ 35 = 23
5. 805 ÷ 23 = 35
6. (35 x 20) = (35 x 3) = 805
7. (23 x 30) + (23 x 5) = 805
8. 805 ÷ 2.3 = 350
9. 805 ÷ 3.5 = 230
10. 8050 ÷ 23 = 350
11. 8050 ÷ 35 = 230
12. (20 x 30) + (20 x 5)+ (30 x 3) + (30 x 20) = 805
13. 23000 x 35000 = 805000000
14. 805000 ÷ 2300 = 35
15. 80500 ÷ 3500 = 23
16. 80500 ÷ 3500 = 23
17. 80500 ÷ 350 = 230
18. 805000 ÷ 350 = 2300
19. 805000 ÷ 230 = 3500
20. 805 ÷ 230 = 3.5
21. 805 ÷ 350 = 2.3
22. 0.023 x 0.035 = 0.000805
23. 85000000 ÷ 23 = 3500000
24. 85000 ÷ 23000000 = 0.00035
25. 85000 ÷ 3500000 = 0.00023
26. 8050 ÷ 23000 = 0.35
27. 8050 ÷ 35000 = 0.23
28. 8050 ÷ 3500 = 2.3
29. 8050 ÷ 2300 = 3.5

finds in the given example a method of deducing other results which is safe and reliable, and he applies it ad infinitum. Pupil 198 however, although he recognises that a given idea can be applied repeatedly, as in his responses 14 - 21, shows much greater willingness to seek alternative approaches. He calls upon a range of ideas, such as decimals, division, brackets, and the original idea of using the distributive law in response 6. Sometimes his adventuresomeness leads him into errors (as in response 12) and slips (as in responses 25, 26 and 27).

Tables 7.6 and 7.7 give the responses of these two pupils to two problem-posing tasks, one (Cross-number) in a numerical domain, and the other (Scattergram) in a spatial domain.

Table 7.6

Clues Provided by Pupils 198 and 172 in Responses to Test 11 (Cross-number)

	Pupil 198	Pupil 172
a Across	If I flew 1197.9 km in 9.9 hours how many km would I fly in 1 hour?	73 + 58
c Across	$100000_2 = \ldots\ldots\ldots\ldots_{10}$	17 + 19 - 14 + 10
e Across	$144_5 = \ldots\ldots\ldots\ldots_{10}$	7^2
g Across	$4_{10} = \ldots\ldots\ldots_2$	100 + 100 + 100 - 200 + 500 - 100 - 400
a Down	½ dozen x 2 dozen x 1 dozen - ½ dozen - 1/6 dozen	12 x 12
b Down	This number is unlucky for some and is 4 times its second digit plus one.	unlucky for some.
d Down	$5^4 - 5^3 - 2$	5 x 50
f Down	$3^8 - 9^2$	$9^2 + 20 + 30 - 50$

Table 7.7

Questions Posed by Pupils 198 and 172 in Responses to Test 12 (Scattergram)

Pupil 172

1. How many families have 2 boys and 4 girls? Answer, 0.
2. How many boys are there than girls? Answer, 7.
3. How many girls have two brothers? Answer, 7.
4. How many boys have three sisters? Answer, 2.
5. How many families had 4 boys and 3 girls? Answer, 1.
6. How many boys are there on the chart? Answer, 46.
7. How many girls are there on the chart? Answer, 39.
8. How many people altogether? Answer, 85.

Pupil 198

1. How many families have more boys than girls? Answer, 14.
2. How many families have more girls than boys? Answer 9.
3. How many girls are there? Answer, 39.
4. How many boys are there? Answer, 46.
5. How many families have equal amounts of boys and girls? Answer, 7.
6. How many families are there in the class? Answer, 30.
7. How many children are there in all the families? Answer, 85.
8. How many families have more than the square of 2 in them? Answer, 3.
9. How many families have less than 4 children in them? Answer, 22.
10. How many families have 4 children? Answer, 5.
11. What is the average number of children are there in a family (to the nearest whole number)? Answer, 3.

(pupil's wording quoted verbatim)

Again the preference for handling complex ideas demonstrated by pupil 198 contrasts with the preference for safe, unchallenging responses shown by pupil 172. This is particularly true in their responses to the Cross-number puzzle, in which they were invited to make up clues to fit given answers. The low creative pupil restricts his thinking to such safe ideas as simple addition, subtraction, multiplication and square numbers. These are mathematical skills and knowledge which a pupil of high attainment knows that he can handle successfully and without difficulty. By contrast, pupil 198 calls upon a range of ideas from his mathematical experience, such as number bases, decimals, cube and higher powers, digits, and speed calculations. It should be remembered that these two pupils have the same mathematical background. But one shows willingness to use this background so much more creatively than the other.

Tables 7.8 and 7.9 show the responses of these two pupils on two of the divergent production tests constructed around the notion of redefinition. It would be expected that if these two boys carried over their tendencies to broad or narrow coding into mathematics then it would be in such tests as these that differences between them would be most acute. Given a set of numbers and asked to make up subsets (Test 3), the broad coder should show greater ability to group the elements of the set together in different ways. This is certainly apparent in the responses in Table 7.8. Pupil 172 gets hold of one idea and uses it repeatedly. Pupil 198 is not averse to using one particular idea to the point of exhaustion (as in responses 12 - 20), but he shows much greater ability to recategorise the elements of the given set, using five different ideas, compared with the two ideas used by his less creative contemporary. Similar comments should apply to their responses in Table 7.9. Given two diagrams to compare, it would be expected that a broad coder would be able

Table 7.8

Responses of Pupils 198 and 172 to Test 3 (Subsets)

Pupil 172

1.	2, 4, 6, 8, 10, 12, 14, 16	(even numbers)
2.	1, 3, 5, 7, 9, 11, 13, 15	(odd numbers)
3.	3, 6, 9, 12, 15	(multiples of 3)
4.	4, 8, 12, 16	(multiples of 4)
5.	5, 10, 15	(multiples of 5)
6.	6, 12	(multiples of 6)
7.	7, 14	(multiples of 7)
8.	8, 16	(multiples of 8)
9.	9	(multiples of 9)
10.	10	(multiples of 10)
11.	11	(multiples of 11)
12.	12	(multiples of 12)
13.	13	(multiples of 13)
14.	14	(multiples of 14)
15.	15	(multiples of 15)
16.	16	(multiples of 16)

Pupil 198

1.	2, 4, 6, 8, 10, 12, 14, 16	(even numbers)
2.	3, 5, 7, 9, 11, 13, 15	(odd numbers)
3.	4, 9, 16	(square numbers)
4.	2, 3, 5, 7, 11, 13	(prime numbers)
5.	4, 8, 12, 16	(multiples of 4)
6.	2, 6, 9, 12, 15	(multiples of 3)
7.	5, 10, 15	(multiples of 5)
8.	7, 14	(multiples of 7)
9.	6, 12	(multiples of 6)
10.	8, 16	(multiples of 8)
11.	2, 3, 4, 5, 6, 7,.....16	(multiples of 1)
12.	2, 3	(numbers which are 3 and under)
13.	2, 3, 4	(numbers which are 4 and under)
14.	2, 3, 4, 5	(numbers which are 5 and under)
15.	2, 3, 4, 5, 6	(numbers which are 6 and under)
16.	2, 3, 4, 5, 6, 7	(numbers which are 7 and under)
17.	2, 3, 4, 5, 6, 7, 8	(numbers which are 8 and under)
18.	2, 3, 4, 5, 6, 7, 8, 9	(numbers which are 9 and under)
19.	2, 3, 4, 5, 6, 7, 8, 9, 10	(numbers which are 10 and under)
20.	2, 3, 4, 5, 6, 7, 8, 9, 10, 11	(numbers which are 11 and under)

Table 7.9

Responses of Pupils 198 and 172 to Test 5b (Similarities: Shapes)

Pupil 172

1. They both have a line going from A to C.
2. If you add up all the angles they both come to the same amount.
3. They both have four sides.
4. They are both labelled with A, B, C, D
5. They both have A and C opposite to each other.
6. They both have B and D opposite to each other.
7. They both have A next to B and D.
8. They both have B next to A and C.
9. They both have C next to B and D.
10. They both have D next to A and C.

Pupil 198

1. They are both quadrilaterals.
2. They are both parallelograms.
3. They both have a line AB.
4. They both have a line BC.
5. They both have a line CD.
6. They both have a line DA.
7. They both have a line AC.
8. They both have a triangle ABC.
9. They both have a triangle ADC.
10. They both have an area ABCD.
11. They both have an angle ABC.
12. They both have an angle DAB.
13. They both have an angle BCD.
14. They both have an angle ADC.
15. They both have an angle DAC.
16. They both have an angle BAC.
17. They both have an angle DCA.
18. They both have an angle ACB.

to see more similarities than a narrow coder. In fact there is not a great difference in the level of creativity, as measured by fluency and originality, between their responses here. Pupil 198 uses slightly more ideas and the reference to angles in responses 11 - 18 is an original idea, although he uses it repeatedly.

Summary of Comparison Between Two Pupils

To summarise, it has been seen that these two pupils show differences in personality traits, particularly in terms of attitudes and category width, which to some extent appear to be consistent with their responses in the mathematical creativity tasks. The high creative pupil is a broad coder and shows more inclination to think in broad categories in divergent production tests and not to restrict his thinking in tests requiring the breaking of mental sets. He is highly self-confident in mathematics and shows no anxiety. These attitudes are consistent with his willingness to try alternative methods, to encounter complexity in problem-solving and not to limit himself to safe, predictable responses. The low creative pupil is a narrow coder and seems to carry over this tendency into his mathematical thinking to some extent. This is consistent with his poor ability to break from mental sets, and his tendency to use the same ideas repeatedly in divergent production tasks. These behaviours are also consistent with his modest self-concept in mathematics. He shows little inclination to depart from safe, predictable responses, and having found a method tends to stick to it.

Summary of Conclusions from Pupil Profiles Within the Highest Attaining Group

From the analysis of profiles of pupils within the highest attaining group undertaken in this chapter it can be concluded that to some extent the tentative description of the high mathematically creative pupil and the less creative counterpart, given at the end of Chapter 6, is realised in the pupils used in this research. Of course, not all the characteristics assigned to the hypothetical high creative will be necessarily present in the real high creative. The analysis of pupils in Tables 7.1, 7.2 and 7.3 showed that there are exceptions. But there is a clear general trend, and the two pupils compared in detail highlight this trend. The high mathematically creative pupil can be expected to be a broad coder, to have a high self-concept in mathematics and low levels of anxiety towards mathematics and tests. The pupil will probably demonstrate willingness to make reasoned judgments involving elements of risk in mathematics, and will trust those judgments. In many of these respects this pupil may be contrasted with the low mathematically creative pupil. In particular, the low creative is likely to be a narrow coder and to have only a modest level of self-concept in mathematics. Consideration of the responses of two such particular individuals have shown that their behaviours in the mathematical creativity tests are consistent with these marked differences in their personality traits.

Profiles of Other Than Very High Attainers

The consideration of correlations in Chapter 6 suggested that the clearest contrasts between the profiles of high and low mathematically creative pupils would emerge in the group of highest mathematical attainment. Consequently the present chapter has concentrated on pupil profiles within this group. In doing this it was noted that no pupil of other than the highest attaining group would have been categorised as high mathematically creative by the criteria used in selecting the six high creatives listed in Table 7.3 and considered earlier. With so much data available for so many tests it has been necessary to be selective in the analysis of profiles and so this has been restricted to the area likely to be most fruitful. Had resources permitted a similar procedure to that carried out in this chapter for the high attainers could have been undertaken for the other groups of pupils in the sample, even though the correlations had suggested that less clear patterns of pupil profiles would emerge.

However, a preliminary analysis along these lines was begun. First Tables 7.10 and 7.11 were compiled to allow comparison of profiles of high and low mathematically creative pupils from other than the highest band of MA. In each of these tables, two pupils have been selected from each of the bands B (above average attainment), C (average) and D (below average), one from the top half of the band, and one from the lower half of the band, who were high scorers on the mathematical creativity measure compared to pupils in that band. (See Figures 5.6 and 5.7). A further two pupils of similar MA scores but low on the mathematical creativity measure were selected from each band. The pupils are ranked in Tables 7.10 and 7.11 by MA scores, for each of the High and Low subsets. This means that in each table the pupil in any given row of the High subset is of similar MA level to the pupil in the corresponding row of the Low subset. If the profiles of these pairs of pupils are compared there is no clear indication of the trends noticed amongst the group of very high attainers being present here. Nor is there any clear group differences in terms of performances on the personality-related measures between the High and Low OF's or DP's. For example, the CW columns do not suggest that these High OF and DP subsets are on the whole broader coders than the Low subset. Nor do the scores in the SCM and ATM columns suggest the trends to higher self-concept and lower anxiety towards mathematics which were found to distinguish the high mathematically creative in the group of very high attainers.

Table 7.10

Profiles of Six Pupils High in OF for Their Level of MA, and Six Pupils Low in OF for Their Level of MA, Taken from Other Than the Highest MA Band

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problems	MA band
High OF													
113	F	130	108	127	108	120	113	99	106	108	118	124	B
141	F	121	-1	118	-1	93	-1	80	77	108	81	79	B
264	M	121	97	114	90	93	90	104	117	87	91	101	C
222	F	113	102	103	81	111	-1	105	86	105	110	107	C
116	F	102	76	99	112	84	73	95	77	123	109	91	D
205	M	101	74	86	112	75	113	104	64	129	121	75	D
Low OF													
267	M	82	101	123	103	102	113	79	122	76	63	99	B
49	M	82	86	116	112	93	104	105	122	97	93	89	B
57	M	79	-1	112	112	80	113	91	103	123	125	81	C
147	F	79	83	106	99	93	92	138	83	97	105	85	C
238	F	82	92	97	90	70	61	109	77	118	125	85	D
254	M	82	75	79	76	89	73	137	86	97	85	77	D

Note: MA band, B = above average,
C = average,
D = below average.

Table 7.11

Profiles of Six Pupils High in DP for Their Level of MA, and Six Pupils Low in DP for Their Level of MA, Taken From Other Than the Highest MA Band

High DP

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problem	MA band
14	F	96	139	128	112	-1	113	92	100	105	102	109	B
236	F	108	119	119	90	102	101	101	83	105	125	99	B
45	M	96	116	112	86	-1	99	-1	97	90	89	83	C
250	M	80	110	102	99	107	-1	96	103	105	91	99	C
283	M	91	103	97	112	107	-1	102	114	87	85	79	D
25	F	82	94	79	99	89	85	-1	97	123	125	83	D

Low DP

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problem	MA band
163	M	-1	92	124	112	120	94	93	114	76	65	93	B
164	M	89	75	117	112	84	92	89	92	87	102	99	B
180	M	108	83	108	112	120	65	104	103	84	91	99	C
115	F	85	77	101	67	93	89	75	89	133	114	83	C
116	F	102	76	99	112	84	73	95	77	123	109	91	D
154	M	82	75	79	76	89	73	137	86	97	85	77	D

Note: MA Bands, B = above average

C = average

D = below average

Tables 7.12 and 7.13 show a second approach to identifying subsets of relatively high and low mathematically creative subsets in other than the high attaining group. In these subsets a high mathematically creative pupil is defined as having both OF and DP scores greater than some minimum value. Similarly a low mathematically creative pupil is defined as having both OF and DP scores less than some stated maximum value. A search was undertaken amongst pupils with MA scores less than 130 with various values for the minimum value of OF/DP. It was necessary to set this as low as 107.5 (half a standard deviation above the mean) before a small group of four pupils was found. These are the relatively high mathematically creative pupils in Table 7.12. The MA scores of these four pupils were found to lie in the above average range, 115 to 129 inclusive. Four pupils in this MA range of low mathematically creative ability were then found by setting a maximum OF/DP level of 90. These relatively high and low mathematically creative subsets can be compared in Table 7.12. Again it seems that the characteristics of broader coding, higher self-concept and lower anxiety towards mathematics which were found to be typical of high mathematically creatives in the high attaining group do not appear here. One observation which can be made is that three of the four in the relatively high mathematically creative subset show particularly high levels of test anxiety. This can be interpreted as test anxiety being associated with pupils who have certain creative competences in mathematics but who find that in conventional assessments styles of performance are demanded of them at which they are less competent and consequently they usually fail to make the highest grades in mathematics.

A second search was undertaken amongst pupils with MA scores less than 119 (119 was the lowest MA score in the high subset in Table 7.12), but setting the minimum OF/DP at lower values. Setting

Table 7.12

Profiles of Relatively High and Low Mathematically Creative Pupils

With MA Scores in the Range 116 - 127

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problems
High												
113	F	130	108	127	108	120	113	99	106	108	118	124
236	F	108	119	119	90	102	101	101	83	105	125	99
253	M	113	120	125	94	84	103	122	100	87	99	85
259	M	113	116	125	112	107	90	124	103	90	117	118
Low												
49	M	82	86	116	112	93	104	105	122	97	93	89
118	F	85	88	121	112	120	103	140	92	118	109	91
164	M	89	75	117	112	84	92	89	92	87	102	99
193	M	85	82	120	86	93	113	89	114	94	99	114

Table 7.13

Profiles of Relatively High and Low Mathematically Creative Pupils

With MA Scores in the Range 108 - 117

Pupil no.	Sex	OF	DP	MA	RT23	RT22	NC	CW	SCM	ATM	TA	Problems
High												
12	F	113	106	114	112	116	113	76	109	94	79	103
202	M	106	104	117	76	93	113	-1	109	80	75	89
213	M	113	106	111	76	120	113	104	112	76	95	-1
221	F	102	103	114	112	98	113	102	69	123	109	95
223	F	108	105	117	76	102	92	141	81	123	114	87
Low												
111	F	85	83	113	112	98	113	96	97	94	118	97
132	F	85	84	108	76	98	97	96	103	94	109	87
138	F	82	84	109	94	93	113	82	66	126	110	81
171	M	85	81	113	112	84	87	109	97	97	85	107
183	M	82	80	109	112	80	85	89	94	118	87	89

this level at 102 produced the subset of five relatively high mathematically creative pupils in the MA range shown in Table 7.13. By setting a maximum OF/DP level of 86 a low mathematically creative subset of five pupils in this MA range was then identified for comparison. Again, consideration of Table 7.13 does not suggest the same pattern of characteristic differences between high and low mathematically creative pupils which was found in the high attaining group. One or two initial observations can be made however. The pupils in the low subset in Table 7.13 are all below average risk-takers in Test 22, and this is not the case with the high subset. This test was based on the construct of pupils showing confidence to risk gaining or losing marks, by exposure to unknown mathematics questions. (In Table 6.7 a highly significant correlation between RT_{22} and DP scores was recorded for the group of average attainers). The least creative pupils in this lower attainment range show much less willingness to enter upon an unknown mathematical situation. It could be suggested that, being not particularly successful in mathematics they have learnt from experience that it is safer to stick with what they know they can do. Such an attitude would be unfavourable to showing creativity in mathematics. A second observation from Table 7.13 is that four out of the five pupils in the high subset score the maximum for nonconformity. This is the first time in the analysis that there has been any indication that nonconformity in mathematics as assessed by Test 26, might be associated with higher levels of mathematical creativity.

Finally two girls (221, 223) in the high mathematically creative subset in Table 7.13 stand out as showing particularly low levels of self-concept in mathematics and high anxiety towards mathematics. Consideration of these two girls suggests why some of the hypotheses being investigated are supported only within the group of very high attainers. For other than the high attainers it may be that having a high self-concept in mathematics and a low level of anxiety facilitates the pupil in thinking flexibly and divergently about the unusual mathematics problems presented in this research. However it may also be the case that pupils, like these two girls, who have the ability to do fairly well in such tasks, but who in conventional assessments do poorly in comparison to higher attaining pupils, have consequently learnt not to expect to succeed in the mathematics they are given to do and are anxious about the subject as it is usually experienced. Such complex interaction between mathematical creativity performance and attitudinal factors like self-concept and anxiety towards mathematics would result in the lack of significant correlations obtained in other than the highest attaining group.

Clearly the data in Appendix 6 could be subjected to further analysis and subsets for profile comparisons selected in many different ways. However a decision was made after this preliminary analysis not to pursue this line of enquiry further at this stage.

Postscript

The reader will have noticed the remarkable similarity in scores obtained by pupils 198 and 199 (see Table 7.3), suggesting that these two pupils have almost identical profiles. It was discovered, after the marking of pupils' scripts had been completed, that in fact they are identical twins.

CHAPTER 8

SUMMARY AND CONCLUSIONS

This study has been an investigation into aspects of mathematical creativity in schoolchildren. The research has had two main emphases. First there has been the development of a battery of tests for the assessment of mathematical creativity in schoolchildren. Secondly there has been the identification of certain personality and attitudinal characteristics of mathematically creative pupils. The main part of the research involved the administration of a series of paper-and-pencil tests to 283 pupils aged 11 - 12 years in three suburban Middle schools, over a period of some 10 weeks. This series included tests designed to assess mathematical creativity and also those designed to probe certain aspects of personality and attitude which were conjectured to be related to creative performance in mathematics.

The Relationship of this Study to Various Areas of Study

Figure 8.1 is a schematic representation of the way in which the author has seen the present study of aspects of mathematical creativity in schoolchildren in relation to other areas of study. Box 1 indicates the two main emphases of the present study: the development of a battery of tests for assessing mathematical creativity and the investigation into certain characteristics of mathematically creative pupils. This work is derived from both the areas of mathematics education (Boxes 5, 7, 8) and general creativity (Boxes 2, 6). The work dealing with the assessment of mathematical creativity uses a framework with two main strands. One of these is the notion of divergent production which has figured in many of the psychometric approaches to studying creativity in general (Box 2). Investigators such as Torrance and Guilford have assessed the creative thinking abilities of students over a wide age range by means

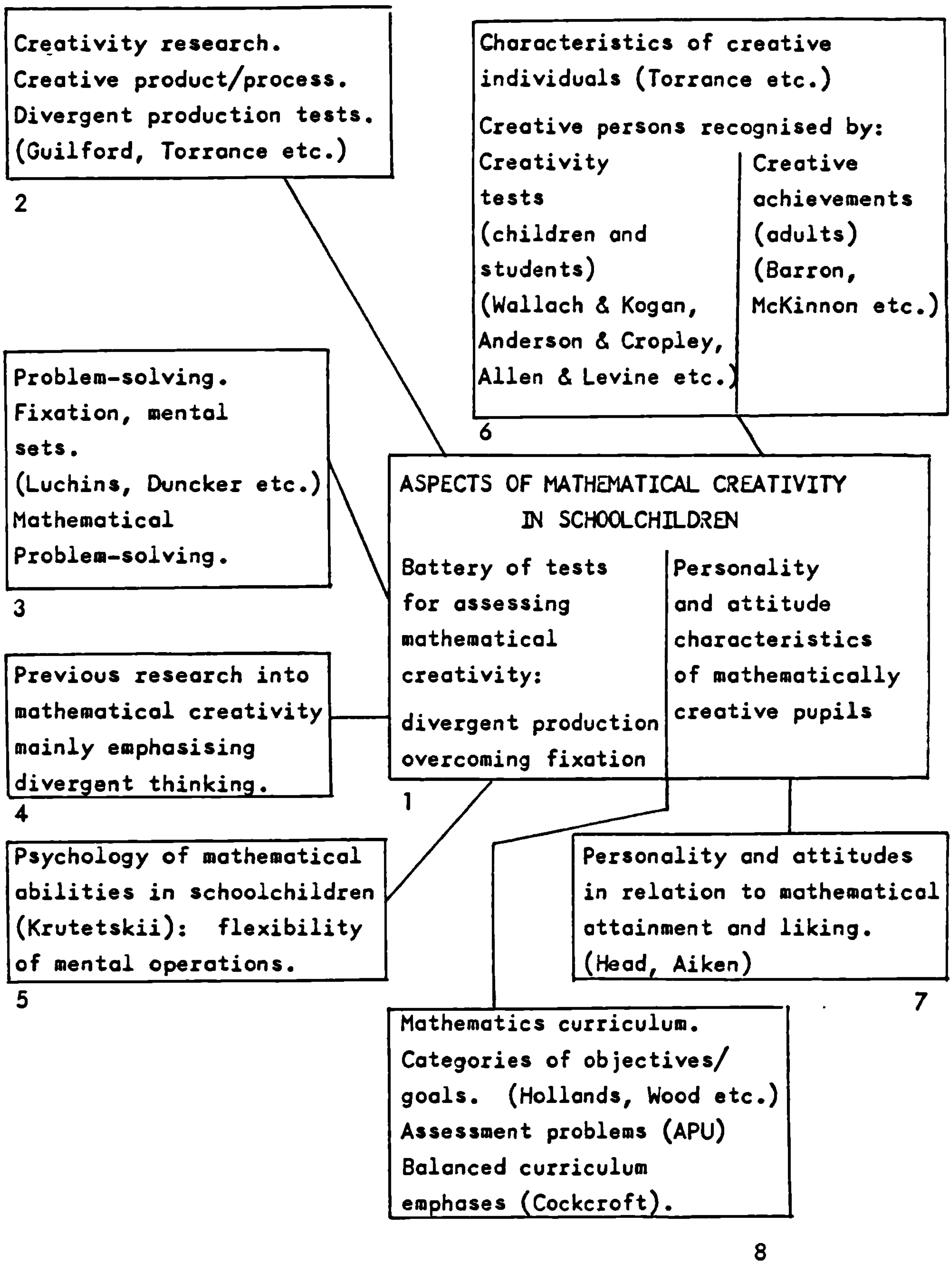


Figure 8.1. A schematic representation of the place of the present research into mathematical creativity in schoolchildren.

of batteries of divergent production tests in which the students' responses are scored for such attributes as fluency, flexibility, originality and elaboration. A number of previous researchers have considered the application of similar styles of tests to mathematics (Box 4) and their work has been reviewed in Chapter 2 of this report (see, in particular, Table 2.1). The notion of flexibility of mental processes recurs in discussions of creative thinking and this is seen by Krutetskii (1976), in his major study of the psychology of mathematical abilities in schoolchildren, as a key component of mathematical ability (Box 5). His discussion of this component leads to the second strand of the assessment of mathematical creativity used in the present study, that of the ability to break from mental sets, to overcome fixation. This has not been much emphasised in previous assessments of mathematical creativity in schoolchildren (Box 4). The assessment procedures developed in relation to this second strand have called particularly upon research into problem-solving, (Box 3) undertaken by Gestalt psychologists such as Luchins (1951) and Duncker (1945), as well as being strongly influenced by the work of Krutetskii (Box 5).

The investigation into aspects of personality and attitudes which might be found to be typical of mathematically creative pupils is principally related to the previous extensive research into characteristics of creative persons (Box 6). This has been reviewed in Chapter 3 of this study. Much of this work has been of a biographical nature, examining the characteristics of individuals, mainly adults, who are judged to be creative in terms of their achievements (e.g. Barron, 1969, McKinnon, 1961, 1962). More directly relevant to the present study is the work which has been undertaken in the area of characteristics of children found to be creative in terms of performance on general creativity tests. In

Chapter 3 of this report particular attention was given to the studies undertaken by Wallach and Kogan (1965), Anderson and Cropley, (1966), Allen and Levine, (1968). Also related to this emphasis in the present study is the existing work on personality and attitudes related to mathematics achievement and appreciation (Box 7),as summarised by Head (1981) and Aiken (1976).

The study not only derives from these areas of research, but also feeds back into them. In Chapter 2 of this report it was suggested that creative ability in mathematics was not necessarily related to creative ability as assessed by more general forms of tests. Those interested in creativity research in general (Box 2) will find the battery of tests developed in this study a useful tool for considering a more domain-specific aspect of creative thinking. The results of the investigation into personality and attitudinal characteristics of mathematically creative schoolchildren serve to complement the existing findings (Box 6) on the characteristics of creative adult mathematicians and scientists. These results also provide a little more information about the role of personality and attitudes in mathematics, a field of study in which the evidence has been seen to be somewhat fragmentary (Box 7). The battery of mathematical creativity tests developed in this study could make contributions to research related to Boxes 3, 4 and 5. The battery possibly provides a more balanced and fuller assessment of mathematical creativity than that provided by previous researchers (Box 4) with the dual emphasis on overcoming fixation and divergent production, and with subcategories within these headings (see Figure 2.1). To the extent that the battery of tests assesses important aspects of mathematical ability the study has provided further insights into the psychology of mathematical abilities in schoolchildren (Box 5), and supplements, for example, the work of Krutetskii, which was mainly individually administered, interview-based problem-solving tasks, with the provision of group-administered, paper-and-pencil tasks. One of the key problems in the area of research into problem-solving in mathematics (Box 3) is the question as to how an investigator can recognise that a period of training in mathematical problem-solving has produced gains in students' ability to solve mathematical problems. It seems to this author that there is a logical problem here, since what constitutes a 'problem' to one student may be no more than an 'exercise' to another. The essence of a problem is that there is no readily accessible algorithm or procedure for its solution (Lester, 1980). It could be argued therefore that a period of training in problem-solving may simply result in certain problems becoming mere exercises, due to familiarity with that type of problem. One way forward here might be to identify key components of the problem-solving process in mathematics and then to seek to assess students' facilities in these respects. The present author would suggest that overcoming fixations and divergent production are two such key aspects in mathematical problem-solving, and that those engaged in research in this field may find the work of the present study to be a fruitful area for consideration in relation to the whole problem-solving process.

Finally the study really starts and finishes by consideration of the school mathematics curriculum (Box 8). In Chapters 1 and 2 it was seen that creativity in mathematics is seen by many mathematics educators as an important goal, albeit a rather vague one. Teachers cannot set out to foster what they cannot recognise, so the provision of an assessment instrument for mathematical creativity contributes to the mathematics curriculum in a positive way. Cockcroft (1982) has recommended a more balanced range of experiences for all pupils in mathematics, including mathematical investigations. The present study would provide mathematics teachers with a means for assessing whether such an approach actually achieves anything in terms of mathematical creativity objectives. Certainly it is possible to draw some tentative practical implications for mathematics teachers from the results of the present study, both from the responses of the pupils to the mathematical creativity tests and from the findings in relation to personality and attitudinal characteristics. That is one of the concerns of this final chapter.

Assessment of Mathematical Creativity in Schoolchildren

The Framework Used

One of the main achievements of the present research has been the development of a battery of tests for the assessment of creativity in mathematics, which has been used in the investigation with children aged 11 - 12 years. Mathematical creativity was not precisely defined in this study, but rather consideration was given to key ideas which recur in creativity literature and their relevance to school mathematics. From consideration of the creative process the key idea of mental set and overcoming fixation emerged as being particularly relevant to children doing mathematics. From consideration of the creative product the notion of divergent production was selected as a second key idea. These two ideas were the basis for a framework within which the battery of tests was constructed (see Figure 2.1). Some tests based on the fixation construct were designed to assess the pupil's ability to overcome what has been termed 'content-universe' fixation, in which failure to solve the problem may be due to self-imposed restrictions concerning the range of elements which may be brought to bear upon the problem. Others were designed to assess the pupil's ability to overcome 'algorithmic' fixation, in which the pupil continues to use a previously successful algorithm or procedure even when it becomes inappropriate or inefficient. Three different, though overlapping, approaches were used for the construction of tests related to divergent production. These were situations requiring problem-solving, problem-posing and redefinition. The first of these entailed giving the pupils an open-ended mathematics problem which had many possible appropriate solutions. The second entailed the provision of a mathematical situation in which many possible problems could be posed. Redefinition implies a mathematical situation in which many different responses can be obtained by continually redefining the elements and relationships within the situation. Tests of overcoming fixation and tests of divergent production were constructed to cover both numerical and spatial domains, in order to reflect the two main content emphases in the school mathematics curriculum.

Principles for the Construction of Mathematical Creativity Tests

During the development of this battery of tests a number of principles to guide in the construction of such tests of mathematical creativity were enunciated. These are restated below, since they may be useful guides for the construction of further tests of these sorts.

Overcoming fixation. For the construction of tests related to the ability to overcome fixation in mathematics, the following principles have been used.

1. It should be expected that on each test some pupils in the sample will succeed and some will fail, and the main reason for some failing should be demonstrably the failure to break from a mental set.

2. The mental set might be either an algorithmic fixation or a content universe fixation. The battery of tests should include examples of both.

3. Tests should be devised in both numerical and spatial

domains.

4. The mathematical skills and knowledge required in the tests should be well within the grasp of the majority of pupils in the sample.

Divergent production. For the construction of tests related to divergent production in mathematics, the following principles have been used.

1. A situation should be given to the pupils upon which they would be able to bring to bear a wide range of mathematical ideas.

2. The situation might involve problem-posing, problem-solving or redefinition.

3. All pupils should be able to make some appropriate responses, but there should be sufficient mathematical restraints within the task so that the original responses will be non-trivial and have some degree of face validity for associating them with the notion of mathematical creativity.

4. The tests should aim to discriminate between pupils on the basis of their ability to use the mathematics they know in many different, varied and original ways.

5. Tests should be devised in both numerical and spatial domains.

Criteria for Evaluating Mathematical Creativity Tests

Arising from the principles which guided the construction and development of the tests of mathematical creativity, and also from analysis of the responses obtained from pupils to such tests, criteria have been stated for the acceptance of a particular test as a valid assessment of the construct in question.

Overcoming fixation. The criteria laid down for a test discriminating between pupils on the basis of their ability to overcome content universe fixation were (a) that some pupils should be successful in solving the given problem; and (b) that some should fail, and the responses of the pupils should provide internal evidence that this failure is due principally to their working within an insufficient and restricted universe. For a test discriminating between pupils on the basis of their ability to overcome algorithmic fixation a number of similar items are required, some designed to establish an algorithm and one or more critical items having a more appropriate solution without the use of that algorithm. The criteria laid down for accepting such a test as valid were (a) that the responses of the pupils should indicate that the algorithm has been established by successful completion by most pupils in the sample of most of the non-critical items; (b) that some pupils should succeed in obtaining the most appropriate solution to the critical itmms; and (c) that the responses of the pupils who fail on the critical items should provide internal evidence that this failure is due to the continued application of the algorithm.

Divergent production. The criteria laid down for a test of divergent production in mathematics to be accepted as valid were: (a) the pupils' responses should use a range of mathematical ideas; (b) at least 20 different appropriate responses are shown to be possible for the pupils in the sample; (c) the pupils' responses should show a fairly consistent interpretation of the instructions in the test; (d) there should be several obvious responses, so that most pupils will obtain these: such responses are zero-rated for originality; (e) there should be a number of appropriate responses which are obtained by relatively fewer (10%, 5%, 2%) pupils, so that credit for originality can be given accordingly; and (f) these original responses should have some degree of face validity for indicating mathematical creativity: in other words, the original response should not be mathematically trivial.

The Battery of Tests of Mathematical Creativity

Using the above criteria, five out of the eight tests of overcoming fixation (Tests 1, 10, 14, 20 question 6, and 24 in Appendix 1), and 10 out of the 14 divergent production tests (Tests 2, 3, 4, 5a, 5b, 7, 11, 12, 16, 18) which were administered in the main part of the research programme were finally accepted as valid. These constitute the battery of tests of mathematical creativity which is now available as a result of the present research. This may be a useful tool in further research into the notions of mathematical creativity. Mathematics teachers interested in recognising creative ability in mathematics may also see this battery of tests as being a useful resource. Evidence has been educed in this investigation to support the view that these tests assess aspects of mathematical ability not assessed by conventional tests of mathematical attainment. Although, as was expected, the level of mathematics attainment was found to be a limiting factor in performance on the mathematical creativity tests, within any band of attainment wide ranges of performance in the mathematical creativity tests were obtained. The spread of scores increased with the level of attainment, so it was suggested that the battery of tests would be most effective in discriminating between pupils in the higher levels of mathematics attainment. However, other pupils, who in conventional assessments may perform only moderately,may be shown by such tests as these to have abilities in mathematics not normally being tapped or recognised. It is suggested therefore that this battery of tests, together with the principles and criteria associated with the two key ideas of overcoming fixation and divergent production, may serve in some way to fill partially the gap in assessment procedures in mathematics acknowledged by the Assessment of Performance Unit (1980, 1981, 1982), discussed in the opening chapter of this report.

Mathematics educators will want to convince themselves that these tests are assessing abilities in mathematics which can be recognised as being worthwhile and important. The detailed analysis of pupils' responses provided in Chapter 4 of this report has been given partly for this purpose. Consideration of these responses according to the specified criteria for validity led to the inclusion or exclusion of particular tests in the battery.

It has been argued in this study that mathematics teachers will recognise that the failure of a pupil to solve a particular problem in mathematics is often due to some sort of fixation. A commitment to the use of inappropriate algorithms or the failure to consider the application of other than a limited range of mathematical ideas to a problem are common phenomena in children's work in mathematics. It seems self-evident that the pupil who shows greater ability to break from such fixations in mathematics is demonstrating a relevant and useful facility. As has been highlighted in this report elsewhere, this sort of flexibility of mental processes is one of the major components of mathematical ability identified by Krutetskii (1976).

Perhaps less obviously worthwhile in terms of mathematical ability is the notion of divergent production, though, as was noted in Chapter 2 (see Table 2.1), a number of researchers have used this notion in considering creativity in mathematics. Again all that can be done is to point to the actual responses provided by the pupils as evidence that the tests are tapping some sort of ability which can justifiably be labelled as mathematical creativity. For this reason, in Chapter 4, catalogues of all responses obtained to the divergent production tasks have been provided. Perhaps more convincing is an examination of the scripts of individual pupils. In Chapter 7 a comparison of the responses of two individual pupils was made. One of these was the pupil showing overall the highest level of performance on the battery of tests, the other a pupil with the same very high score for mathematics attainment, but showing overall in the group of highest attainers the lowest level of performance on the battery of tests. Appendix 7 contains further examples of individual pupils' responses to a number of the divergent production tests. Placed side by side are the responses of two pupils for each test. One pupil scored highly on the test, and the other is a pupil of the same level of mathematics attainment but who scored low marks on the test. The reader is invited to judge whether the performance of the one is justifiably referred to as being mathematically creative in comparison with the uncreative performance of the other. It should be emphasised that in each example the pair of pupils is of equal mathematics attainment in conventional terms. In each case it can be seen that the high scorer calls upon a wide range of mathematical ideas, brings several novel or original ideas to bear upon the situation, shows an ability to switch freely from one productive idea to another, and does not restrict the responses to the problem to safe, predictable, unchallenging paths. By contrast, the low scorer, in each case a pupil with the same mathematics attainment score, tends to use a very limited range of ideas from the available store of mathematical skills, concepts and principles, often gets onto a particular train of thought about the problem, which may be appropriate or inappropriate, sticks with it however dull the resulting responses may be, and shows an inclination to stay with safe, predictable, unchallenging procedures. Of course, it is somewhat circuitous reasoning to point to these contrasts, because it is precisely because of these qualities that the sets of responses in question have been scored high or low for fluency and originality. What matters in the end is whether the individual mathematics educator values the qualities of thinking which are demonstrated by the high scores on these tests and recognises them as showing creative ability in mathematics.

Some Limitations in the First Emphasis of This Study

A satisfactory split-half coefficient of reliability (0.89) between the ten divergent production tests in the battery was obtained in Chapter 5 of this report. However no consideration has been given to the reliability of the tests of overcoming fixation. There were only five of these accepted as valid, and the distributions of pupils' scores on these tests showed considerable variations.

The Study did not include any consideration of the stability of either of these measures of mathematical creativity.

Finally it needs to be borne in mind that the sample used for the investigation was very top-heavy in terms of mathematical attainment.

Further work in the assessment of mathematical creativity could be undertaken in three directions therefore: (a) the provision of normative data for performance on these tests; (b) the development of additional tests related to the overcoming fixation construct; (c) data regarding the reliability and stability of such mathematical creativity measures.

Some Characteristics of Mathematically Creative Pupils

The Hypotheses Used in the Investigation

The second emphasis in the study was on identifying some significant characteristics of pupils found to be highly mathematically creative. Six hypotheses were formulated in Chapter 3 of this report and examined by means of correlations in Chapter 6 and by means of pupil profiles in Chapter 7. The hypotheses were derived from a number of hunches outlined in Chapter 1 concerning aspects of personality and attitudes which were considered likely to be related to creative ability in mathematics. They are summarised below.

"Pupils who show higher levels of mathematical creativity than their contemporaries with similar levels of mathematical attainment will tend to:

1. be more willing to take risks in mathematics;
2. be more nonconformist in mathematics;
3. be broader categorizers;
4. have higher self-concepts in mathematics;
5. have lower levels of anxiety towards mathematics;
6. have lower levels of anxiety towards tests in general."

In these hypotheses, "similar levels of attainment" has been taken to mean pupils within a particular band of scores on the NFER Mathematics Attainment Test EF. Four bands were defined and the hypotheses tested within them: A, very high attainers;B, above average attainers; C, average attainers; D, below average attainers. It was considered necessary to formulate and test the hypotheses in this way because of the anticipated relationship between mathematical creativity measures and mathematics attainment. This relationship was confirmed in Chapter 5 with correlations of 0.69 and 0.65 between mathematics attainment (MA) and the composite measures of overcoming fixation (OF) and divergent production (DP) respectively. However, although it would appear that the level of mathematical skills and knowledge attained by a pupil is inevitably a limiting factor on performance in mathematical creativity measures, there were still considerable spreads of OF and DP scores within each band of MA scores. The hypotheses were designed to investigate whether these variations might be related to personality and attitudinal factors.

Results from the Consideration of Correlations

The first hypothesis was examined by means of three investigator-designed instruments based on a risk-taking-willingness construct

(Tests 23, 27, 22). Indices of willingness to take risks were obtained from these three instruments and have been referred to as RT_{23} , RT_{27} and RT_{22}. RT_{23} is considered to be an indicator of a pupil's willingness to risk making a reasoned guess in a mathematical situation of some uncertainty; RT_{27} is considered to be an indication of the pupil's willingness to risk a wild guess in mathematics on the basis of little evidence in order to procure a prize; RT_{23} is considered to be an indicator of the pupil's willingness to risk exposure to unknown mathematics questions, with the risk of losing or gaining marks. The second hypotheses was also examined by means of a newly devised, exploratory instrument, designed to assess the pupil's level of nonconformity. The index obtained from this instrument (NC) is considered to be an indication of the pupil's willingness to trust personal judgments in mathematics even when these are at variance with the group consensus.

Hypothesis 3 was investigated using a modification of the category width instrument used by Wallach and Kogan (1965). The CW index obtained from this is thought to indicate a pupil's tendency to think in broad rather than narrow categories in the coding of information.

Hypotheses 4, 5 and 6 were investigated by means of an attitude questionnaire containing well tested items measuring self-concept in mathematics (SCM), anxiety towards mathematics (ATM) and test anxiety (TA).

Table 8.1 gives a list of all significant correlation coefficients obtained within the four attainment bands between the various indices mentioned above and the mathematical creativity measures of OF and DP. These are taken from various tables in Chapter 6 of this report.

It can be seen that no significant correlations were obtained for the index of nonconformity, so that on the basis of the results obtained from Test 26 there was no support found for hypothesis 2.

Table 8.1

Significant Correlations Between Mathematical Creativity Measures and Other Indices Used in the Investigation

Index	Hypothesis tested	OF	DP
RT_{23}	1	0.30* (A+)	
RT_{27}	1		-0.35**(B)! and -0.24*(C)!
RT_{22}	1	0.27* (B)	0.33**(C)
CW	3	0.34*(A)	0.32*(A)
SCM	4		0.45**(A)
ATM	5	-0.30*(A)	-0.41**(A)
TA	6		0.31**(B)!

Notes: letters in parentheses refer to attainment bands

* indicates significant at 5% level

** indicates significant at 1% level

! indicates that the correlation is in the opposite direction to that hypothesised

A+ refers to a slightly smaller group of very high attainers (MA scores >130, rather than $\geqslant 130$).

A review of previous work in assessing risk-taking had led to the conviction that a great variety of behaviours might be described by this term. Hence three risk-taking instruments, of very different natures, had been included in the testing programme. The behaviour on Test 27 (that involving risking a wild guess on the basis of very little evidence) was found to be unrelated to the behaviour on the other two risk-taking instruments (Tests 23 and 22). In fact RT_{27}

was significantly correlated in a negative direction with DP scores in bands B and C. There was only slight support for hypothesis 1 in terms of RT_{23} (risk-taking involving a reasonable guess in a mathematics situation of some uncertainty) and overcoming fixation scores amongst the very high attainers. There was some support for the hypothesis in terms of RT_{22} in band B (correlated with OF) and band C (correlated with DP). It is apparent from these results that risk-taking in mathematics is not a single clearly indentifiable type of behaviour, and its relation to mathematical creativity performance is still unclear.

The clearest results in the investigation arose from the use of modifications of established instruments for assessing category width and attitudes. All the significant correlations obtained from these which supported the hypotheses were within the group of very high attainers. Within this group there is support for hypothesis 3 (for both OF and DP), for hypothesis 4 (for DP especially) and for hypothesis 6: in fact, in one group (band B) a highly significant correlation in the opposite direction to that hypothesised was obtained. This was interpreted as indicating a complex interaction between test anxiety and mathematical creativity performance. Sometimes the level of test anxiety may facilitate or inhibit creative thinking in mathematical situations. Other times high or low test anxiety may be seen as the result of a mismatch or a close match, as the case may be, between the pupil's competencies and the demands of conventional assessments in mathematics.

The correlations obtained suggested that within the group of very high attainers - the group within which the mathematical creativity measures discriminated to the greatest extent - it might be possible to draw a tentative picture of some of the characteristics of mathematically creative pupils. Such pupils would have high self-concepts in mathematics, expecting to be able to succeed with the mathematics they are given to do, and would show low levels of anxiety towards the subject. Thus they are not put off by the unfamiliar nature of the divergent production tasks, for example, and are able to call upon a wide range of mathematical ideas with confidence. They are not therefore predisposed by their attitudes to stay with safe, predictable approaches to the problems. This would be consistent with less rigidity in their thinking in the overcoming fixation problems. Furthermore, being broad coders, they are more predisposed to think in terms of similarities rather than differences. Such an attitude is favourable to the demands of the mathematical creativity tests. They may see the possibility of a greater range of mathematical ideas being applicable to a given problem, and hence would show less fixation and greater flexibility. Finally such pupils may be more willing than their less creative counterparts in the group of very high attainers to hazard a guess in a mathematical situation of some uncertainty. Such a willingness to take risks may be seen as favourable in the unfamiliar style of mathematical creativity tests when the pupils may not be too sure of what is being demanded of them.

Results from the Consideration of Pupil Profiles

In Chapter 7 of this report consideration has been given to the profiles of individual pupils based on their performances on the personality and attitude related instruments. The intention here was to see whether the tentative picture of the mathematically creative pupil arrived at from the consideration of correlations was realised in actual pupils. The main focus in this chapter was upon the group of very high attainers, because the correlations had suggested that the clearest pattern would emerge within this group and because the mathematical creativity measures discriminated to the greatest extent within this group. In a comparison between the six highest mathematically creative pupils and the six lowest within the group of very high attainers a clear picture emerged. Four out of the six high creatives were high risk-takers, in terms of both RT_{23} and RT_{22}, were broad coders, and showed high self-concepts in mathematics and low levels of anxiety towards mathematics and tests in general. In many of these respects there were contrasts with the low creatives. Each of them deviated markedly in one or more ways from this description. Two pupils of the same very high level of mathematics attainment, but one the highest mathematically creative and the other the lowest in that band, were then compared in more detail. The most marked differences between them were that the high creative was a much broader coder, showed a considerably higher self-concept in mathematics and lower levels of anxiety. It was seen, in a comparison of their responses on the mathematical creativity tests, that these differences in characteristics were consistent with the differences in their performances in overcoming fixation and divergent production.

Some Limitations in the Second Emphasis of the Study

First it should be re-emphasised that the instruments used for assessing risk-taking and nonconformity in mathematics were of a very exploratory nature. The results from the first part of the investigation described in Chapter 6 were therefore very tentative.

Secondly, it must be conceded that the characteristics selected for investigation - risk-taking, nonconformity, category width, self-concept, anxiety towards mathematics and tests - although based on hunches arising from consideration of previous work in the fields of both creativity and mathematics education, were nevertheless a somewhat arbitrary collection of factors. There may well be other more significant personality factors not considered which should be taken into account in terms of mathematical creativity.

Thirdly, the nature of the research situation, based on the use of a fairly large sample of pupils, necessitated the use of paper-and-pencil tests. Pupil profiles have been based entirely on the performances on these tests. No use has been made of individual interviews, which could have added valuable insights into factors affecting mathematical creativity performance.

Finally, the data (which are available in Appendices 4, 5 and 6) have not been subjected to all possible forms of analysis. The hypotheses were tested one by one within the four bands of attainment. Such an approach takes no account of the possibility of moderator factors other than mathematics attainment. Clearly other ways of defining the attainment bands could have been considered and may have produced a picture with different emphases.

Areas for Further Research

There is clearly a case for further research into the notion of risk-taking in a mathematical context. The instruments used in this research, although of a very exploratory nature, did nevertheless discriminate between the pupils in the sample. The relationships with mathematical creativity performance were variable and unclear, but this area is probably worth pursuing further, particularly since the majority of high creatives considered in the profile approach were found to be high risk-takers on two of the measures. There is a need for the development of more refined instruments here and a systematic consideration of the components of a risk-taking situation in a mathematical context. There is no support from the present study for further exploration of nonconformity, in the sense of trusting one's judgments against the group consensus, in relation to mathematical creativity.

As mentioned above further research could be undertaken into the relationship of other personality factors to mathematical creativity, and consideration given to the influence of moderator factors other than mathematics attainment.

Some Conflicts in Mathematical Creativity

In the course of this study a number of potential conflicts have emerged between the notions associated with creativity in mathematics and other behaviours in mathematics which may be considered desirable.

For example, in marking the divergent production tests a difficulty arose over the question of how to react to inaccurate responses. In general the principle was adhered to that a response was not accepted if it was not mathematically correct, on the grounds that it was little use having creative ideas if they could not be applied appropriately. This procedure inevitably sets up a conflict in the mind of the assessor, particularly when a pupil produces an original or clever mathematical idea but makes a small error in its application. This conflict between accuracy and creativity is well illustrated by the example, shown in Table 8.2, of one pupil's responses to the divergent production task in Test 3 (Subsets). In finding subsets from the given set of integers from 2 to 16 inclusive this pupil uses an impressive range of appropriate and original mathematical ideas, but makes minor errors in no less than eight of the 17 responses.

Clearly this is not just a difficulty in assessment of mathematical creativity, but an issue of relevance to the mathematics teacher. Accuracy in doing mathematics is, of course, important, but there is an obvious danger that in over-emphasising this aspect of the subject the teacher may cause the pupil to perceive accuracy as being of greater importance than creative thinking. If what matters to the pupils is getting mathematics right and avoiding errors, because presumably that is what matters to the teacher, then it would seem that pupils would be unlikely to risk straying from safe, predictable paths where learnt routines can be applied, even if opportunities for more creative mathematical activity are provided. Teachers of mathematics are therefore challenged by this conflict to consider carefully the relative values they might place upon the notions of creativity and accuracy in their assessment of their pupils and in their responses to pupils' contributions in mathematics lessons.

Table 8.2

Responses of One Pupil to Test 3(Subsets), Illustrating the Conflict Between Creativity and Accuracy

Subset	Rule	Error
1,3,5,7,9,11,13,15	odd numbers	1 included
3,6,9,12,15	divisible by 3	
4,8,12,16	divisible by 4	
5,10,15	divisible by 5	
1,2,3,4,5,6,7,8,9,10	below 10	1, 10 included
11,12,13,14,15,16	over 10	
12,14,16	even nos. over 10	
11, 13, 15	odd nos. over 10	
2,4,8	go into 16	16 excluded
2,5,10,4	go into 20	
10,5,2,4,8	go into 40	
2,4,5,10	go into 100	
2,4,6,8	factors of 24 < 10	3 excluded
2,3, 7,11,13	prime numbers	5 excluded
4,5,6,8,9,10,12,14,15,16	not prime numbers	5 excluded
4,5,6,8,9	not prime below 10	5 excluded
7,8,9,10,11,12	between 7 and 12	7,12 included

A second possible cause of conflict is between creative thinking in mathematics and the pupil's perception of mathematics as a body of rules to be learnt and obeyed. Such a perception would clearly militate against flexible or divergent thinking. This was apparent in a number of the overcoming fixation tasks: For example, in Test 14 (Sum and Difference), the stimulus of the word 'sum' for many pupils seemed to produce the response of applying the rule of addition. In Test 10 (Jugs) the large number of pupils who showed fixation were simply applying a rule which had been found to work. It could be suggested that their previous experience of mathematics might have led them to expect that a rule once learnt could be relied upon. Of course, in much of mathematics this is precisely so. But there is clearly a case here for pupils having occasional experiences of mathematics which do not consist merely of learning and applying rules.

A related problem is the possible conflict between being systematic and being creative in the approach to a mathematics problem. Very often teachers of mathematics will want to emphasise to pupils the importance of being systematic. This may involve the careful control of variables in an investigation, so that an underlying pattern may emerge or in order that all possibilities of a particular kind are considered. This is clearly desirable mathematical behaviour in many situations, and, in fact, something which many pupils are slow to learn. And yet, in several of the divergent production tasks used in the present research, a systematic approach used by many pupils produced dull, predictable and narrow sets of responses. For example, in Test 12 (Scattergram), a pupil might list all possible questions of the form, "How many families had x boys and y girls?, by systematically letting x and y run sequentially through all values from 0 to 6 in turn. There is commendable mathematical thinking here, but not thinking which is credited with marks for creativity. Somehow, presumably, pupils need to be able to evaluate their responses and to learn when it is appropriate to say: "and so on".

One of the most important facets of mathematical thinking is the formation of generalizations. In the sense that a generalization in mathematics may consist of combining a number of individual, previously unrelated statements into one single generalized statement, it could be argued that such thinking could be justifiably described as essentially creative. Koestler (1964) describes the essence of a creative act in science as the fusion of previously unrelated ideas. But in the present research it has been noted that such generalized thinking may conflict with the demands of, for example, a problem requiring the breaking from a mental set. In Test 24 (Cuts) it was noted that the behaviour of pupils who found an underlying pattern in the way the rectangles could be cut into a given number of parts and then proceeded to use this pattern in successive instances were merely showing a type of generalized thinking which would normally be encouraged and applauded. There are many patterns to be found in mathematical situations, and many generalizations to be formed and applied - in fact, it might even be argued that patterns and generalizations are the essence of mathematics. And so the fact that some mathematical problems may not be solved by pupils in the most efficient way because they are looking to apply such generalized thinking is again a source of conflict for the learner.

In evaluating pupils' responses to Test 3 (Subsets) a distinction was suggested between sequential and global generalizations. Some pupils gave a subset such as $\{3,6,9,12,15\}$ and stated that the rule was, "they go up by 3"; whereas others would state that the rule was, "they are all multiples of 3". Both responses indicate that a pattern is being recognised, but in the first case the stimulus for the pattern is essentially the way in which successive items differ, the transformation that has taken place between one item and the next, whereas in the second case the stimulus for the pattern is consideration of what all the elements have in common. The present author has coined the terms "sequential" and "global" to refer to these two types of generalizations. Clearly the global type is likely to be the more powerful of the two. It is tempting to associate the sequential type of thinking with the rule-obeying behaviour and the systematic approaches discussed above, which have been seen to conflict to some extent with the demands of creative thinking in mathematics. The global type of generalization is conceptually related to the notion of broad categorization, since the essence of it is the association of a number of different entities on the basis of the ways in which they are similar. If this is so, then, in view of the link established between broad categorization and mathematical creativity in this investigation, it seems possible that there would be less conflict between the global type of generalized thinking and mathematical creativity than might be the case with sequential generalizing. These ideas are very much in their infancy and are put forward as a possible area for further consideration in the realm of mathematical thinking. The suggestion is made that perhaps investigational work in mathematics might not produce gains in creative thinking if the major emphasis in seeking patterns is on the sequential type of generalization rather than the global. If however pupils are encouraged to seek global generalizations, that is to find ways in which all the entities in a situation are the same, then they will be required to let their minds range over all available mathematical principles and concepts seeking some means of associating together the entities in question. Thus they would be experiencing that type of flexible thinking about mathematics which is assessed in the divergent production tests.

Promoting Mathematical Creativity in the Classroom

The final question that remains is what the implications are of this present research for the mathematics teacher who wishes to promote creative thinking in mathematics in the classroom. Some obvious suggestions arise from this study, and some tentative pointers for practice may be made. The suggestions that follow should be considered as the basis for further research into this important area of the promotion of mathematical creativity in schoolchildren.

First it seems obvious that the teacher who would like to achieve gains in performance on the types of mathematical creativity tests used in this research will have to provide pupils with experiences in mathematics where fixations have to be overcome and where divergent production is encouraged. Many of the tests used in this programme could be the basis for class lessons, particularly those related to divergent production. In fact, some of these such as Test 5b (Similarities: shapes), Test 7 (Nine Dot Areas) and Test 12 (Scattergram) were developed from class lessons. Similarly, Test 1 (Areas), related to overcoming content-universe fixation, was the outcome of a class lesson activity. A teacher who wishes to retain these tests for assessment purposes will, of course, need to devise similar situations for class activities. The notion of redefinition is a productive one for appropriate activities. Various sets of two or three dimensional shapes may be provided and pupils invited to make up subsets and try to guess each other's rules for doing this. Various pairs of numbers, two or three dimensional shapes, or attribute blocks, would provide experience of investigating the ways in which two entities are the same. Teachers may easily build problem-posing activities into their work in mathematics. Graphical work is a useful source of such activities. Pupils can be invited to pose each other questions about a graph (bar chart, line graph, pie chart, scattergram etc), and credit given for novel or clever ideas. The teacher who is interested in fostering creative thinking in mathematics will also provide occasional problems in mathematics which have many possible solutions or approaches.

Thus it is suggested that opportunities for overcoming fixations and divergent production in mathematics should be given to pupils, even though, as has been suggested earlier, these behaviours might appear to conflict to some extent with other behaviours in mathematics which are valued by the teacher such as accuracy, rule-obedience, being systematic and pattern-seeking.

Although it has been noted in this study that pupils with high mathematics attainment are in a better position to demonstrate higher levels of mathematical creativity, it should not be inferred that activities fostering creativity in mathematics would only be appropriate to the high attainers. Indeed such activities may well serve to give recognition to abilities possessed by moderate attainers which are not conventionally recognised. Tagg (1972), because of a concern for the needs of university students in the social sciences taking a compulsory mathematics course, argues that such students should have the opportunity to function in mathematics in ways common to other fields of study. Hence he has devised experiences and assessment methods for such students which emphasise divergent as well as convergent thinking, and which are not merely such as would be given to future mathematicians. Similar arguments could be put forward for mathematical work at a school level. The opportunity to work on open-ended investigations, as Cockcroft (1982) recommends for all pupils in mathematics would seem to be highly desirable from the point of view of encouraging creativity in the subject.

Jensen (1976) advocated the encouragement of varied, alternative approaches to mathematics questions, rather than giving the impression that there is always one right way of tackling them. So, for example, pupils would be frequently asked to find three different ways of doing a mental arithmetic calculation. She further suggests that it would be appropriate in developing creative thinking in mathematics for the teacher to vary the embodiment of simple concepts and principles, so that when they are learnt they have a more powerful potential applicability. Even with young children, Jensen argues that it is possible to encourage them to formulate their own questions in a mathematical situation and to find their own individual methods of solution. Sturgess (1971) suggests that the typical experiences of many pupils learning mathematics would lead them to the conclusion that to do well in the subject one must do as one is told and not ask questions. Arguing the case that at any level of mathematics it is possible for students to be engaged in creativity of a kind, he opposes the notion of mathematics teaching as merely the passing on of pre-digested ideas.

Many authors, such as Davies (1980), Brown (1980), and Zeddies (1981), have provided anecdotal evidence that young children are capable of work in mathematics which can be considered creative, inventive or original, if they are given the opportunity and encouragement. They provide illustrations of children's individual approaches to subtraction and division problems, for example, which show much inventiveness. The point is frequently made that teachers should avoid implanting in the pupil's mind the notion that only standard methods of working in mathematics are acceptable. If children are rewarded or praised for showing a creative approach to their mathematics then they may change their view of what is acceptable and show more inclination to develop their own individual and novel techniques. Cockcroft (1982) suggests that there is evidence that such individual, informal approaches to standard mathematics problems may in the long run be of more use to students anyway.

Borenson (1981) describes three high school students' experiences and impressions of learning mathematics in a more creative way. One student values the emphasis on method rather than information and describes how all the class would be involved in making conjectures in mathematics and seeking exceptions to these generalizations before attempting to prove them. Such activity is another facet of divergent thinking in mathematics. A second student comments that part of the teacher's role is to encourage students to make observations beyond those found in the textbook, though he warns that this might have an initial traumatic effect upon students conditioned to learning by memorization. The third student speaks of the misconceptions that many high school students have of mathematics, particularly the conception of its orderliness and structure, and criticises the presentation of mathematical topics always in their final, highly polished form which gives the student no experience of the progressive changes, modifications and mistakes which may have occurred on the way to the topic's present status. Borensom argues that the teacher must be a risk-taker also, venturing into the unknown in mathematics with the students, with no guarantee of success or anything worthwhile emerging. By taking such a stance the teacher may encourage similar attitudes and perceptions in the pupils.

Allinger (1982, a and b) makes a number of suggestions about classroom practice designed to minimise the formation of negative mind sets in mathematics learning in elementary and secondary schools. These include the provision of geometric problems and puzzles about figures with interpenetrating elements (such as triangles within triangles) and the avoidance of always labelling or alligning geometric shapes in the same way, so as not to engender fixations in visualization. To avoid algorithmic fixations (Einstellung effect) he suggests that the teacher should encourage different methods and approaches to mathematics questions, challenge processes that have become mechanical and occasionally include in a series of exercises of one sort an example which cannot be solved by the set method. Teachers should discuss with pupils the many facets of mathematical items, provide and discuss problems with superfluous information which has to be disregarded , and provide opportunities for divergent thinking, particularly via problem-posing situations.

Certain implications may be drawn from the findings of the second emphasis of the present study. For example, it would be reasonable to conclude that teachers wishing to foster mathematical creativity should adopt teaching styles which are designed to encourage pupils' self-concepts in mathematics and to decrease their level of anxiety about the subject. Experiences designed to encourage broad coding, such as multi-base work in arithmetic and work with attribute blocks, provided appropriate discussion is used to draw attention to the underlying similarities of differing mathematical experiences or embodiments of a principle, could be expected to produce some pay-off in terms of mathematical creativity performance. The encouragement of certain types of risk-taking in mathematics, such as willingness to make a reasoned, calculated guess in spite of some uncertainty, might also be a useful teaching approach. All this suggests, of course, a classroom atmosphere in which there is no censure for the pupil who has a go even though the response may not be always correct. In fact, it would be an interesting research question to investigate in the context of school mathematics whether open discussion of risk-taking situations in mathematics between teacher and pupils might produce similar gains in mathematical creativity performance as that observed by Glover (1977) in general creativity.

All the above suggestions are, of course, very tentative. But they could provide the basis for further investigation. This study has provided a battery of tests which could be used now to assess whether particular teaching approaches and emphases do actually produce gains in mathematical creativity performance.

Of course, the relationships between mathematical creativity and characteristics of personality and attitudes may be looked at in two ways. It is implicit in the above discussion that such factors as self-concept in mathematics and anxiety towards the subject may be facilitating or inhibiting in mathematical creativity performance. They may also be viewed as consequences rather than as antecedents. Thus a further research question might be whether teaching styles designed to foster creativity in mathematics might produce gains in terms of increasing self-concept and lowering of anxiety. In view of the known relationships of these factors to mathematics attainment it might then appear that a more creative approach to mathematics teaching could, in the long run, produce gains in conventional aspects of mathematics attainment. As Tammadge (1979) has said:

> "The slogans of the revolution in school mathematics include the words Discovery and Understanding. They should explicitly have included Creativity."

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APPENDICES 1 - 7

APPENDIX 1

ASSESSMENT INSTRUMENTS

Test 0 (NFER Mathematics Attainment Test EF) is reproduced here by kind permission of the National Foundation for Educational Research in England and Wales.

Test 21 contains items taken from the Mathematics Attitude Inventory developed by the National Science Foundation Research Project, at the University of Minnesota. These are used by kind permission of the Director.

The test papers reproduced here are photoreductions of the actual test papers used in the research, by a linear scale factor of $1/\sqrt{2}$.

Timetable for the Administration of the Battery of Tests

Test no.	Name	Purpose of test	Approx. timetable
		OF=Overcoming fixation	
		DP=Divergent production	
		S=Spatial domain	
		N=Numerical domain	
0	NFER EF	Mathematical attainment	before week 1
1	Areas	OF: content: S	week 1
2	Counters	DP: problem-posing N	week 1
3	Subsets	DP: redefinition: N	week 2
4	Shape-finding	DP: redefinition: S	week 2
5a	Similarities: number	DP: redefinition: N	week 2
5b	Similarities: shape	DP: redefintions: S	week 2
6	9-Dot Routes	DP: problem-solving: S	week 3
7	9-Dot Areas	DP: problem-solving: S	week 3
8	Multiplication	OF: algorithmic: N	week 3
9	Fractions	OF: algorithmic: N	week 3
10	Jugs	OF: algorithmic: S	week 5
11	Cross-number	DP: problem-posing: N	week 5
12	Scattergram	DP: problem-posing: S	week 5
13	Classroom	DP: problem-posing: S/N	week 5
14	Sum and Difference	OF: content: N	week 6
15	Double and Add	OF: algorithmic: N	week 6
16	Results	DP: problem-solving: N	week 6
17	What Can You See?	DP: redefinition: S	week 6
18	Three Cards	DP: problem-solving: N	week 7
19	Factory	DP: problem-posing: N	week 7
20	Problems	Solving unusual problems	week 8

Test no.	Name	Purpose of test	Approx. timetable
21	Questionnaire	Anxiety to maths self-concept in maths test anxiety	week 8
22	Self-Confidence	Self-confidence in maths	week 9
23	Multi-Choice A	Risk-taking	week 9
24	Cuts	OF: algorithmic: S	week 9
25	C-W	Category-width	week 9
26	Multi-choice B	Nonconformity	week 10
27	Clues	Risk-taking	week 10

Test O (NFER EF Mathematics Attainment Test)

3

1. What is the largest whole number that can be made from the digits 1, 6, 3, 2 if all the digits are to be used?

(a) 6321 (b) 3621 (c) 2163 (d) 1623 (e) 1236

2–3. Which of the following shapes best describe these common things?

(a) a rectangle (b) a square (c) a triangle (d) a circle (e) a rhombus

2. A gramophone record

3. A tennis court

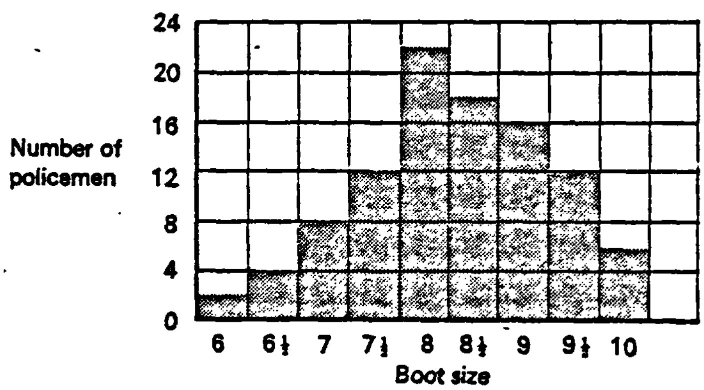
---

4. In this addition one figure was replaced by * wherever it appeared. What does * stand for?

$$\begin{array}{r} 1* \\ *1 \\ + \; ** \\ \hline 165 \end{array}$$

(a) 2 (b) 3 (c) 5 (d) 7 (e) 8

5–7. The bar chart below shows how many policemen wear boots in each of the sizes from 6 to 10.



5. How many policemen wear a size 9 boot or larger?

(a) 16 (b) 20 (c) 25 (d) 30 (e) 34

6. Which two sizes of boots are worn by the same number of policemen?

(a) 6 and 6½ (b) 6½ and 10 (c) 7½ and 9½ (d) 8½ and 9 (e) 8½ and 9½

7. Which size of boot is worn by most policemen?

(a) 6 (b) 7 (c) 8 (d) 9 (e) 10

GO ON TO THE NEXT PAGE

8. A boy thinks of a number, doubles it and adds two. If the number he thinks of is x, what is the final number?

(a) $x + 4$ (b) $2x + 2$ (c) $2x + 3$ (d) $2x + 4$ (e) none of these

---

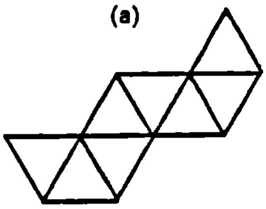
9. A man walks round the perimeter of a rectangular field. Which of the following describes his action?

(a) walks along one side of the field
(b) walks half way round the field
(c) walks along both shorter sides of the field
(d) walks diagonally from corner to corner of the field
(e) walks along all sides of the field

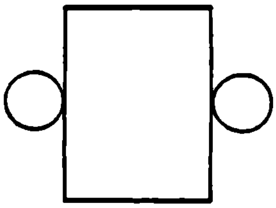
10–12. Which of these nets could be used to make the following solids?

10. Cylinder
11. Pyramid
12. Cone

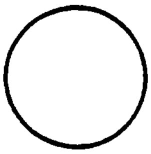
(a)



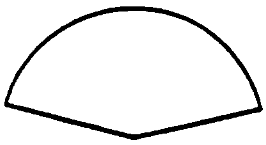
(b)



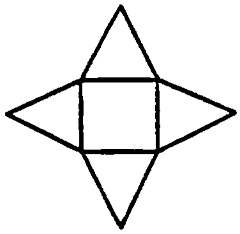
(c)



(d)



(e)



---

13. If an odd number is multiplied by an even number, the answer will be:

(a) always an odd number
(b) usually an odd number
(c) an odd number half of the time
(d) usually an even number
(e) always an even number

GO ON TO THE NEXT PAGE

14–16. To wh'ch decimal is each of the following percentages equal?

	Percentages	Decimals
14.	5%	(a) 0·005
15.	50%	(b) 0·03
		(c) 0·05
16.	30%	(d) 0·30
		(e) 0·50

---

17. Of the 360 men and women at a party, one third were women. How many men were present?

(a) 60 (b) 120 (c) 240 (d) 1080 (e) 2160

18–19. What are the results of doing the following operations on the number 6?

18.	Adding -5	(a) 1
		(b) 4
19.	Subtracting $+2$	(c) 8
		(d) 9
		(e) 11

20–22. To which of the answers (a) to (e) is each of the statements 20, 21 and 22 equivalent?

20.	2 out of 6	(a) 1
21.	19 out of 19	(b) $\frac{5}{9}$
22.	10 out of 18	(c) $\frac{1}{3}$
		(d) $\frac{1}{19}$
		(e) $\frac{1}{2}$

---

23. If $B = A$ and $3A + 2B = 10$

What are the values of A and B?

(a) $A = 2$ and $B = 2$

(b) $A = 3$ and $B = 3$

(c) $A = 4$ and $B = 4$

(d) $A = 5$ and $B = 5$

(e) $A = 10$ and $B = 10$

GO ON TO THE NEXT PAGE

6

24. The square of a number is always:

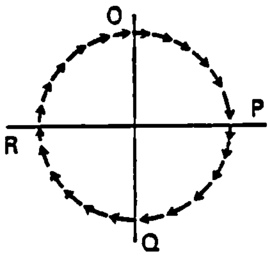
(a) the number added to itself
(b) the number multiplied by itself
(c) the number plus 2
(d) the number multiplied by 2
(e) none of these

25–26. The population of a town is 74 938. What is the population to the nearest:

25. ten?
26. hundred?

(a) 75 000
(b) 74 950
(c) 74 940
(d) 74 930
(e) 74 900

27–29.



An aircraft flies clockwise in a circle from O. Where will it be if it turns through:

27. 130°
28. 270°
29. 660°

(a) O
(b) P
(c) Q
(d) R
(e) none of these

---

30. Which of the following fractions is the smallest?

(a) $\frac{11}{12}$ (b) $\frac{5}{6}$ (c) $\frac{3}{8}$ (d) $\frac{1}{9}$ (e) $\frac{1}{4}$

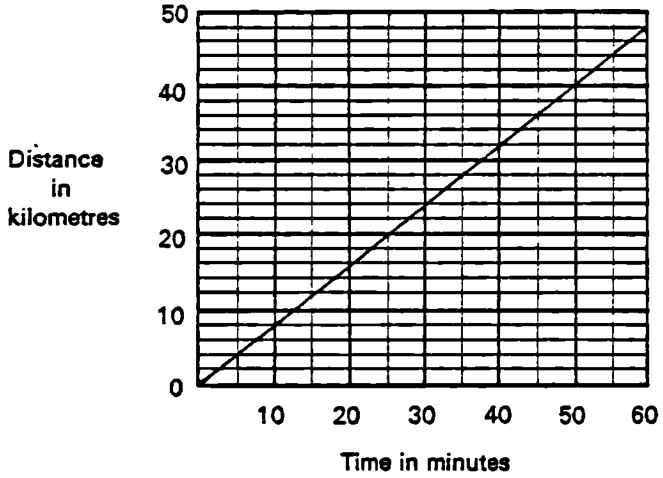
GO ON TO THE NEXT PAGE

31–33.

The following table shows the time taken by a car to travel various distances at the same speed.

Time in minutes	0	15	25	30	55
Distance in kilometres	0	12	20	24	44

This information is plotted on the axes below, in the form of a straight line graph.



31. How far has the car gone in 45 minutes?

(a) 32 km (b) 36 km (c) 40 km (d) 44 km (e) 48 km

32. How long does it take the car to travel 40 kilometres?

(a) 50 min (b) 45 min (c) 40 min (d) 35 min (e) 30 min

33. What is the speed of the car, in kilometres per hour?

(a) 75 (b) 48 (c) 43 (d) 38 (e) 30

---

34. The square root of 90 lies between:

(a) 2 and 4
(b) 5 and 7
(c) 8 and 10
(d) 11 and 20
(e) 21 and 40

---

35. It costs 40 centimes to send a postcard from France to England.

How much does it cost to send 9 postcards? (100 Centimes = 1 Franc)

(a) 3·06 Fr (b) 3·60 Fr (c) 30·6 Fr (d) 36·0 Fr (e) 306 Fr

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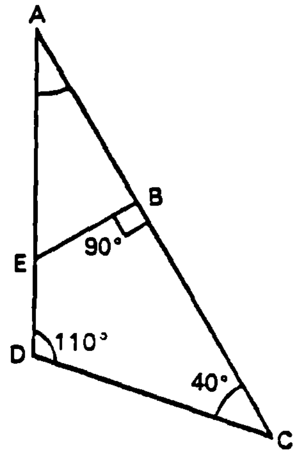
36–37. Use the sketch below to answer the following:



36. How big is angle BED?

(a) 90° (b) 100° (c) 110° (d) 120° (e) 130°

37. How big is angle BAE?

(a) 70° (b) 60° (c) 50° (d) 40° (e) 30°

38–40. To which of the following fractions is each of the decimals equivalent?

	Decimals	Fractions
38.	0·60	(a) $\frac{9}{25}$
39.	0·45	(b) $\frac{4}{15}$
40.	0·36	(c) $\frac{4}{9}$
		(d) $\frac{3}{5}$
		(e) $\frac{9}{20}$

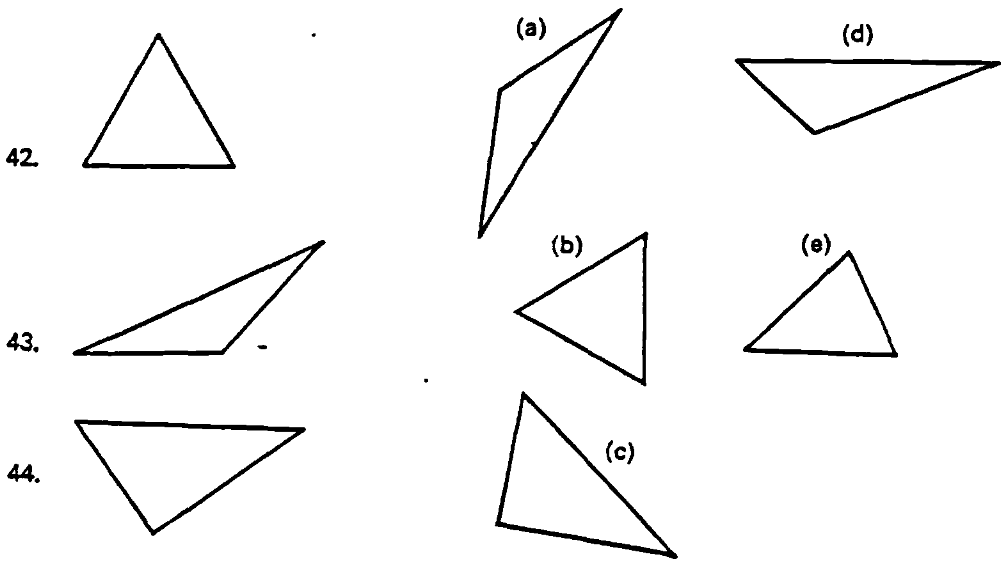
---

41. Which of the following has the same value as 3^3?

(a) 3×3 (b) $3 + 3$ (c) $3 + 3 + 3$ (d) $3 \div 3$ (e) $3 \times 3 \times 3$

GO ON TO THE NEXT PAGE

42–44. To which of the triangles (a) to (e) is each of the triangles 42, 43 and 44 similar?



45–46. A pile of sand weighing 2·520 kg is divided successively into 2, 3, 4, 5, 6, 7, ,8, 9, and 10 equal smaller piles. The table below shows the weight of these piles.

Number of small piles	1	2	3	4	5	6	7	8	9	10
Weight of each pile in grams (g)	2 520	1 260	840	630	504	420	360	315	280	252

How many piles would there be if each pile weighed:

45. 180 g?

46. 42 g?

(a) 14
(b) 40
(c) 60
(d) 280
(e) 420

47. Some time ago it was said that an average family had 2·4 children. What does this mean?

(a) all families had 2·4 children
(b) all families had either 2 or 3 children
(c) some families had a child for only 0·4 of the time
(d) the total number of children was 2·4 times the total number of families.
(e) the average was wrongly calculated

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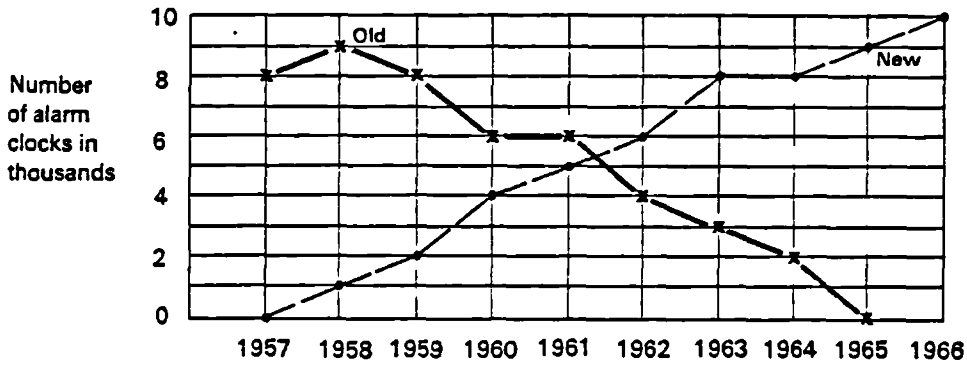
48–49. The graph below shows the number of one manufacturer's old and new alarm clocks in use during each year from 1957 to 1966.



48. What was the largest number of old clocks in use during any one year during this period?

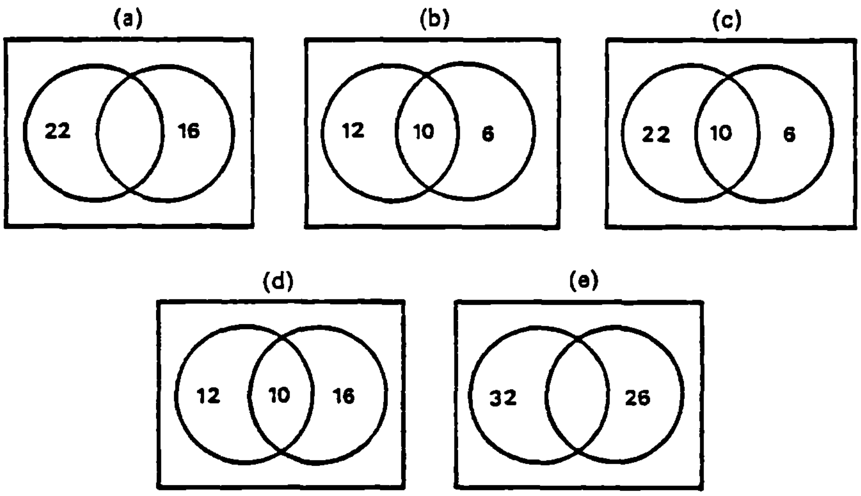
(a) 10 000 (b) 9 000 (c) 8 000 (d) 7 000 (e) 6 000

49. How many clocks (old and new) were in use in 1963?

(a) 7 000 (b) 8 000 (c) 9 000 (d) 10 000 (e) 11 000

50–51. In a class of children, 22 liked apples, 16 liked oranges, and of these, 10 liked both apples and oranges.

50. Which one of the following Venn diagrams illustrates these facts?

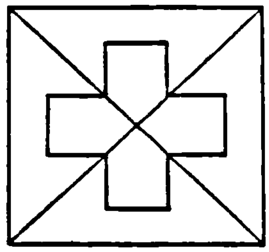


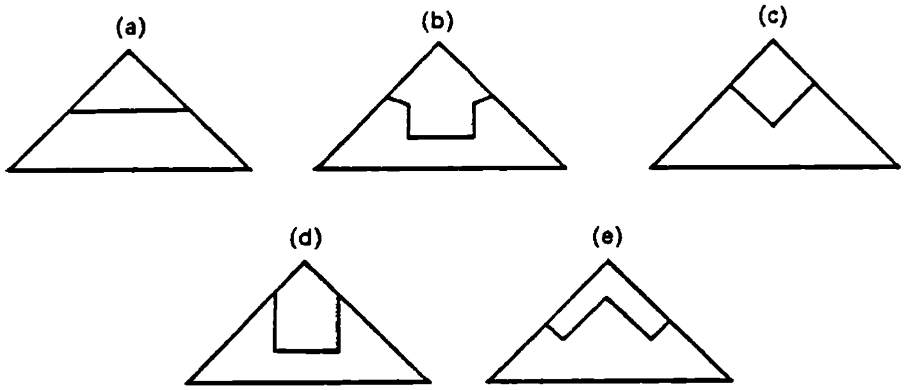
51. There were 35 children altogether in the class. How many did not like apples or oranges?

(a) 7 (b) 10 (c) 13 (d) 19 (e) 21

GO ON TO THE NEXT PAGE

52. Which one of the shapes below could be used four times to make this diagram?





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53. Which one of these statements best conveys the idea of the volume of an object?

(a) the amount of space filled by an object
(b) the weight of an object
(c) the surface area of an object
(d) the weight of water an object will hold
(e) all of these

---

54. Which one of the following numbers has 7 as a factor?

(a) 51 (b) 61 (c) 71 (d) 81 (e) 91

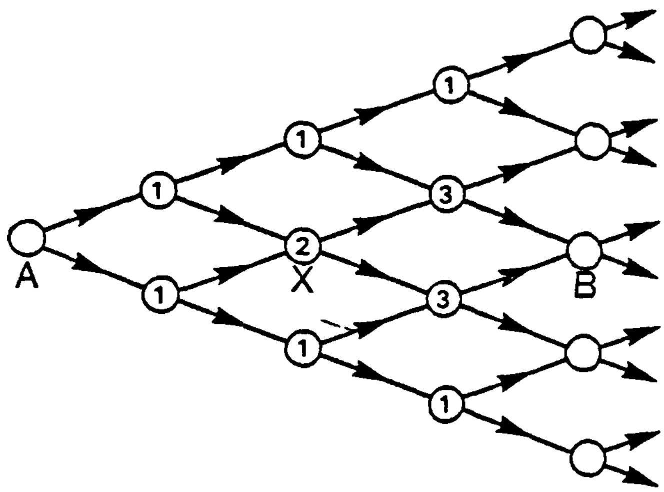
---

55. A man photographed two trees, both the same distance from the camera. The trees on the photograph were 3 cm and 4 cm high. If the height of the smaller tree is in fact 12 metres, what is the height of the other tree?

(a) 14 m (b) 15 m (c) 16 m (d) 18 m (e) 24 m

GO ON TO THE NEXT PAGE

56–57. The diagram below shows the number of routes for getting to various parts of the diagram starting at A and going **only in the direction of the arrows.**



56. How many routes are there for getting from A to B ?
(a) 4 (b) 5 (c) 6 (d) 7 (e) 8

57. On how many of these routes do you pass through X ?
(a) 4 (b) 5 (c) 6 (d) 7 (e) 8

---

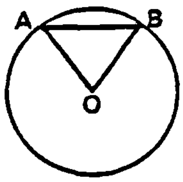
58. The population of a city is given as approximately 2 400 000. Which one of the following figures has been rounded off ?

(a) 2 040 950
(b) 2 114 158
(c) 2 240 876
(d) 2 446 314
(e) 2 475 912

GO ON TO THE NEXT PAGE

59. O is the centre of the circle.
AB is equal in length to the radius of the circle.
What is angle AOB ?

(a) 30° (b) 60° (c) 75° (d) 90° (e) you can't tell

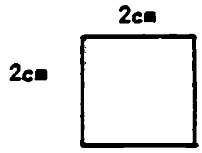


60. The length of one side of a square is y cm. What is the **area** of the square ?

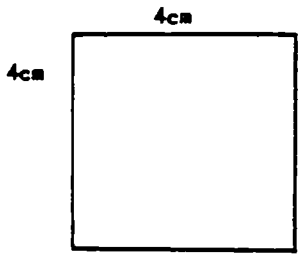
(a) $2y\ cm^2$ (b) $3y\ cm^2$ (c) $4y\ cm^2$ (d) $y^2\ cm^2$ (e) $4y^2\ cm^2$

END OF TEST

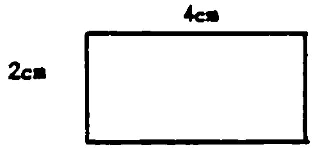
TEST 1 AREAS

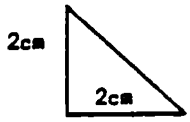


The area of this square is 4 cm^2.



Complete the sentences below.





The area of this rectangle is .....cm^2

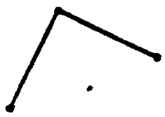
The area of this triangle is .....cm^2

The area of this square is .....cm^2

DO NOT GO ON TO THE NEXT QUESTIONS UNTIL YOU HAVE BEEN TOLD

YOU MAY USE A RULER IN THESE QUESTIONS

1) These are two sides of a four-sided figure.

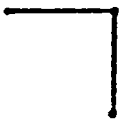


Draw the other two sides so that the area of the figure is 4 cm^2

2) This is one side of a four-sided figure.

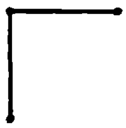
Draw the other three sides so that the area of the figure is 8 cm^2

3) These are two sides of a four-sided figure.



Draw the other two sides so that the area of the figure is more than 4 cm^2

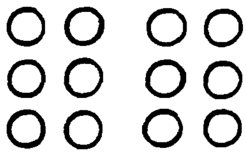
4) These are two sides of a four-sided figure.



Draw the other two sides so that the area of the figure is less than 2 cm^2.

TEST 2 COUNTERS

Walking round a classroom one day I looked over Jane's shoulder. I saw her putting out some counters like this, to help her do a maths question.



After looking at the counters for some time Jane wrote down the answer to the question.
I wonder what the question was that she was doing and what answer she got.
It might have been 6 + 6 = 12, for example.

Think of as many different questions as you can that Jane might have been doing. Try to use lots of different and clever ideas.
In each case write down the question and the answer that Jane should have got. Here is my first idea to get you started.

1) 6 + 6 = 12

TEST 3 SUBSETS

Here is a set of numbers to use in this question :

(2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16)

There are lots of different rules that you could use to make up SUBSETS from this set.

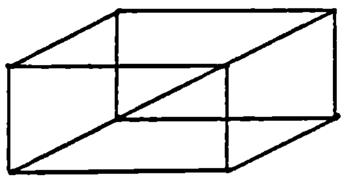
For example, if my rule is that the numbers are even, I get this subset :

(2, 4, 6, 8, 10, 12, 14, 16)

Think of as many different ways as you can for making subsets from the given set.
In each case state briefly what your rule is.
Don't use subsets with only one number in them.
Write your answers like this :

1) (2, 4, 6, 8, 10, 12, 14, 16) (even numbers)

TEST 4 SHAPE-FINDING



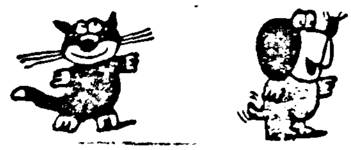
What shapes can you see in this diagram? Draw the shapes you can see and say how many of each shape there are in the diagram.

Example :

I can see two of these rectangles -

TEST 5 SIMILARITIES

These two creatures are very different, of course.



But there is still a lot which is the same about them.
For example,

they both have tails
they both have whiskers
they are both standing on two legs, etc...

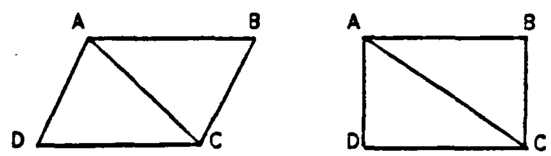
(a) NUMBERS
Here are two different numbers. Although they are different, there are lots of things which are the same about them.

$\boxed{16}$ $\boxed{36}$

Write down as many things as you can think of that these two numbers have in common.

Your answers should start with the words, "They are both....." or, "They both......"

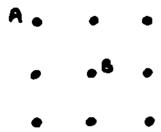
(b) SHAPES
Now do the same for these two figures



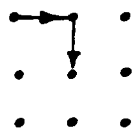
Write down as many things as you can think of that these two figures have in common.
"They are both....." "They both....."

TEST 6 NINE DOT ROUTES

How many different routes can you find for getting from A to B?



Use only straight line paths from one dot to another, for example, like this :



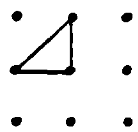
No route should pass through the same dot more than once.

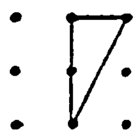
Each route should start at A and finish at B.
Once a route arrives at B it must stop.

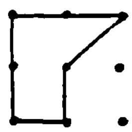
TEST 7 NINE DOT AREAS

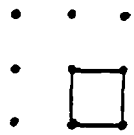
(a) You can make lots of different shapes by joining up dots on a nine dot grid.

Here are some examples -









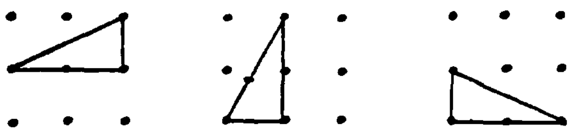
The area of this small square is 1 cm^2.

What are the areas of the other three shapes?

DO NOT GO ON TO THE NEXT PART UNTIL YOU ARE TOLD.

(b) In this part notice that we do not count two shapes as being different if they are just the same shape but in different positions.

For example -



We will count all three of these to be the same shape.

What you have to do -

by joining up dots on a nine dot grid, with straight lines, draw as many <u>different</u> shapes as you can find which have an area of 2 cm^2.

ANSWER SHEET FOR TESTS 6 AND 7

TEST 8 MULTIPLICATION

Find the answers to these multiplication questions.
Do all your working on this sheet.
The first one is done for you as an example.

QUESTION	SPACE FOR WORKING	ANSWER
a) 23 x 24	23 24 460 92 552	552
b) 21 x 12		
c) 32 x 22		
d) 34 x 23		
e) 20 x 10		
f) 16 x 21		

TEST 9 FRACTIONS

Find the answers to these fraction sums.
Do all your working on this sheet.
The first one is done for you as an example.

QUESTION	SPACE FOR WORKING	ANSWER
a) $\frac{1}{2} + \frac{2}{5}$	$\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \frac{9}{10}$	$\frac{9}{10}$
b) $\frac{2}{5} + \frac{1}{3}$		
c) $\frac{1}{4} + \frac{2}{3}$		
d) $\frac{1}{3} + \frac{2}{7}$		
e) $\frac{1}{2} + \frac{1}{4}$		
f) $\frac{1}{3} - \frac{1}{8}$		

TEST 10 JUGS

You have three jugs, A, B and C.
The problem is to find the best way of measuring out a given quantity of water, using just these three jugs.
You are told how much water each jug holds. There are no marks on the sides of the jugs, so the only way to make accurate measurements is to fill a jug to the brim.

Here is an example.

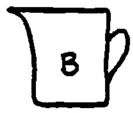
Problem - measure out 55 units.

Jug A holds 10 units.
Jug B holds 63 units.
Jug C holds 2 units.

Solution - fill B, pour off A, fill C and add to B.

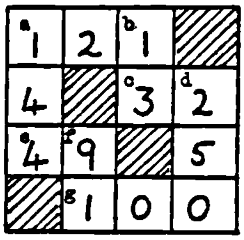
We can write the solution down like this : B - A + C
or if you prefer it : 63 - 10 + 2

Now try these six problems - you can use a jug more than once.

	Measure out this number of units	 Jug A holds this number of units	 Jug B holds this number of units	 Jug C holds this number of units	Solution
1)	52	10	64	1	
2)	14	100	124	5	
3)	3	10	17	2	
4)	100	21	127	3	
5)	20	23	49	3	
6)	5	50	65	5	

TEST 11 CROSS-NUMBER

Here is a cross-number puzzle, with the answers already written in. It's like a crossword, only using numbers instead of words.



The competition is to see who can make up the cleverest clues! My clue for (a) across is - "the number of pence is £1.21", but I expect you can do better than that. Try to use lots of different ideas.

Clues

ACROSS

(a)

(c)

(e)

(g)

DOWN

(a)

(b)

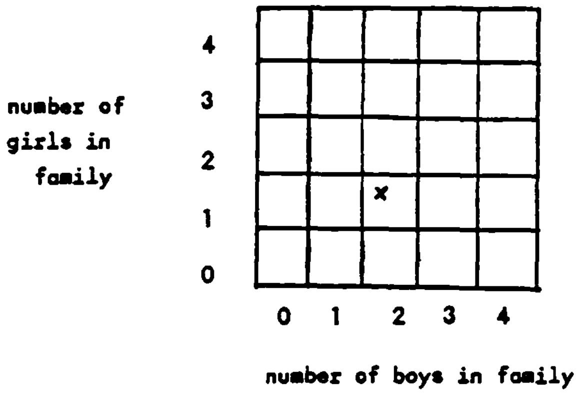
(d)

(f)

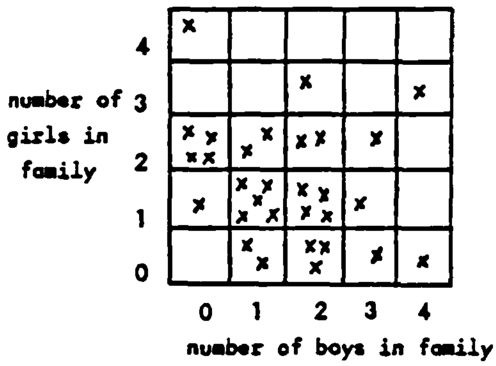
TEST 12 SCATTERGRAMS

The children in a class drew a sort of graph (called a scattergram) to show the number of boys and the number of girls they each had in their families.

One boy had 2 boys and 1 girl in his family, so he put a cross on the graph like this -



When the graph was finished it looked like this -



(a) Answer these questions from the graph.

(i) how many families had 3 boys and 2 girls?

(ii) how many families had 2 boys and 1 girls?

(b) Make up as many interesting and different questions as you can that can be answered from the graph.

Give the answers to your own questions, like this -

<u>Question 1</u> How many families had 2 boys and 1 girl? <u>Answer 4</u>

You get extra marks for thinking of questions that no-one else thinks of.

TEST 13 CLASSROOM

Imagine that you are going to do a mathematics project all about your classroom.
I want you to think of as many interesting questions as you can that you might try to answer.
Remember that it is supposed to be a maths project, so make sure the questions involve using some mathematics.
Also, make sure your questions could actually be answered. But, don't worry, you will not have to answer them!

Here is an example of a question you could use -
"What is the height of the room in metres?"
That's OK as a question because it's something to do with mathematics and it could actually be answered.

Here is a question which is no use, because it's nothing to do with maths -
"Find out where the furniture was manufactured".

And here is another question which is no use, this time because it would be impossible to answer it -
"How long would it take to burn all the desks in the classroom?".

So, there you are, we want questions that could be answered, and that involve some mathematics.

So, imagine you are going to do a mathematics project all about your classroom. Make up as many interesting and clever mathematical questions as you can about your classroom that you might be able to answer.

TEST 14 SUM AND DIFFERENCE

(a) Here are two numbers

70 and 30

Complete the following:

their sum is ______ their difference is ______

DO NOT GO ON TO THE NEXT PART UNTIL YOU ARE TOLD

(b) Find two numbers

		ANSWERS
(i)	whose sum is 10 and whose difference is 4	______ and ______
(ii)	whose sum is 12 and whose difference is 8	______ and ______
(iii)	whose sum is 23 and whose difference is 1	______ and ______
(iv)	whose sum is 19 and whose difference is 17	______ and ______
(v)	whose sum is 25 and whose difference is 15	______ and ______
(vi)	whose sum is 16 and whose difference is 0	______ and ______
(vii)	whose sum is 14 and whose difference is 10	______ and ______
(viii)	whose sum is 10 and whose difference is 10	______ and ______
(ix)	whose sum is 22 and whose difference is 14	______ and ______
(x)	whose sum is 9 and whose difference is 2	______ and ______

TEST 15 DOUBLE AND ADD

Imagine a machine into which you can feed pairs of numbers, like (4,3)
The machine always doubles the first and adds the second, and then gives you the answer, like this :

(4,3) —— [ ] ——→ 11

(a) Fill in the missing numbers below

(2,8) —— [ ] ——→

(5,5) —— [ ] ——→

DO NOT GO ON TO THE NEXT PART UNTIL YOU ARE TOLD

(b) Same machine.

Write in the missing numbers.

(__, 4) —— [ ] ——→ 10

(__,7) —— [ ] ——→ 9

(__,12) —— [ ] ——→ 16

(__,13) —— [ ] ——→ 23

(__, 8) —— [ ] ——→ 30

(3, __) —— [ ] ——→ 15

(8,__) —— [ ] ——→ 16

TEST 16 RESULTS

If you are given a result like

$53 \times 21 = 1113$

you can work out all sorts of other results without having to do a lot of hard work.

For example, if I know $53 \times 21 = 1113$, I can deduce very easily this result -

$53 \times 210 = 11130$

Do you see how I have done that?

Notice that I did not have to do a lot of difficult calculations to get the second result.

Now you are given this result

$23 \times 35 = 805$

Write down other results which you can work out from this given result.

Try to use lots of different ideas in your answers.

Remember - you want results which don't require a lot of hard work.

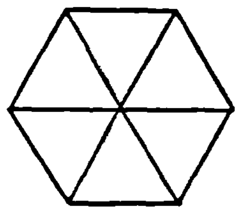
You shouldn't have to do any long multiplication, for example.

Here is one result to get you started -

1) $23 \times 350 = 8050$

TEST 17 WHAT CAN YOU SEE?

What can you see in this diagram?



Write down everything that you can think of to say about this diagram.

TEST 18 THREE CARDS

Three cards that I will call A, B and C have numbers written on one side of them.
If you add the number on card A to the number on card B and then multiply the answer by the number on card C you get 9.

$$(\boxed{A}+\boxed{B})\times\boxed{C} = 9$$

What do you think the numbers on the cards A, B and C might be?
List as many different possibilities as you can think of.

A	B	C

A	B	C

TEST 19 FACTORY

A factory employs 200 craftsmen, 40 apprentices and 10 foremen.
A craftsman earns £3 per hour for a 40 hour week.
If a craftsman does any overtime he gets £5 an hour for this.
An apprentice is paid £60 a week.
A foreman is paid £180 a week.
Apprentices and foremen do not do overtime.

Make up as many questions as you can about this factory which can be answered from this information. Give the answers to your questions. Make your questions as varied and as interesting as possible.

Here's one to get you started -

1) If a craftsman works 42 hours one week, how much is he paid?
   Answer £130

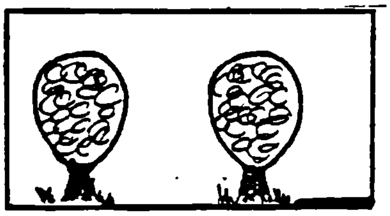
TEST 20 PROBLEM-SOLVING TEST

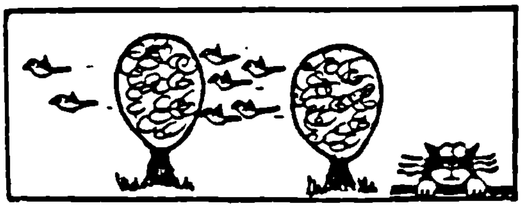
1) Find all the ways in which 32p postage can be made up using only 2p and 3p stamps.

---

2) There were 16 sparrows hidden in these two bushes.

When the cat appeared 2 sparrows flew away from the first bush, and 5 flew from the second bush to the first.



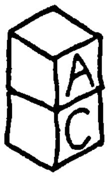


Then there were the same number of sparrows in each bush.
How many sparrows were in each bush to start with?

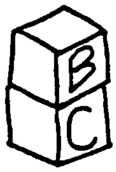
Answer - There were ..... sparrows in the first bush and ..... in the second bush.

TEST 20 (continued)

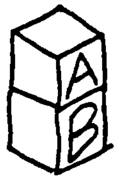
3) John wanted to weigh three boxes, A, B and C. But, because his weighing scales would not weigh anything less than 100g, he had to weigh them two at a time. This is what he found -



A and C together weigh 105g



B and C together weigh 120g



A and B together weigh 125g

Can you work out the weight of each box?

Answer - A weighs .... B weighs .... C weighs ....

---

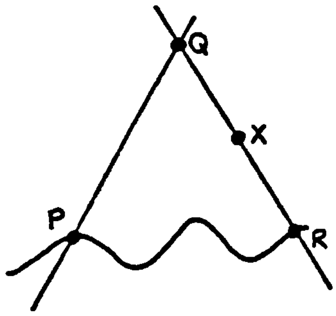
4) Look at this map showing the roads connecting three towns P, Q and R.

The distance from P to Q is the same as the distance from Q to R.

X is a service station exactly half-way between Q and R.

There are two ways of getting from P to X.
One way is 12 miles.
The other way is 9 miles.

How far is it from P to R along the wiggly road?



TEST 20 (continued)

5) Two boys each have money, but no half-pennies.
They each want to buy a ruler.

But they find that one pupil needs 24p more to buy the ruler, and the other pupil needs 2p more to buy it.

So they decide to put their money together and buy one ruler between them.

But even then there is not enough money to pay for the ruler.

What is the price of the ruler?

Answer - The ruler costs .... pence.

---

6) In a magic square the numbers in the rows, columns and diagonals all add up to the same answer. Like this :

8	1	6
3	5	7
4	9	2

in this example, each row, column and diagonal adds up to 15.

Find the missing numbers in the following magic squares.

	3	
1	3	5
4		

	2	
4	3	2

	2	
0	3	6

	6	
1	3	5

TEST 21 QUESTIONNAIRE

Over the page you will find some sentences about various things to do with school. Read each sentence carefully and decide whether it describes how you feel. Then put a ring round one of the symbols next to the sentence.

If you STRONGLY AGREE with the sentence, ring SA
If you AGREE with the sentence, ring A
If you are NOT SURE whether you agree, ring NS
If you DISAGREE with the sentence, ring D
If you STRONGLY DISAGREE with the sentence, ring SD

Try these sentences first with your teacher -

(i)	I like cream cakes	SA	A	NS	D	SD
(ii)	I like having pins stuck in me	SA	A	NS	D	SD
(iii)	I don't like football	SA	A	NS	D	SD
(iv)	School dinners are great	SA	A	NS	D	SD

Be sure to answer every question.
Remember to answer according to how you feel at present.
Work quickly and don't spend a lot of time thinking about any question.

TEST 21 (continued)

SA STRONGLY AGREE
A AGREE
NS NOT SURE
D DISAGREE
SD STRONGLY DISAGREE

1)	In the winter I like playing in the snow at playtime.	SA	A	NS	D	SD
2)	I feel at ease in a maths lesson.	SA	A	NS	D	SD
3)	I worry a lot while I am taking a test.	SA	A	NS	D	SD
4)	I like watching TV when I get home from school.	SA	A	NS	D	SD
5)	I don't do very well in maths.	SA	A	NS	D	SD
6)	When the teacher says we are going to have a test I become afraid that I will do badly.	SA	A	NS	D	SD
7)	When I am taking a hard test I forget some things that I knew very well before I started the test.	SA	A	NS	D	SD
8)	Maths is easy for me.	SA	A	NS	D	SD
9)	I enjoy watching films in lessons at school.	SA	A	NS	D	SD
10)	When I am in bed at night I sometimes worry about how I am going to do in a test at school the next day.	SA	A	NS	D	SD
11)	I like going on outings with my class.	SA	A	NS	D	SD
12)	I feel nervous when the teacher asks me a question about maths.	SA	A	NS	D	SD
13)	I usually understand what we are talking about in maths lessons.	SA	A	NS	D	SD
14)	I am afraid of school tests.	SA	A	NS	D	SD
15)	While I am on the way to school I sometimes worry that the teacher will give us a test.	SA	A	NS	D	SD
16)	No matter how hard I try I cannot understand mathematics.	SA	A	NS	D	SD
17)	I sometimes dream at night that the teacher is angry because I cannot do my work.	SA	A	NS	D	SD
18)	I enjoy games lessons at school.	SA	A	NS	D	SD
19)	It doesn't disturb me to do maths problems.	SA	A	NS	D	SD
20)	When the teacher says we are going to have a test I get a nervous or funny feeling.	SA	A	NS	D	SD
21)	I am good at doing maths problems.	SA	A	NS	D	SD
22)	Just thinking about maths makes me feel nervous.	SA	A	NS	D	SD
23)	I enjoy school time more than the school holidays.	SA	A	NS	D	SD
24)	When I am in bed at night I sometimes worry that I have done badly on a test we had that day at school.	SA	A	NS	D	SD
25)	Working with numbers upsets me.	SA	A	NS	D	SD
26)	I remember most of the things we learn in maths.	SA	A	NS	D	SD
27)	I would rather be told how to do a maths problem than have to work it out for myself.	SA	A	NS	D	SD
28)	I often feel worried in maths lessons.	SA	A	NS	D	SD
29)	When the teacher asks a question in class I hope someone else and not me will be asked to answer it.	SA	A	NS	D	SD
30)	I worry more about maths tests than most other things at school.	SA	A	NS	D	SD

TEST 22 SELF-CONFIDENCE

Script for administrators

How confident are you in your mathematics ability?
I am going to give you 20 maths questions starting with some easy ones and gradually getting harder.
Write your name and the question numbers 1 to 20 on your papers please.
You will be given 10 seconds only for each question.

Now listen carefully to the marking scheme.
Question 1 is worth 1 mark, question 2 worth 2 marks, question 3 worth 3 marks and so on up to question 20 which is worth 20 marks.
BUT if you get a question wrong you lose the marks.
For example, if you got questions 1, 2 and 3 correct and then got question 4 wrong you would score 1 + 2 + 3 - 4 marks = 2 marks in all.

Now, you can stop and hand in your paper whenever you like before a question is given. Otherwise you have to take the question and take a chance of either gaining the marks for that question or losing them.

Of course, once you say you want to stop, you answer no more questions.
(Please repeat marking scheme instructions).

Ready? (Allow 10 seconds after reading each question for them to work it out).

Question 1 worth 1 mark. "6 + 6" write down your answer.
Anyone want to stop now?
If so, write STOP against question 2 and hand in your paper.

Question 2 worth 2 marks. "2 x 5" write down your answer.
Anyone want to stop now?
If so, wrtie STOP against question 3 and hand in your paper.

Question 3 worth 3 marks. Remember you lose 3 marks if you get this wrong.
"What is the cost of 4 apples at 5p each?" write down your answer.
Anyone want to stop now?
If so, write STOP against question 4 and hand in your paper.

Question 4 worth 4 marks. Remember you lose 4 marks if you get this wrong.
"How many centimetres in a metre?" write down your answer.
Anyone want to stop now?
If so, write STOP against question 5 and hand in your paper.

Question 5 worth 5 marks. "What is the area of a rectangle 6cm long and 3cm wide?" write down your answer.
Anyone want to stop now?
If so, write STOP against question 6 and hand in your paper.

(Continue with same form of words for questions 6 to 20)

6) 2.2 + 3.3
7) What is the area of a triangle with base 6cm and height 5cm?
8) 4 x 13
9) Multiply the number of sides in a hexagon by the number of sides in a triangle.
10) 4.5 x 3
11) Add up the numbers from 10 to 15
12) a half plus a third
13) 0.2 x 0.3
14) 10 per cent of two pounds fifty
15) two-fifths of 2 metres, in centimetres
16) what is the average of 25, 27, and 32?
17) what change do I get from a hundred pounds if I buy 3 articles at seven pounds forty pence each?
18) 0.4 x 0.4 x 0.4
19) 12½ per cent of twelve pounds
20) How many degrees in three-eighths of a complete rotation?

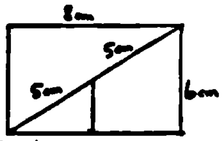
TEST 23 MULTI-CHOICE TEST A

Do not write on this question paper. Answer by ringing the correct letter on the answer sheet provided. Give just one answer for each question. If you don't know the answer ring the letter E.

1) What is the perimeter of this rectangle?

A 10cm B 12cm C 20cm D 24cm E don't know

2) What is the length of the conjugal in this rectangle?



A 3cm B 4cm C 5cm D 10cm E don't know

3) What is the next number in the sequence? 5, 6, 8, 11, ...

A 12 B 13 C 14 D 15 E don't know

4) What is the next number in this sequence? 3, 26, 28, ...

A 29 B 30 C 4 D 51 E don't know

5) What is the sum of the two principal factors of 60?

A 17 B 8 C 50 D 60 E don't know

6) The median of the numbers 3, 4, 8, 9, 12, 16, 25, 26, 30, is 12.
What is the median of these numbers - 4, 6, 9, 15, 21, 34, 35?

A 15 B 35 C 4 D 12 E don't know

7) What sign is missing from this calculation? 35 $\square$ 7 = 5

A + B - C x D ÷ E don't know

8) What sign is missing here? 598 $\square$ 8947 = 5350306

A + B - C x D ÷ E don't know

9) Which of these calculations is wrong?

A 35 + 5 = 40 B 35 x 5 = 40 C 40 - 3 = 37 D 100 ÷ 10 = 10
E don't know

10) Which of these calculations is wrong?

A 4 x 3 = 12 B 65 ÷ 5 = 13 C 592 x 874 = 571408 D 81 ÷ 9 = 9
E don't know

TEST 23 (continued)

11) Which of the following is not equivalent to $\frac{1}{2}$?

_A_ $\frac{2}{3}$ _B_ $\frac{2}{4}$ _C_ $\frac{6}{12}$ _D_ $\frac{4}{8}$ _E_ don't know

12) Which of the following is equivalent to $\frac{6}{7}$?

_A_ $\frac{1}{7}$ _B_ $\frac{3}{7}$ _C_ $\frac{2}{3}$ _D_ $\frac{29.46}{34.37}$ _E_ don't know

13) Which of the following is the zero matrix?

_A_ $\begin{bmatrix} 0 & 1 \\ 2 & 3 \end{bmatrix}$ _B_ $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ _C_ $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ _D_ $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ _E_ don't know

14) Which matrix is equal to $\begin{bmatrix} 3 & 5 \\ 9 & 6 \end{bmatrix} + \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$? _E_ don't know

_A_ $\begin{bmatrix} 4 & 6 \\ 10 & 7 \end{bmatrix}$ _B_ $\begin{bmatrix} 2 & 4 \\ 8 & 5 \end{bmatrix}$ _C_ $\begin{bmatrix} 3 & 5 \\ 9 & 7 \end{bmatrix}$ _D_ $\begin{bmatrix} -1 & 4 \\ 10 & 7 \end{bmatrix}$

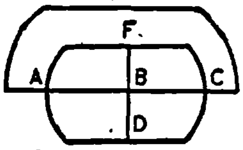
15) Find the value of $2^2 + 3^2$

_A_ 25 _B_ 13 _C_ 10 _D_ 16 _E_ don't know

16) What is the sum of the vectors $\begin{pmatrix} 2 \\ 4 \end{pmatrix}$ and $\begin{pmatrix} -1 \\ 5 \end{pmatrix}$?

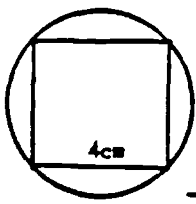
_A_ $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ _B_ $\begin{pmatrix} 1 \\ 9 \end{pmatrix}$ _C_ $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ _D_ $\begin{pmatrix} 3 \\ 9 \end{pmatrix}$ _E_ don't know

17) In the diagram, apart from F, which other point is an odd-node?



_A_ A _B_ B _C_ C _D_ D _E_ E

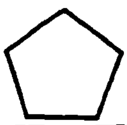
18) In the diagram the area of the square is $16cm^2$. What is the area of the circle?



_A_ about $10cm^2$ _B_ about $12cm^2$ _C_ about $25cm^2$

_D_ about $100cm^2$ _E_ don't know

19) Which of the following is an equilateral pentagon?

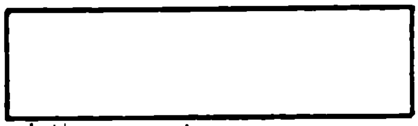
_A_  _B_  _C_  _D_  _E_ don't know

20) What is the answer to this calculation? $4586 \div 7895 = ?$

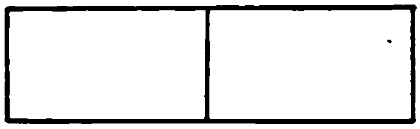
_A_ 1.5 _B_ about 0.6 _C_ 8 _D_ more than 300 _E_ don't know

TEST 24 CUTS

These problems ask you to find out how many lines you need to draw to cut this rectangle into a given number of equal parts. The parts must be the same shape and the same size.



For example, to cut it into 2 equal parts, you would need to use just 1 line, like this.



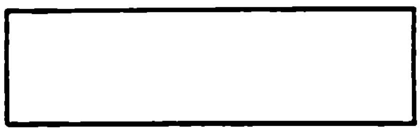
To cut it into 3 equal parts, you would need 2 lines.

Do these.

To cut it into 5 equal parts you would need ...... lines.
(on the diagram show roughly where your lines would be)

To cut it into 7 equal parts you would need ...... lines.

To cut it into 9 equal parts you would need ...... lines.



TEST 25 C-W TEST

This game asks you to guess about a lot of things in our world. For instance, if you knew that most men in the world are around 170 cm tall, you might guess that the tallest man in the world is 200 cm tall, or 225 cm tall, or only 100 cm tall. In this game you get a chance to guess about things like that. Just put a tick next to your guess for each of the questions below.

1) Most birds fly at the speed of about 17 miles per hour.

(a) How fast does the fastest bird fly?
- 30 miles per hour
- 21 miles per hour
- 60 miles per hour
- 18 miles per hour

(b) How fast does the slowest bird fly?
- 15 miles per hour
- 5 miles per hour
- 10 miles per hour
- 2 miles per hour

2) Most whales are about 20 metres long.

(a) How long is the longest whale?
- 21 metres
- 50 metres
- 25 metres
- 30 metres

(b) How long is the shortest whale?
- 12 metres
- 2 metres
- 15 metres
- 18 metres

3) Most dogs are about 105 cm long.

(a) How long is the longest dog?
- 140 cm
- 120 cm
- 170 cm
- 200 cm

(b) How long is the shortest dog?
- 30 cm
- 15 cm
- 75 cm
- 60 cm

4) Most cars are able to go about 100 miles per hour.

(a) How fast will the fastest car go?
- 223 miles per hour
- 105 miles per hour
- 424 miles per hour
- 150 miles per hour

(b) How fast will the slowest car go?
- 3 miles per hour
- 18 miles per hour
- 9 miles per hour
- 1 mile per hour

5) Most roads are about 9 metres wide.

(a) How wide is the widest road?
- 52 metres
- 27 metres
- 12 metres
- 36 metres

(b) How wide is the narrowest road?
- 8 metres
- 3 metres
- 1 metre
- 6 metres

TEST 25 (continued)

6) Most buildings are about 18 metres high.
   (a) How high is the tallest building?
   140 metres
   450 metres
   50 metres
   20 metres

   (b) How high is the shortest building?
   15 metres
   2 metres
   10 metres
   5 metres

7) Most windows are about 85 cm wide.
   (a) How wide is the widest window?
   350 cm
   90 cm
   110 cm
   145 cm

   (b) How wide is the narrowest window?
   5 cm
   60 cm
   30 cm
   75 cm

8) Most sailing boats go at about 9 miles per hour.
   (a) How fast will the fastest sailing boat go?
   23 miles per hour
   44 miles per hour
   11 miles per hour
   15 miles per hour

   (b) How fast will the slowest sailing boat go?
   7 miles per hour
   8 miles per hour
   5 miles per hour
   3 miles per hour

9) Every year about 300 new school text books are written (since 1950)
   (a) What is the largest number of school text books written in one year? (since 1950)
   524 books
   330 books
   392 books
   980 books

   (b) What is the smallest number of school text books written in one year? (since 1950)
   94 books
   25 books
   9 books
   180 books

10) Most people spend about 55 minutes out of a whole day eating meals.
   (a) What is the longest time anyone spends eating meals in a whole day?
   60 minutes
   105 minutes
   240 minutes
   73 minutes

   (b) What is the shortest time anyone spends eating meals in a whole day?
   3 minutes
   29 minutes
   47 minutes
   11 minutes

TEST 26 MULTI-CHOICE TEST B

Do not write on this paper. Answer by ringing the correct letter on the answer sheet. Give just one answer for each question.

1) $7 \times 8 = ?$
A 65 B 63 C 42 D 56 E 58

2) $\frac{3}{4} + \frac{3}{4} = ?$
A $\frac{6}{8}$ B $1\frac{1}{2}$ C $\frac{9}{4}$ D $\frac{3}{8}$ E $3\frac{1}{2}$

3) $(0.1)^2 =$ A 0.1 B 0.01 C 0.2 D 0.02 E 0.001

4) What is the cost of 15 apples at 6p each?
A 90p B 65p C 76p D £1.20 E 80p

5) Which of these fractions is the largest?
A $\frac{3}{7}$ B $\frac{1}{3}$ C $\frac{1}{2}$ D $\frac{3}{4}$ E $\frac{3}{20}$

6) If $41 \times 32 = 1312$, what is the value of 410×320?
A 1312 B 13120 C 131200 D 1312000 E 13120000

7) How many centimetres are there in 3 metres?
A 30 B 300 C 3000 D 0.3 E 0.03

8) How many square centimetres are there in 3 square metres?
A 300 B 900 C 30000 D 90000 E 6000

9) How long does a TV programme last that starts at 11.30 am and finishes at 12.10 pm?
A 40 minutes B 12 hrs 40 mins C 80 mins D 20 mins E 50 mins

10) A boy goes to bed at 9.40 pm . Which of these times in 24-hour clock notation is closest to his bed-time?
A 09.30 hours B 19.00 hours C 20.05 hours D 21.00 hours E 22.00 hours

TEST 26 (continued)

11) The monthly repayment on a new bicycle works out at £2.551 - what is this to the nearest ten pence?

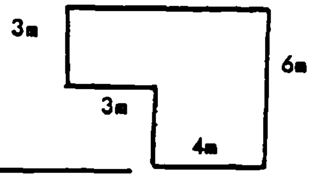
A 2550p B 300p C 200p D 250p E don't know

12) £1.20 + £2.80 = ?

A £3 B £3.28 C £4 D £4.10 E £5

13) What is the area in square metres of this room?

A 16 B 23 C 55 D 30 E 33



14) Which of these numbers is the largest?

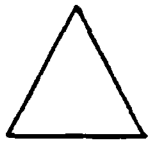
A 4962 B 38295 C 30999 D 9998 E 3986.5

15) Which of these numbers is the smallest?

A 0.00009 B 0.9 C 0.85 D 0.000085 E 0.001

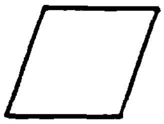
16) This triangle is called

A scalene B equilateral C isosceles
D right-angled E obtuse



17) This shape is called a

A rhombus B pentagon C trapezium
D rectangle E parallel



18) What is the product of 4 and 8?

A 4 B 6 C 12 D 2 E 32

19) Which of these is <u>not</u> a multiple of 3?

A 20 B 21 C 24 D 27 E 36

20) What is the difference between 2 and $\bar{1}.9876$?

A 0.0123 B 3.9876 C 0.0234 D 0.0124 E 0.1234

TEST 27 CLUES

Clues for round 1 -

(1) it is a whole number
(2) less than 60
(3) odd
(4) it has 2 digits
(5) not a multiple of 3
(6) second digit greater than first
(7) does not end in 7
(8) it ends in 9
(9) it is less than 50
(10) more than 20
(11) equals 7 x 7

(ANSWER : 49)

Clues for round 2 -

(1) one of the shapes on this sheet
(2) not a circle
(3) no curved bits
(4) less than 6 sides
(5) not a triangle
(6) not a rectangle
(7) on lower half of the sheet
(8) a quadrilateral
(9) not concave (doesn't go in on itself)
(10) not a parallelogram
(11) not Q

(ANSWER : W)

TEST 27 (continued)

Clues for round 3 -

(1) a whole number
(2) less than 60
(3) even
(4) greater than 20
(5) does not have 3 as the first digit
(6) not a multiple of 11
(7) it is not 56 or 58
(8) a multiple of 6
(9) not a multiple of 7
(10) a multiple of 12
(11) half way between 22 and 26

(ANSWER : 24)

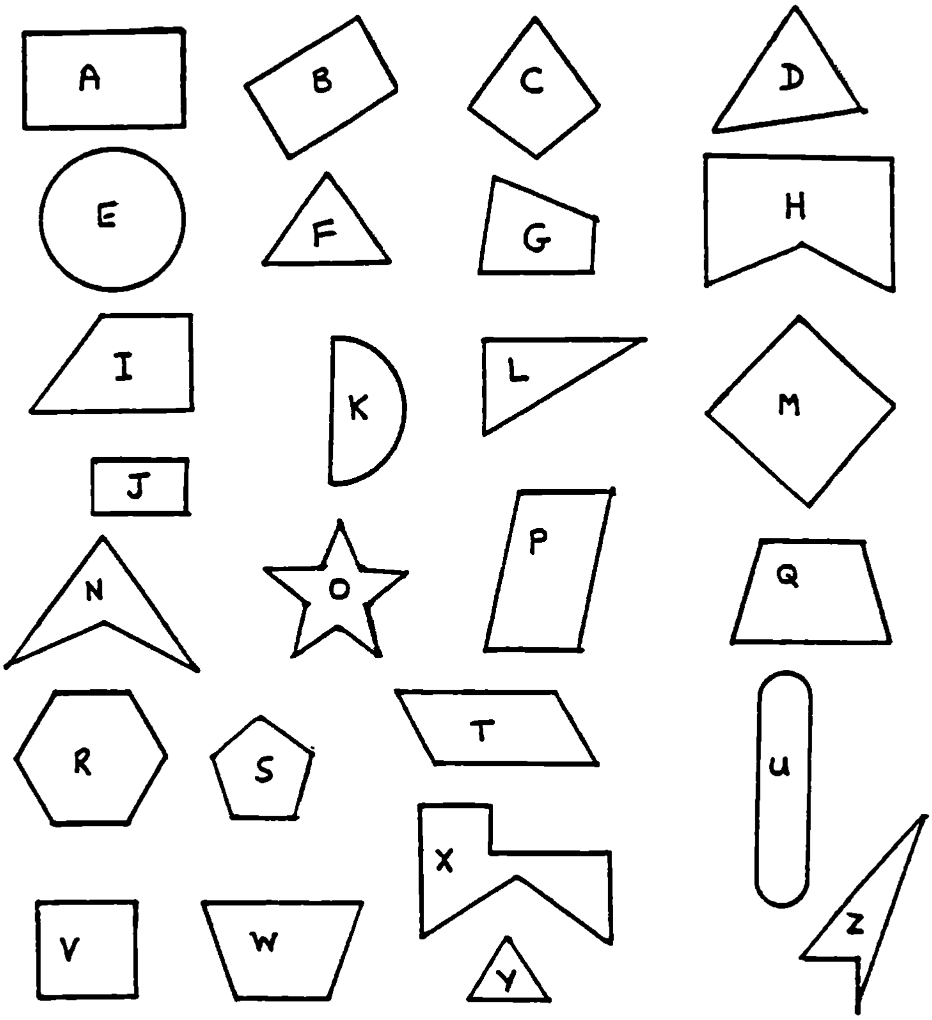
Clues for round 4 -

(1) on this sheet
(2) not a star shape
(3) not all sides equal
(4) straight edges only
(5) 3 or 4 sides
(6) not a rectangle
(7) no parallel sides
(8) on top half of sheet
(9) a triangle
(10) no right angles
(11) two sides equal length

(ANSWER : F)

TEST 27 (continued)

Set of shapes displayed on sheet :



Scoring scheme displayed :

No: of clues	1	2	3	4	5	6	7	8	9	10	11
Points	60	59	57	54	50	45	39	32	24	15	5

APPENDIX 2

INSTRUCTIONS TO TEST ADMINISTRATORS

Test 1 AREAS

(on same sheet as Test 2 COUNTERS)

Distribute papers. Children will need rulers.
Names on papers please, printed clearly.
Give class about a minute to complete the sentences, then give and explain the answers to these three simple questions. Let the children mark their own work.
Then allow them 5 minutes to complete the remaining questions on the test (questions numbered 1, 2, 3, 4).
After this time tell children to turn over to Test 2.

Test 2 COUNTERS

Names on papers please, printed clearly.
Please read the preamble to the children. Repeat and emphasise the second sentence - "I saw her putting out some counters like this, to help her do a maths question." Please make sure the children understand what they have to do, but try to prevent them asking questions which might give away ideas to other children (e.g. can we use fractions?). Obviously you should not give them any other ideas, apart from the example given in the question.
Allow 10 minutes for this test.
Answers to be written on the test paper, but extension sheets can be supplied to children if necessary. In this case please ensure that names are written on such sheets.

Test 3 SUBSETS

(on same sheet as Test 4 SHAPE-FINDING and Test 5 SIMILARITIES)
Distribute papers and paper for writing answers on. Names on paper please.

If class are not familiar with the term 'subset' please explain it using examples from the set of children in the room - e.g. the subset consisting of the children with birthdays in September, the subset of children sitting at this table, the subset of children who come to school on bicycles, etc...
Then read the instructions in the question to the children. Please do not give away any other ideas, apart from the given example. Allow <u>10 minutes</u> for this test. Then tell the class to leave that question and go on to the next.

<u>Test 4 SHAPE-FINDING</u>

Allow <u>10 minutes</u> for this question. Then tell the class to stop and listen while you explain the next question.

<u>Test 5 SIMILARITIES</u>

Please read the preamble to the class and ask for one or two more examples of similarities for the cat and dog shown. Emphasise that the statements begin with either "they both..." or "they are both..."
Allow <u>5 minutes</u> for part (a) NUMBERS and then <u>5 minutes</u> for part (b) SHAPES.

<u>Test 6 NINE DOT ROUTES</u>

(on same sheet as Test 7 NINE DOT AREAS)

Test 6 and 7 are to be answered on the dotty paper provided, one side for each test. Name on dotty paper please. Rulers are not required in this test.
Please read the instructions. Emphasise that a route must not pass through the same point twice. Try not to give away ideas

like using diagonal paths, but make sure the children understand that a straight line can be drawn from one dot to any other dot in constructing a route.

Allow 5 minutes for this test, then ask the children to turn over both the question paper and the dotty paper for Test 7 NINE DOT AREAS.

Test 7 NINE DOT AREAS

Please give the class a minute or so to think about the questions in part (a), then ask for answers from the class. Explain how the correct answers are arrived at.

Then read the instructions for part (b) to the class.

Allow 10 minutes for this part. Answers on dotty paper may be drawn freehand.

Test 8 MULTIPLICATION

(on same sheet as Test 9 FRACTIONS)

Names on papers please.

Please go carefully through the working for question (a) on the blackboard. If you do not usually set this out as shown on the sheet ask the children to cross out my working and copy down your usual way of doing it from the board. Spend as much time as you think is necessary in getting the process in example (a) clearly understood.

This test should take about 10 minutes, but please allow enough time for all the children to have at least started part (f).

Please no comments about part (e) !

Test 9 FRACTIONS

Same procedure as for Test 8 MULTIPLICATION
Please allow all children enough time to get to the end of this test.
You may need some unrelated activity for those who finish quickly.
The test should take no longer than 10 minutes.

Test 10 JUGS

(on same sheet as Test 11 CROSS NUMBER)

This requires careful explanation, so please read the instructions to the class and answer any questions they may have. I will answer any questions you may have in the briefing session. Go through the example with them on the board - problem to measure out 55 units, if Jug A holds 10, B holds 63, C holds 2. Either method of writing down the solution is acceptable.
Names on paper please.
Solutions to be written on the sheet in the spaces provided.
Time allowed - as long as it takes, probably about 10 minutes.
After about a minute, please give the class the solution for the first question. This is B - A - C - C, 64 - 10 - 1 - 1 , with suitable explanation.
It is important that children do not give the game away if they spot the best ways of doing parts 5 and 6. So please instruct the class to keep their ideas to themselves, and when they finish to turn over and go straight on to Test 11 CROSS NUMBER.

Test 11 CROSS NUMBER

No explanation required. No definite time limit, but 10 minutes should be sufficient for most children to compile their clues.

Test 12 SCATTERGRAM

(on same sheet as Test 13 CLASSROOM)

Children will need paper on which to answer this test.

Names on papers.

Please read the preamble to the class and use the questions in (a) to check that children understand how to read the graph. The answers are (i) 1 and (ii) 4. Allow 10 minutes for (b).

Test 13 CLASSROOM

Please read through the whole of this question to the class, explaining where necessary what they have to do, without giving away any ideas.

They will need paper on which to write their answers. Allow 10 minutes.

Test 14 SUM AND DIFFERENCE

(on same sheet as Test 15 DOUBLE AND ADD)

Use part (a) to ensure familiarity with 'sum' and 'difference'. Then allow sufficient time for all the children to have had a go at the last part of (b). This test should take no longer than about 10 minutes.

Test 15 DOUBLE AND ADD

Use part (a) to ensure that they understand how the 'machine' works. Then allow sufficient time (probably about 10 minutes) for the class to complete (b).

Test 16 RESULTS

(on same sheet as Test 17 WHAT CAN YOU SEE?)

Please read and explain as necessary the preamble. Don't give away any other ideas! Time 10 minutes for this test. Provide extension sheets if necessary (unlikely). Names on extension sheets, of course.

Test 17 WHAT CAN YOU SEE?

Time 10 minutes for this question. Again you may possibly need to provide extension sheets for some creative children.

Test 18 THREE CARDS

(on same sheet as Test 19 FACTORY)

Names on papers please.
Just to check that the children understand the idea of the cards, please say, "If A were 5, B were 1 and C were 2, what would the answer be? (take suggestions) - That's right (!) 12, because we get 5 add 1 equals 6, multiply by 2, answer 12. Now you've got to choose numbers for A, B and C to make the answer 9. Write your answers in the columns on your paper."
Please allow 5 minutes for this test.

Test 19 FACTORY

Names on papers please.
Read the preamble to the class, and explain how the answer 130 is obtained in the example, so that they understand about overtime. Do not give away any other ideas. Please allow 10 minutes for this test.

Test 20 PROBLEM-SOLVING

Please allow 45 minutes for this test. Answers written on the papers. Give no explanations of the questions.

Test 21 QUESTIONNAIRE

Please go carefully through the front page with the class, give them a few minutes to ring their answers for the trial items, then discuss their answers to make sure they understand the system. It will probably take them about 15 minutes to complete the questionnaire.

Analysis of items -

Anxiety towards mathematics	12, 22, 25, 27, 28, 30
Self-concept in mathematics	2, 5, 8, 13, 16, 19, 21, 26
Test anxiety	3, 6, 7, 10, 14, 15, 17, 20, 24, 29
Fillers	1, 4, 9, 11, 18, 23

Test 22 SELF-CONFIDENCE

Children will need a piece of paper on which they write their names and question numbers 1 to 20. Administrators are provided with a script for this test (see Appendix I). At the end of the test you may like to let the children mark each other's work according to the described scoring scheme. Then collect scripts again

Test 23 MULTI-CHOICE TEST A

Children will need the answer sheets for this test. Please do not answer any questions about the content of this test. The test is designed deliberately to put the children into situations where they may have to take risks. Allow 20 minutes for this test.

When you have collected the answer sheets in you may feel free to go through the test with the children and tell them how you would have answered.

Please would each teacher complete the questionnaire enclosed about this test.

Test 24 CUTS

This should only take 5 minutes.

Test 25 C-W Test (Category-width)

This will take about 10 minutes, but please allow all children to finish.

Read the preamble to them and make sure they understand what they have to do. Apologies, but I cannot supply the correct answers to these questions!

Test 26 MULTI-CHOICE TEST B

Children will need the answer sheets that they used for Test 23 again.

Allow 20 minutes for this test. Do not go over the questions after you have collected in the papers.

Two days later, hand back both question papers and answer sheets and tell the children the following.

"Mr Haylock, the man who is making up all these tests for you to do, had a look through the answers you had given to this test the other day, and he was very surprised at some of the silly mistakes some children had made. So he has asked us to give you another chance at this particular test paper. And what is more he has asked us to give you some help this time, in a rather unusual way. He has given this test to children a year older than you at the High School, and for each question I am going to tell you which was the most popular answer chosen by those older children. It need not necessarily be right - that's up to you to decide - but the answer I call out is the one that was chosen by most of those older children. If you decide to change any of your answers on your answer sheet, just put a cross through your old answer, like this (demonstrate) A ~~B~~ C $\boxed{D}$ E, and put a little square box round your new answer."

"Now look again at the questions and the answers you gave. Make sure you are looking at the answers for TEST B. That's the lower half of the answer sheet.

Remember, I am going to call out the answer that was chosen by most children a year older than you. That is, the most common, the most popular answer.

Ready, question 1, the most popular answer by the High School children was "D". (Allow half a minute for thought).

If you want to change your answer, just cross out your old answer and put a box round your new answer.

Question 2, the most popular answer was "B". (Allow half a minute)" .....and so on.....

question	most popular answer	(correct answer)
1	D	D
2	B	B
3	C	B
4	A	A
5	E	D
6	C	C
7	B	B

question	most popular answer	(correct answer)
8	B	C
9	A	A
10	C	E
11	D	E
12	C	C
13	E	E
14	B	B
15	A	D
16	B	B
17	C	A
18	E	E
19	A	A
20	E	D

Collect in answer sheets.

You may then like to go through and give the children the correct answers.

Test 27 CLUES

Before you start this test please revise the following, if necessary -

meaning of 'digit', 'multiple',
how to test for divisibility by 3,
6-times, 7-times, 12-times tables
meaning of 'triangle', 'quadrilateral', 'parallelogram'
'concave', 'right angle', 'rectangle'.

Each child will require four small pieces of paper.

Ask them to write their names on each piece.

"We are going to play a sort of guessing game. There are four rounds to this game. In each round you have to guess a number or a shape from the clues I give you. There will be eleven clues for each round. You can make your guess whenever you like, by writing it on your piece of paper and handing it in. The sooner you make your guess the more points you score if it is right. Of course, if you get it wrong you score no points.
This is the scoring scheme. (Display scoring scheme prominently).
If you guess correctly after only 1 clue, you score 60 points.
Then if you guess correctly after 2 clues you score 59 points.
After 3 clues 57 points, after 4 clues, 54 points and so on.
Notice how the number of points you can score goes down more and more quickly. So that if you wait until you have had 10 clues you only score 15 points, and then if you wait until you've had the last clue you can only score 5 points.
Let's see who can do best in this game. We're going to play four rounds, then afterwards we'll add up your scores and see who is the winner. There will be a small prize of some sweets (display!) for the winner in this class.

Right, first round is 'guess my number'. Write 'round 1' on one of your pieces of paper. Remember I'm going to give you eleven clues. You can make your guess whenever you like.
Just write your guess on your paper and raise your hand and I will collect in your paper and write on it how many clues you have had.
First clue.....(Please write clues on the board. Do <u>not</u> erase the clues as you go along. Eventually all eleven clues should be written on the board.
Allow half a minute for thought between each clue. When a pupil

want to hand in a guess, remember to write on his paper the number of clues received. You may remind pupils of the meanings of any terms used in the clues as you go along. Do not give scores until after round 4).

Second round is 'guess my shape'. Write 'round 2' on one of your pieces of paper. (Display shape sheet). (Proceed as in round 1).

To make your guess just write the letter of the shape on your paper.

Third round is 'guess my number' again. Write 'round 3' on one of your pieces of paper.

(Proceed as in round 1).

Fourth round is 'guess my shape'. Write 'round 4' on your last piece of paper. Ready. (Proceed as in round 2).

(You may then go through the four piles of guesses, announcing who has guessed correctly and what scores they obtain. The answers are 49, W, 24 and F. Please retain the pieces of paper with the children's guesses and the number of clues received for me. Thank you).

APPENDIX 3

Additional data related to Test 23 (Multi-Choice Test A)
Test 27 (Clues)
Test 22 (Self-Confidence)
Test 26 (Multi-Choice Test B)
Test 25 (Category Width)
Test 21 (Questionnaire)

TEST 23

Table A3.1

Percentages of Pupils Choosing Various Options for the Items in Test 23 (Multi-Choice Test A)

	Options:				
Question:	A	B	C	D	E
1	2.9	0.8	53.7	42.6	0
2	30.2	11.2	7.9	33.9	16.9
3	7.9	3.7	10.7	77.3	0.4
4	6.2	17.8	21.5	43.4	11.2
5	21.9	13.2	10.3	31.8	23.6
6	47.5	7.0	11.2	5.4	20.7
7	0.8	0	3.3	95.5	0.4
8	2.5	0.8	92.1	1.2	3.7
9	0	96.3	3.3	0	0.4
10	0	7.4	85.6	2.5	4.5
11	81.4	7.9	5.4	2.5	2.9
12	1.7	8.3	26.4	47.9	15.7
13	1.7	1.7	77.7	6.2	12.8
14	68.2	3.7	7.9	6.2	14.0
15	4.1	55.0	36.4	3.3	1.2
16	2.5	55.8	13.2	14.5	14.0
17	1.7	10.3	5.8	68.6	13.6
18	5.8	11.6	73.1	2.5	7.0
19	71.9	2.9	6.2	15.3	3.7
20	38.4	35.5	5.0	10.7	10.3

Table A3.2

Means and Standard Deviations of RT_{23} Indices for Various Levels of Mathematical Attainment

Maths attainment	Mean	RT_{23} indices: Standard deviation	Number
A. very high	86.2	18.2	51
B. above average	88.1	18.0	96
C. average	72.0	23.0	82
D. below average	79.4	26.6	17

Table A3.3

Means and Standard Deviations for RT_{23} Indices, Boys and Girls Separately

	Mean	RT_{23} indices: Standard deviation	Number
Boys	82.5	21.2	129
Girls	83.6	20.5	114
All	83.0	21.0	243

Table A3.4

Correlations Between RT_{23} Indices and Mathematical Creativity Measures, Boys and Girls Separately

	Correlation coefficient between:	
	RT_{23} and OF	RT_{23} and DP
Boys	0.14 (119 pupils)	0.12 (120 pupils)
Girls	0.14 (102 pupils)	0.18 (117 pupils)

TEST 27

Table A3.5

Means and Standard Deviations for RT_{27} Indices for Various Levels of Mathematics Attainment

	RT_{27} indices:		
Maths attainment	Mean	Standard deviation	Number
A. very high	22.0	15.4	43
B. above average	25.5	18.1	65
C. average	36.4	18.5	78
D. below average	47.5	14.5	12

Table A3.6

Means and Standard Deviations for RT_{27} Indices, Boys and Girls Separately

	RT_{27} indices: Mean	Standard deviation	Number
Boys	30.6	18.9	111
Girls	30.1	19.1	88
All	30.4	19.0	199

Table A3.7

Correlations between RT_{27} Indices and Mathematical Creativity Scores (OF/DP), Boys and Girls Separately

	Correlation coefficient between: RT_{27} and OF	RT_{27} and DP
Boys	-0.24 (101 pupils)*	-0.32 (102 pupils)**
Girls	-0.23 (73 pupils)*	-0.54 (80 pupils)**

* significant at 5% level ** significant at 1% level

TEST 22

Table A3.8

Means and Standard Deviations for RT_{22} Indices for Various Levels of Mathematics Attainment

	RT_{22} indices:		
Maths attainment	Mean	Standard deviation	Number
A. very high	90.1	10.5	50
B. above average	80.3	15.1	71
C. average	69.8	15.6	79
D. below average	66.2	13.0	16

Table A3.9

Means and Standard Deviations for RT_{22} Indices, Boys and Girls Separately

	RT_{22} indices:		
	Mean	Standard deviation	Number
Boys	76.5	17.8	122
Girls	79.0	14.7	95
All	77.6	16.5	217

Table A3.10

Correlations Between RT_{22} Indices and Mathematical Creativity (OF/DP) Scores, Boys and Girls Separately

	Correlation coefficient Between:	
	RT_{22} and OF	RT_{22} and DP
Boys	0.49 (112 pupils)	0.47 (114 pupils)
Girls	0.43 (85 pupils)	0.41 (88 pupils)

(all correlations significant at 1% level

TEST 26

Table A3.11

Means and Standard Deviations for NC Indices for Various Levels of Mathematics Attainment

	NC Indices:		
Maths attainment	Mean	Standard deviation	Number
A. very high	90.3	13.9	49
B. above average	86.8	16.5	87
C. average	71.0	26.8	78
D. below average	60.3	30.8	16

Table A3.12

Means and Standard Deviations for NC Indices,

Boys and Girls Separately

	NC indices:		
	Mean	Standard deviation	Number
Boys	80.0	23.3	124
Girls	80.4	23.5	108
All	80.2	23.4	232

Table A3.13

Correlations Between NC Indices and Mathematical Creativity

Scores, Boys and Girls Separately

	Correlation coefficient between:	
	NC and OF	NC and DP
Boys	0.31 (111 pupils)	0.30 (115 pupils)
Girls	0.29 (96 pupils)	0.29 (102 pupils)

(all correlations significant at 1% level)

TEST 25

Table A3.14

Means and Standard Deviations for CW Scores for Various Levels of Mathematics Attainment

	CW scores:		
Maths attainment	Mean	Standard deviation	Number
A. very high	34.9	8.9	49
B. above average	29.7	11.0	87
C. average	29.0	9.7	85
D. below average	31.2	11.2	17

Table A3.15

Means and Standard Deviations for CW Scores, Boys and Girls Separately

	CW scores:		
	Mean	Standard deviation	Number
Boys	32.9	9.5	130
Girls	28.0	10.7	111
All	30.6	10.4	241

Table A3.16

Correlations Between CW Scores and Mathematical Creativity Scores (OF/DP), Boys and Girls Separately

	Correlation coefficient between: CW and OF		Correlation coefficient between: CW and DP	
Boys	0.22*	(118 pupils)	0.25**	(121 pupils)
Girls	-0.01	(100 pupils)	0.11	(105 pupils)

* significant at 5% level ** significant at 1% level

TEST 21

Table A3.17

Means and Standard Deviations of Attitude Indices from Test 21 for Various Levels of Mathematics Attainment

	SCM		ATM		TA		
Maths attainment	Mean	St. dev	Mean	St. dev	Mean	St. dev	No.
A.	70.5	13.9	28.6	15.2	39.3	19.3	54
B.	60.5	14.6	35.1	14.5	44.3	16.7	89
C.	54.2	14.9	42.5	18.5	51.5	18.3	85
D.	43.5	18.3	49.0	18.1	60.2	15.7	15

Table A3.18

Means and Standard Deviations for Attitude Indices from Test 21, Boys and Girls Separately

	SCM		ATM		TA		
	Mean	St. dev	Mean	St. dev	Mean	St. dev	No.
Boys	62.2	16.9	33.9	18.1	44.6	20.0	135
Girls	55.9	15.4	39.5	16.8	49.8	17.2	109
	**		*		*		

** difference in means significant at 1% level

* difference in means significant at 5% level

Table A3.19

Correlations Between Attitude Indices from Test 21 and Mathematical Creativity Scores (OF/DP), Boys and Girls Separately

	OF		DP	
	Boys	Girls	Boys	Girls
SCM	28**	38**	46**	39**
ATM	-32**	-29**	-40**	-20*
TA	-25**	-25**	-26**	-14
Numbers:	(124)	(97)	(126)	(101)

* significant at 5% level ** significant at 1% level

APPENDIX 4

RAW SCORES ON OVERCOMING FIXATION TESTS AND OVERALL OF SCORES

Notes

The pupils from the three Middle Schools used are numbered 1 - 102, 103 - 207 and 208 - 283, respectively.

A pupil's absence for a particulqr test is indicated by -1.

The overall OF scores are standardised to a mean of 100 and a standard deviation of 15.

Pupil	Sex	Test Number 1	14	20q6	10	24	OF Score
1	F	4	0	0	0	0	85
2	F	7	0	0	-1	0	96
3	F	7	10	0	0	-1	108
4	F	7	10	0	6	0	118
5	F	-1	-1	0	-1	0	-1
6	F	7	0	0	6	-1	101
7	F	7	0	0	-1	0	96
8	F	7	0	-1	6	0	101
9	F	-1	0	0	-1	-1	-1
10	F	10	0	0	0	0	96
11	F	4	0	0	0	0	85
12	F	4	10	0	6	0	113
13	F	7	-1	-1	0	0	-1
14	F	10	0	0	0	0	96
15	F	7	0	0	0	0	91
16	F	4	0	0	0	0	85
17	F	10	-1	0	0	0	101
18	F	7	-1	0	0	0	96
19	F	4	10	0	0	0	102
20	F	7	-1	0	0	0	96
21	F	2	0	0	0	0	82
22	F	-1	-1	0	-1	0	-1
23	F	4	0	0	0	0	85
24	F	7	10	0	0	0	91
25	F	2	0	0	0	0	82
26	F	5	0	0	0	-1	87
27	F	7	-1	0	0	0	96
28	F	7	10	0	0	0	108
29	F	10	10	10	10	0	146
30	F	4	-1	0	-1	-1	-1
31	F	4	0	-1	6	0	96
32	F	7	0	-1	6	0	101
33	F	7	10	10	0	0	108
34	F	5	-1	0	0	0	92
35	F	-1	-1	0	0	0	-1
36	F	7	-1	0	0	-1	-1
37	F	2	0	0	6	0	92
38	F	7	0	0	0	0	91
39	F	4	0	0	-1	0	91
40	F	4	0	0	0	0	85
41	F	4	10	10	0	0	119
42	F	7	-1	0	0	0	96

Pupil	Sex	Test Number................ 1	14	20qs	10	24	OF Score
43	M	10	10	0	6	0	123
44	M	-1	0	0	-1	0	-1
45	M	10	0	0	0	0	96
46	M	7	-1	0	0	0	96
47	M	4	0	0	0	0	85
48	M	4	0	0	0	0	85
49	M	2	0	0	0	0	82
50	M	5	0	0	0	0	87
51	M	4	10	0	0	-1	102
52	M	7	10	0	0	0	108
53	M	10	10	0	10	0	130
54	M	4	0	0	10	0	102
55	M	7	-1	-1	0	0	-1
56	M	7	10	10	0	0	108
57	M	-1	0	0	0	0	79
58	M	4	10	10	0	0	119
59	M	10	10	0	0	0	113
60	M	7	-1	-1	0	0	-1
61	M	7	0	0	0	0	91
62	M	7	0	0	6	0	101
63	M	10	-1	0	0	0	101
64	M	7	10	0	6	0	118
65	M	4	10	10	0	-1	119
66	M	7	10	5	10	0	133
67	M	7	-1	0	0	0	96
68	M	10	10	0	10	0	130
69	M	10	0	0	6	0	106
70	M	10	10	10	6	-1	140
71	M	7	0	0	0	0	91
72	M	7	0	0	0	0	91
73	M	4	0	0	10	0	102
74	M	4	0	0	0	0	85
75	M	10	-1	0	0	0	101
76	M	2	0	-1	6	-1	-1
77	M	7	10	0	0	0	108
78	M	-1	10	0	-1	0	-1
79	M	-1	-1	0	-1	0	-1
80	M	4	0	0	0	0	85
81	M	7	0	0	0	0	91
82	M	7	10	0	0	0	108
83	M	10	0	0	-1	0	111
84	M	10	10	0	10	0	130
85	M	10	10	-1	0	0	113
86	M	7	10	0	0	0	108
87	M	7	0	0	0	0	91
88	M	7	-1	0	0	-1	-1
89	M	10	0	0	-1	-1	-1
90	M	4	0	0	0	-1	85
91	M	10	10	0	0	0	113
92	M	7	0	0	0	0	91
93	M	8	10	0	0	-1	109
94	M	7	0	0	0	0	91
95	M	10	0	0	0	0	96
96	M	-1	0	0	-1	0	-1
97	M	10	10	0	0	0	113
98	M	4	-1	10	0	0	108
99	M	7	10	10	0	0	124
100	M	4	0	0	6	-1	96
101	M	7	10	0	0	0	108
102	M	10	10	0	6	-1	123

Pupil	Sex	Test Number................ 1	14	20q6	10	24	OF Score
103	F	7	10	0	10	-1	124
104	F	10	10	0	0	0	113
105	F	4	0	0	0	0	85
106	F	5	0	0	0	0	87
107	F	7	10	0	0	0	108
108	F	4	0	0	-1	0	91
109	F	-1	-1	-1	-1	-1	-1
110	F	10	0	0	6	0	106
111	F	4	0	0	0	0	85
112	F	4	10	0	0	0	102
113	F	10	10	10	0	0	130
114	F	-1	0	0	0	-1	-1
115	F	4	-1	0	0	0	85
116	F	4	0	0	10	0	102
117	F	7	0	0	0	0	91
118	F	4	0	0	0	0	85
119	F	2	0	-1	0	0	82
120	F	4	0	0	0	0	85
121	F	4	0	0	10	0	102
122	F	4	0	0	0	0	85
123	F	10	-1	-1	-1	0	-1
124	F	7	0	0	0	0	91
125	F	10	10	0	0	0	113
126	F	4	0	-1	0	-1	-1
127	F	-1	0	0	0	-1	-1
128	F	-1	10	0	0	0	101
129	F	3	0	0	0	0	84
130	F	7	0	0	0	0	91
131	F	7	10	-1	0	0	108
132	F	4	0	0	0	0	85
133	F	7	-1	0	0	0	96
134	F	7	-1	0	-1	0	-1
135	F	2	0	-1	0	0	82
136	F	2	-1	-1	-1	0	-1
137	F	10	10	10	0	-1	130
138	F	2	0	0	0	0	82
139	F	4	10	10	0	0	119
140	F	10	10	0	0	0	113
141	F	-1	10	0	10	0	121
142	F	4	0	0	0	0	85
143	F	10	10	0	0	0	113
144	F	10	0	0	0	0	96
145	F	4	0	0	0	0	85
146	F	4	0	-1	0	0	85
147	F	-1	0	0	0	0	79
148	F	7	-1	0	0	0	96
149	F	10	10	0	0	0	113
150	F	2	0	0	6	0	92
151	F	-1	0	-1	0	-1	-1
152	F	10	0	10	0	0	113
153	F	-1	0	-1	-1	-1	-1
154	F	4	10	-1	0	0	102
155	F	-1	0	0	-1	0	-1
156	F	2	-1	-1	0	0	-1
157	F	2	0	-1	0	-1	-1
158	F	10	10	0	0	0	113

Pupil	Sex	Test Number... 1	14	2006	10	24	OF Score
159	M	7	10	10	0	0	124
160	M	10	0	0	10	0	113
161	M	10	10	0	0	0	113
162	M	-1	10	0	-1	0	-1
163	M	7	-1	0	-1	0	-1
164	M	6	0	0	0	0	89
165	M	4	10	0	0	0	102
166	M	7	0	-1	0	-1	-1
167	M	10	10	0	10	0	113
168	M	4	0	0	0	-1	85
169	M	7	10	0	0	0	108
170	M	10	0	0	0	0	96
171	M	4	0	0	0	0	85
172	M	10	0	0	0	0	96
173	M	7	10	0	10	0	124
174	M	10	10	0	6	10	140
175	M	4	0	0	0	0	85
176	M	10	0	0	0	0	96
177	M	10	0	0	10	0	113
178	M	7	-1	10	0	0	113
179	M	-1	10	0	10	0	121
180	M	7	10	0	0	0	108
181	M	7	10	0	10	0	124
182	M	7	0	0	0	0	91
183	M	2	0	0	0	0	82
184	M	-1	10	0	-1	0	-1
185	M	4	-1	0	0	0	85
186	M	2	0	0	6	0	92
187	M	-1	-1	-1	-1	0	-1
188	M	10	10	0	0	-1	113
189	M	7	10	0	0	0	108
190	M	4	-1	-1	0	0	-1
191	M	6	10	0	0	0	106
192	M	0	0	0	6	0	89
193	M	4	0	0	0	0	85
194	M	4	0	0	0	0	85
195	M	4	0	-1	0	-1	-1
196	M	10	0	0	0	0	96
197	M	-1	-1	-1	-1	-1	-1
198	M	10	10	10	10	10	163
199	M	10	10	10	10	0	146
200	M	4	0	0	-1	0	85
201	M	10	0	0	0	0	96
202	M	10	0	0	6	0	106
203	M	4	0	0	10	0	102
204	M	10	10	10	10	0	130
205	M	10	0	0	-1	0	101
206	M	7	10	0	0	0	108
207	M	7	10	0	0	0	108

Pupil	Sex	Test Number................ 1	14	21q6	10	24	OF Score
208	F	4	10	0	0	0	102
209	F	4	0	0	0	0	85
210	F	10	-1	-1	0	-1	-1
211	F	7	0	0	0	0	91
212	F	7	10	0	0	0	108
213	F	10	10	-1	0	0	113
214	F	5	-1	0	0	-1	-1
215	F	7	0	0	0	0	91
216	F	4	0	0	0	0	85
217	F	4	0	0	0	0	85
218	F	10	-1	0	0	0	101
219	F	-1	10	0	0	0	101
220	F	7	0	0	0	0	91
221	F	4	10	0	0	0	102
222	F	10	0	0	10	0	113
223	F	7	10	0	0	0	108
224	F	7	0	0	0	0	91
225	F	4	0	0	0	0	85
226	F	1	0	0	0	0	80
227	F	4	10	0	0	0	102
228	F	4	0	0	6	0	96
229	F	10	-1	0	6	0	111
230	F	7	10	10	0	-1	124
231	F	7	0	0	0	0	91
232	F	10	10	10	0	-1	130
233	F	5	10	10	0	0	121
234	F	4	0	0	0	0	85
235	F	4	0	0	-1	0	85
236	F	7	10	0	0	0	108
237	F	4	0	0	0	0	85
238	F	2	0	0	0	0	82
239	F	10	-1	0	0	0	101

Pupil	Sex	Test Number 1	14	20q6	10	24	OF Score
240	M	5	10	0	-1	0	109
241	M	10	10	0	10	0	130
242	M	2	0	0	0	0	82
243	M	4	0	0	0	0	85
244	M	7	0	0	6	0	101
245	M	4	0	0	0	0	85
246	M	7	0	0	-1	0	96
247	M	4	0	0	0	0	85
248	M	4	0	0	-1	0	85
249	M	4	-1	-1	0	0	-1
250	M	-1	0	0	0	0	80
251	M	4	0	0	0	0	85
252	M	4	0	0	0	0	85
253	M	10	0	0	10	0	113
254	M	2	0	0	0	0	82
255	M	2	0	-1	0	0	82
256	M	10	10	0	6	0	123
257	M	2	0	0	0	0	82
258	M	4	-1	0	0	0	85
259	M	4	10	0	6	0	113
260	M	7	0	0	0	0	108
261	M	4	0	0	0	-1	85
262	M	10	0	0	0	0	96
263	M	8	0	0	0	0	92
264	M	5	10	0	10	0	121
265	M	10	10	0	0	-1	113
266	M	4	-1	-1	10	0	-1
267	M	2	0	0	0	0	82
268	M	2	0	0	0	0	82
269	M	6	0	0	0	0	89
270	M	7	0	0	0	0	91
271	M	2	10	0	0	0	99
272	M	-1	10	0	-1	0	-1
273	M	4	10	0	0	-1	102
274	M	10	10	0	0	0	113
275	M	2	10	0	10	0	116
276	M	4	0	0	0	0	85
277	M	4	0	0	0	0	85
278	M	7	0	0	0	0	91
279	M	4	0	0	0	0	85
280	M	7	0	0	-1	0	96
281	M	2	0	0	0	0	82
282	M	10	0	0	0	0	96
283	M	7	0	0	0	0	91

APPENDIX 5

RAW SCORES ON DIVERGENT PRODUCTION TESTS AND OVERALL DP SCORES

Notes

The pupils from the three Middle Schools used are numbered 1 - 102, 103 - 207 and 208 - 283, respectively.

A pupil's absence for a particular test is indicated by -1.

The overall DP scores are standardised to a mean of 100 and a standard deviation of 15.

Pupil	Sex	Test Number: 2	3	4	5a	5b	7	11	12	16	18	DP Score
1	F	10	8	11	9	11	7	4	3	2	11	100
2	F	12	6	18	9	8	0	-1	4	10	10	103
3	F	10	7	11	5	6	5	6	4	7	11	96
4	F	5	13	12	5	7	7	11	6	8	11	102
5	F	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
6	F	17	8	12	5	10	8	15	4	8	7	107
7	F	7	1	11	6	7	8	-1	4	0	3	89
8	F	10	3	21	4	4	15	8	4	0	11	99
9	F	-1	-1	-1	-1	-1	-1	-1	3	4	2	-1
10	F	16	7	27	9	6	5	15	7	10	10	113
11	F	18	7	9	11	7	7	6	14	13	13	112
12	F	13	8	15	7	12	5	9	10	2	11	106
13	F	10	0	16	6	7	7	9	-1	-1	-1	98
14	F	30	16	28	16	17	7	15	14	11	12	139
15	F	11	7	8	2	6	4	5	4	8	8	92
16	F	10	0	18	3	6	9	4	4	0	4	90
17	F	16	7	17	17	9	7	8	-1	27	15	128
18	F	15	11	12	4	7	7	7	-1	-1	-1	102
19	F	17	32	12	20	18	8	7	4	13	19	134
20	F	27	10	17	5	8	26	5	-1	13	18	128
21	F	10	1	14	6	9	7	4	4	0	3	92
22	F	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
23	F	13	1	18	5	6	8	5	4	0	8	95
24	F	10	14	13	5	9	5	20	3	6	12	107
25	F	14	6	20	3	5	3	4	6	1	8	94
26	F	14	2	11	13	13	3	8	4	0	2	99
27	F	10	9	13	4	7	5	9	-1	12	13	104
28	F	6	4	6	4	4	5	12	3	2	12	90
29	F	24	10	18	3	3	9	10	16	-1	-1	112
30	F	11	0	20	7	9	-1	-1	-1	0	1	95
31	F	9	8	12	7	10	7	4	4	6	11	100
32	F	27	6	10	16	14	7	12	7	-1	-1	123
33	F	14	13	11	6	7	7	13	4	8	12	107
34	F	13	7	8	6	4	6	4	-1	2	5	91
35	F	-1	-1	-1	-1	-1	4	3	-1	0	1	-1
36	F	18	13	15	5	8	8	9	-1	5	12	109
37	F	8	1	8	7	6	3	6	4	0	0	85
38	F	24	0	15	11	11	7	9	7	5	5	108
39	F	16	15	13	10	9	5	-1	4	0	10	105
40	F	16	3	11	3	5	8	8	8	12	8	101
41	F	10	14	21	5	7	8	9	14	11	3	109
42	F	2	7	14	3	7	5	4	-1	-1	-1	89

Pupil	Sex	Test Number....... 2	3	4	5a	5b	7	11	12	16	18	DF Score
43	M	12	12	9	5	7	5	11	6	12	31	112
44	M	-1	-1	-1	-1	-1	-1	-1	4	-1	-1	-1
45	M	12	8	16	8	14	12	14	4	7	18	116
46	M	14	3	8	7	7	8	7	-1	2	6	96
47	M	15	7	5	0	8	10	4	6	8	9	97
48	M	8	4	11	4	6	6	5	7	0	3	89
49	M	12	1	7	4	3	5	7	4	0	6	86
50	M	12	1	12	5	6	4	10	5	1	4	91
51	M	4	8	12	11	-1	7	21	3	11	5	107
52	M	12	16	23	9	15	10	19	7	15	12	127
53	M	9	30	22	18	9	17	15	26	15	26	148
54	M	15	23	10	14	11	7	19	21	9	15	129
55	M	11	5	13	2	4	4	12	-1	2	6	92
56	M	20	16	13	14	14	6	23	8	11	20	130
57	M	-1	-1	-1	-1	-1	-1	4	5	2	6	-1
58	M	10	7	14	5	5	7	10	4	7	9	99
59	M	8	17	14	12	10	7	12	8	8	23	118
60	M	8	6	11	5	10	-1	2	-1	-1	-1	94
61	M	18	9	10	3	5	9	8	4	1	5	96
62	M	5	6	11	8	4	3	9	4	0	5	89
63	M	6	13	11	13	6	5	11	-1	-1	-1	106
64	M	11	11	12	10	9	4	12	7	8	2	104
65	M	12	7	13	5	5	3	6	4	7	7	95
66	M	20	21	16	11	7	10	29	15	12	16	134
67	M	9	7	7	9	5	3	4	-1	2	6	90
68	M	8	9	20	6	8	13	8	4	0	10	103
69	M	15	6	12	6	12	3	15	5	7	8	105
70	M	5	15	18	7	12	7	21	12	9	6	115
71	M	5	10	11	7	9	7	4	4	2	9	96
72	M	16	15	11	7	9	6	10	4	4	8	105
73	M	1	7	16	7	9	9	13	17	5	11	108
74	M	9	4	11	5	3	2	5	4	4	10	89
75	M	11	1	31	2	3	-1	15	-1	-1	-1	104
76	M	12	1	14	6	9	3	9	2	1	5	93
77	M	7	0	12	5	4	5	10	3	8	11	94
78	M	-1	-1	-1	-1	-1	-1	-1	10	7	25	-1
79	M	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
80	M	11	1	9	4	3	7	8	4	0	9	89
81	M	4	4	9	6	6	7	5	3	0	4	87
82	M	3	1	8	3	8	6	13	3	10	3	92
83	M	13	1	-1	-1	-1	-1	-1	4	0	5	86
84	M	12	12	14	12	8	12	8	30	15	29	132
85	M	20	5	37	12	7	11	5	3	8	10	117
86	M	13	12	14	11	14	3	15	6	1	7	109
87	M	16	7	8	8	11	8	11	-1	-1	-1	109
88	M	2	0	9	3	4	11	4	7	0	4	85
89	M	10	4	6	3	3	5	-1	3	8	5	88
90	M	4	5	10	2	6	-1	5	3	5	4	86
91	M	15	18	15	8	12	9	8	12	11	7	117
92	M	10	6	7	9	12	3	4	4	8	9	99
93	M	8	0	11	6	7	6	12	4	6	8	96
94	M	15	0	12	7	9	3	3	4	3	9	94
95	M	13	1	12	5	5	7	7	3	5	6	93
96	M	-1	-1	-1	-1	-1	-1	-1	4	3	6	-1
97	M	29	19	12	15	16	8	19	20	16	18	143
98	M	19	12	10	5	8	4	5	-1	-1	-1	101
99	M	26	14	14	12	9	6	11	5	13	24	124
100	M	12	9	15	7	12	7	13	9	7	11	111
101	M	10	8	8	14	8	9	11	3	10	23	113
102	M	27	12	10	5	2	5	19	17	8	19	117

Pupil	Sex	Test Number........ 2	3	4	5a	5b	7	11	12	16	18	DP Score
103	F	10	9	9	7	10	-1	10	22	11	2	110
104	F	13	9	5	7	8	6	6	11	1	4	97
105	F	9	1	3	4	3	4	5	3	7	1	83
106	F	6	7	5	4	3	5	5	3	5	2	85
107	F	12	15	14	4	11	8	12	13	11	10	113
108	F	11	6	7	7	2	5	-1	4	2	2	89
109	F	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
110	F	7	6	6	4	5	7	14	11	4	9	97
111	F	17	3	3	3	2	3	3	4	2	2	83
112	F	7	5	1	5	6	7	10	7	9	1	92
113	F	21	9	9	4	11	13	8	6	3	12	108
114	F	-1	-1	-1	-1	-1	-1	5	11	-1	-1	-1
115	F	2	7	3	1	2	4	1	-1	7	0	77
116	F	4	1	4	3	0	3	4	3	4	0	76
117	F	9	6	8	5	13	6	7	7	8	9	101
118	F	8	7	3	4	5	2	9	3	8	1	88
119	F	10	1	8	5	6	8	4	0	8	12	93
120	F	7	7	3	4	4	3	5	4	1	0	82
121	F	8	5	3	6	4	3	5	3	6	0	85
122	F	13	6	8	5	5	4	3	4	-1	-1	90
123	F	6	6	14	6	8	-1	-1	-1	-1	-1	98
124	F	11	7	6	5	11	8	13	5	9	15	106
125	F	8	7	0	6	6	8	8	10	7	2	94
126	F	4	3	6	1	1	-1	4	3	-1	-1	76
127	F	-1	-1	-1	-1	-1	7	8	3	4	9	96
128	F	-1	-1	-1	-1	-1	3	4	3	9	10	94
129	F	7	3	2	4	6	5	9	4	0	1	84
130	F	8	4	5	4	6	8	5	2	5	7	90
131	F	8	9	9	5	5	6	13	4	-1	-1	96
132	F	7	4	6	4	4	3	8	3	2	2	84
133	F	8	0	10	3	4	4	5	-1	0	7	84
134	F	7	12	8	3	4	-1	-1	-1	4	9	92
135	F	11	4	7	10	4	5	3	4	-1	-1	92
136	F	13	6	4	3	1	-1	-1	-1	-1	-1	83
137	F	18	14	14	7	9	13	4	3	18	23	119
138	F	9	7	7	5	3	4	4	3	2	0	84
139	F	12	7	6	6	8	6	12	4	8	13	102
140	F	12	12	8	6	9	8	8	12	4	14	106
141	F	-1	-1	-1	-1	-1	3	4	8	-1	-1	-1
142	F	8	1	7	2	0	4	3	3	-1	-1	78
143	F	18	7	4	6	5	6	13	9	13	7	104
144	F	9	9	4	5	4	2	13	8	1	5	91
145	F	5	1	7	10	4	5	0	4	4	0	85
146	F	9	0	4	4	3	5	5	3	-1	-1	83
147	F	-1	-1	-1	-1	-1	1	7	4	3	2	83
148	F	6	4	6	4	7	7	8	-1	2	1	87
149	F	15	10	11	8	3	8	6	11	5	6	101
150	F	7	1	3	3	3	8	8	5	9	2	88
151	F	-1	-1	-1	-1	-1	4	7	4	-1	-1	-1
152	F	12	10	11	5	7	7	6	8	9	8	102
153	F	-1	-1	-1	-1	-1	-1	-1	3	-1	-1	-1
154	F	15	9	7	5	4	4	6	9	-1	-1	96
155	F	-1	-1	-1	-1	-1	-1	-1	10	3	12	-1
156	F	12	1	7	6	2	4	3	-1	-1	-1	85
157	F	6	0	3	9	1	5	3	3	-1	-1	82
158	F	13	9	5	6	9	9	10	21	10	3	108

Pupil	Se	Test Number... 2	3	4	5a	5b	7	11	12	16	18	DF Score
159	M	13	7	8	5	6	5	13	0	8	18	101
160	M	3	5	5	5	2	7	8	4	5	7	88
161	M	1	5	5	4	4	9	5	2	8	7	88
162	M	-1	-1	-1	-1	-1	-1	-1	4	5	4	-1
163	M	4	8	11	3	3	-1	-1	9	7	6	92
164	M	4	0	3	4	0	3	4	3	1	0	75
165	M	9	0	6	7	3	-1	7	12	2	6	90
166	M	8	1	10	0	2	8	5	3	-1	-1	83
167	M	11	8	7	10	9	8	11	10	9	0	104
168	M	12	7	5	3	3	7	7	3	0	3	87
169	M	10	14	19	9	10	10	1	4	13	7	109
170	M	6	4	3	7	6	9	7	3	4	2	90
171	M	7	0	3	3	0	9	4	3	2	5	81
172	M	2	7	9	4	9	7	6	12	4	17	99
173	M	28	12	8	7	9	7	10	23	9	9	119
174	M	20	25	17	10	12	18	24	30	22	15	151
175	M	7	1	8	1	0	4	6	4	5	0	80
176	M	10	1	11	3	4	9	5	4	2	0	87
177	M	7	0	6	6	5	5	5	13	4	3	90
178	M	19	7	9	2	11	8	8	-1	8	8	104
179	M	-1	-1	-1	-1	-1	5	11	17	10	4	110
180	M	10	6	8	6	0	5	3	3	1	0	83
181	M	14	14	11	2	7	11	7	4	6	6	101
182	M	9	3	4	3	2	6	6	4	4	0	83
183	M	7	0	4	4	0	5	7	3	3	1	80
184	M	-1	-1	-1	-1	-1	-1	-1	3	2	5	-1
185	M	3	10	6	3	4	5	11	-1	4	0	87
186	M	9	0	0	5	0	7	4	4	0	4	80
187	M	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
188	M	5	6	3	13	6	8	19	4	3	0	98
189	M	8	4	4	2	4	1	2	3	7	1	81
190	M	8	7	6	3	3	5	11	-1	-1	-1	89
191	M	13	8	11	7	5	7	17	4	13	9	107
192	M	5	4	5	5	0	3	5	8	3	0	81
193	M	8	1	3	5	4	3	6	4	3	0	82
194	M	11	7	3	4	6	2	14	3	9	10	95
195	M	2	0	3	3	2	3	1	2	-1	-1	73
196	M	13	7	9	5	4	5	10	9	4	17	100
197	M	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1
198	M	24	17	28	10	17	39	32	29	17	4	165
199	M	35	15	44	11	15	35	27	34	16	9	172
200	M	12	0	-1	-1	-1	5	-1	3	3	3	86
201	M	8	7	3	7	4	5	6	3	6	2	89
202	M	14	15	15	4	3	9	11	8	3	10	104
203	M	8	9	5	6	8	8	9	8	3	1	95
204	M	13	18	14	14	8	11	21	4	4	7	117
205	M	7	0	3	1	1	-1	-1	3	-1	-1	74
206	M	15	11	14	4	8	12	10	10	9	9	110
207	M	7	1	3	6	1	7	6	7	3	2	85

Pupil	Sex	Test Number.. 2	3	4	5a	5b	7	11	12	16	18	DP Score
208	F	10	11	15	7	3	7	7	3	5	9	98
209	F	14	9	12	6	5	5	3	9	7	13	101
210	F	13	18	38	5	7	7	12	-1	-1	-1	121
211	F	7	15	8	5	5	4	9	4	7	10	97
212	F	15	16	17	7	8	6	4	4	8	13	106
213	F	15	20	12	6	8	6	9	7	9	2	106
214	F	25	11	13	8	7	7	9	-1	-1	9	112
215	F	8	7	12	9	9	1	0	4	6	6	93
216	F	22	18	9	6	7	7	10	8	7	6	108
217	F	13	10	6	9	-1	5	9	4	10	3	100
218	F	24	15	19	14	11	6	12	-1	-1	13	127
219	F	-1	13	10	5	8	9	6	4	17	5	103
220	F	9	10	35	6	7	8	9	8	9	8	111
221	F	12	18	8	6	2	6	11	-1	7	12	103
222	F	8	13	14	3	6	9	13	5	10	4	102
223	F	13	9	14	6	9	10	5	4	9	9	105
224	F	14	18	15	7	11	7	11	9	8	12	114
225	F	18	9	10	5	3	4	10	4	3	8	96
226	F	8	12	5	6	5	5	8	3	1	5	91
227	F	17	17	10	8	8	7	5	6	10	9	106
228	F	11	7	5	7	5	4	6	8	2	4	91
229	F	10	16	8	4	7	7	8	-1	-1	8	100
230	F	10	14	8	8	4	3	7	4	6	4	95
231	F	9	15	6	10	3	3	8	4	7	7	97
232	F	26	23	31	15	8	17	19	15	4	11	139
233	F	18	22	15	16	5	5	8	3	4	12	112
234	F	7	13	8	5	8	8	7	4	7	7	98
235	F	5	-1	-1	-1	-1	-1	-1	4	9	1	-1
236	F	16	12	16	8	8	8	6	13	18	17	119
237	F	17	13	8	10	10	8	15	8	10	4	111
238	F	6	8	5	5	7	7	2	5	5	10	92
239	F	15	19	8	8	9	8	5	-1	-1	13	110

Pupil	Sex	Test Number........ 2	3	4	5a	5b	7	11	12	16	18	DF Score
240	M	9	11	15	5	6	-1	-1	4	2	11	98
241	M	9	25	19	9	9	6	6	16	6	19	118
242	M	12	12	8	4	10	5	12	3	6	4	99
243	M	11	6	5	6	5	3	10	3	6	7	93
244	M	11	15	6	10	10	11	14	4	8	11	111
245	M	11	1	6	4	4	3	3	4	2	5	84
246	M	10	7	6	6	8	-1	-1	3	6	9	96
247	M	13	15	13	10	6	6	4	4	4	8	101
248	M	13	15	12	11	5	-1	-1	9	8	11	111
249	M	2	6	6	5	5	4	0	-1	-1	-1	81
250	M	-1	-1	-1	-1	-1	5	18	3	12	8	110
251	M	11	0	9	4	6	6	6	3	8	10	93
252	M	15	15	16	8	7	10	6	15	10	7	113
253	M	6	20	13	6	8	13	8	13	20	16	120
254	M	10	4	3	1	1	2	1	3	0	0	75
255	M	10	8	10	6	7	8	0	3	2	-1	92
256	M	14	14	19	4	5	6	8	4	11	12	106
257	M	13	8	10	4	6	7	5	4	2	6	93
258	M	8	7	6	6	7	2	4	-1	-1	9	91
259	M	16	18	12	15	9	6	8	8	6	16	116
260	M	8	5	7	12	7	4	4	3	6	23	101
261	M	10	1	12	7	11	3	9	4	8	8	99
262	M	24	20	11	7	6	7	11	4	4	10	109
263	M	6	23	8	11	11	7	10	3	2	16	108
264	M	6	11	8	6	4	5	9	3	7	16	97
265	M	13	20	13	15	9	9	16	4	6	15	119
266	M	9	2	9	2	8	6	6	-1	-1	-1	90
267	M	4	16	10	4	6	10	5	5	8	15	101
268	M	12	7	7	4	3	5	2	3	6	-1	88
269	M	12	0	9	5	5	3	4	5	8	11	92
270	M	10	1	10	6	3	2	7	3	11	12	93
271	M	15	6	8	6	9	4	3	4	0	9	94
272	M	-1	-1	-1	-1	-1	-1	-1	3	5	12	-1
273	M	16	6	9	7	10	6	8	4	2	15	102
274	M	7	11	15	9	10	8	13	4	8	21	112
275	M	18	0	13	6	4	6	8	4	6	13	97
276	M	6	15	15	6	11	6	5	3	5	8	101
277	M	9	7	11	13	10	6	5	3	0	8	99
278	M	8	18	6	12	12	8	9	7	10	11	112
279	M	11	7	12	11	7	5	4	3	5	7	98
280	M	15	-1	-1	-1	-1	-1	-1	4	12	18	-1
281	M	6	9	8	1	3	2	5	3	2	1	81
292	M	10	8	20	7	5	7	5	14	10	16	109
283	M	15	5	13	5	6	14	2	4	10	12	103

APPENDIX 6

SCORES ON ALL MEASURES USED IN THE INVESTIGATION IN CHAPTERS 6 & 7

<u>Key</u>

OF: overall score on overcoming fixation tests

DP: overall score on divergent production tests

MA: mathematics attainment score from Test 0 (NFER EF Test)

RT23: risk-taking index from Test 23

RT22: risk-taking index from Test 22

RT27: risk-taking index from Test 27

NC: index of nonconformity from Test 26

CW: measure of category width from Test 25

SCM: measure of self-concept in mathematics from Test 21

ATM: measure of anxiety towards mathematics from Test 21

TA: measure of test anxiety from Test 21

P-S: problem-solving score from Test 20

<u>Note</u>

All scores (except MA which is standardised according to national norms) have been standardised to a mean of 100 and a standard deviation of 15.

Pupil	Sex	OF	DF	MA	RT23	RT22	RT27	NC	CW	SCM	ATM	TA	P-S
1	F	85	100	115	112	98	104	113	96	103	90	81	95
2	F	96	103	124	72	-1	-1	113	85	92	108	117	114
3	F	108	96	119	-1	-1	-1	113	-1	97	97	91	124
4	F	118	102	131	112	107	84	113	104	112	94	97	120
5	F	-1	-1	111	112	120	100	113	93	106	90	81	99
6	F	101	107	126	-1	-1	-1	-1	-1	100	101	101	114
7	F	96	89	112	94	84	88	102	112	92	112	91	99
8	F	101	99	108	86	107	112	85	112	103	101	102	-1
9	F	-1	-1	119	-1	-1	-1	113	-1	97	118	122	103
10	F	96	113	140	99	107	84	113	101	112	94	95	103
11	F	85	112	122	112	102	80	113	92	94	112	113	101
12	F	113	106	114	112	116	96	113	76	109	94	79	103
13	F	-1	98	-1	86	-1	-1	87	115	-1	-1	-1	-1
14	F	96	139	128	112	-1	-1	113	92	100	105	102	109
15	F	91	92	113	99	-1	-1	102	96	94	105	110	83
16	F	85	90	116	72	-1	88	113	72	-1	-1	-1	83
17	F	101	128	129	112	107	108	108	95	97	112	129	99
18	F	96	102	115	112	120	96	113	89	109	97	102	85
19	F	102	134	129	112	107	-1	113	119	103	101	97	126
20	F	96	128	140	112	107	84	108	92	120	87	101	128
21	F	82	92	105	90	89	100	81	101	103	97	91	83
22	F	-1	-1	110	72	98	88	113	66	103	105	102	89
23	F	85	95	108	86	98	88	97	76	103	105	97	91
24	F	91	107	128	112	-1	-1	113	70	131	84	93	126
25	F	82	94	79	99	89	104	85	-1	97	123	125	83
26	F	87	99	106	-1	89	100	-1	-1	72	123	117	85
27	F	96	104	124	112	-1	-1	102	89	100	94	97	107
28	F	108	90	123	99	-1	-1	97	62	92	112	77	99
29	F	146	112	137	112	111	80	113	102	100	101	109	122
30	F	-1	95	103	-1	107	-1	113	-1	100	112	102	83
31	F	96	100	115	112	-1	108	113	82	-1	-1	-1	-1
32	F	101	123	125	81	-1	88	113	98	-1	-1	-1	-1
33	F	108	107	-1	112	-1	-1	85	101	-1	-1	-1	134
34	F	92	91	107	99	98	96	101	102	94	126	126	95
35	F	-1	-1	-1	76	84	101	-1	92	69	136	142	85
36	F	-1	109	116	112	120	96	113	88	101	101	99	95
37	F	92	85	98	54	89	119	-1	81	83	126	121	85
38	F	91	118	112	99	-1	-1	84	89	103	87	95	111
39	F	91	115	119	112	120	96	113	72	89	118	109	91
40	F	85	101	117	99	-1	-1	108	95	89	105	102	87
41	F	119	109	140	54	107	84	113	109	94	112	106	120
42	F	96	99	111	99	93	-1	85	118	103	80	67	93

Pupil	Sex	OF	DP	MA	RT23	RT22	RT27	NC	CW	SCM	ATM	TA	P-S
43	M	123	112	135	81	107	100	113	108	106	101	93	116
44	M	-1	-1	106	76	93	143	113	78	109	90	99	91
45	M	96	116	112	86	-1	-1	99	-1	97	90	89	83
46	M	96	96	104	112	70	112	113	104	100	94	85	95
47	M	85	97	112	103	84	119	113	99	114	76	85	101
48	M	85	89	112	-1	84	92	113	-1	112	90	79	93
49	M	82	86	116	112	93	131	104	105	122	97	93	89
50	M	87	91	102	54	-1	112	84	96	112	118	125	89
51	M	102	107	122	-1	-1	-1	100	-1	114	108	99	97
52	M	108	127	140	81	80	84	69	116	114	87	71	112
53	M	130	148	140	112	-1	88	113	121	117	80	87	126
54	M	102	129	140	72	116	96	102	109	126	76	69	124
55	M	-1	92	125	76	-1	-1	113	-1	-1	-1	-1	-1
56	M	108	130	140	90	120	96	113	114	117	76	83	122
57	M	79	-1	112	112	80	96	113	91	103	123	125	81
58	M	119	99	128	99	-1	-1	113	91	106	94	83	134
59	M	113	118	140	112	120	96	113	109	131	84	97	120
60	M	-1	94	107	112	70	119	113	101	112	118	121	-1
61	M	91	96	104	76	-1	108	85	102	-1	-1	-1	95
62	M	101	89	112	67	84	100	100	98	77	129	113	99
63	M	101	106	133	90	107	100	103	105	109	97	101	118
64	M	118	104	127	112	107	84	99	108	100	90	75	114
65	M	119	95	111	-1	-1	-1	106	-1	103	112	102	134
66	M	133	134	137	112	116	100	97	89	100	87	89	89
67	M	96	90	114	76	80	96	113	73	103	105	118	79
68	M	130	103	132	112	116	96	113	108	92	105	126	97
69	M	106	105	119	76	-1	-1	104	79	74	126	106	95
70	M	140	115	125	-1	-1	-1	-1	-1	94	94	93	124
71	M	91	96	121	112	70	116	113	114	92	94	101	93
72	M	91	105	117	112	-1	104	113	109	-1	-1	-1	91
73	M	102	108	132	94	111	96	113	114	126	101	91	112
74	M	85	89	115	112	102	104	113	106	103	94	99	91
75	M	101	104	111	94	80	112	76	106	97	123	126	93
76	M	-1	93	100	-1	-1	-1	-1	-1	-1	-1	-1	-1
77	M	108	94	120	112	-1	-1	102	101	100	80	91	126
78	M	-1	-1	129	112	-1	100	113	101	112	90	83	112
79	M	-1	-1	112	94	120	104	113	65	109	90	105	99
80	M	85	89	113	112	84	92	102	-1	131	97	118	91
81	M	91	87	104	45	84	92	65	93	103	108	95	93
82	M	108	92	121	112	-1	-1	99	95	94	80	95	107
83	M	101	86	101	86	-1	100	94	89	-1	-1	-1	93
84	M	130	132	140	112	120	100	113	112	97	94	105	118
85	M	113	117	129	72	-1	-1	103	92	-1	-1	-1	-1
86	M	108	109	136	112	120	108	113	-1	86	129	132	99
87	M	91	109	121	86	-1	-1	97	-1	97	94	99	85
88	M	-1	85	100	-1	84	-1	-1	106	100	90	113	89
89	M	-1	88	113	-1	98	-1	113	-1	100	105	105	91
90	M	85	86	103	-1	102	-1	-1	-1	89	112	114	81
91	M	113	117	134	112	-1	-1	107	102	114	87	81	112
92	M	91	99	115	103	120	123	113	91	114	97	95	95
93	M	109	96	130	-1	-1	-1	88	-1	114	80	85	89
94	M	91	94	119	112	80	112	113	93	100	105	95	85
95	M	96	93	118	112	107	80	113	82	122	87	79	97
96	M	-1	-1	108	108	84	96	113	80	92	112	99	107
97	M	113	143	136	112	120	92	113	79	131	76	83	112
98	M	108	101	124	103	-1	-1	113	101	94	108	91	134
99	M	124	124	133	112	111	100	94	128	114	101	105	128
100	M	96	111	124	112	102	80	113	99	109	87	114	95
101	M	108	113	131	81	89	84	113	104	106	101	134	124
102	M	123	117	140	-1	107	92	-1	-1	112	101	71	128

Pupil	Sex	OF	DP	MA	RT23	RT22	RT27	NC	CW	SCM	ATM	TA	P-S
103	F	124	110	130	112	-1	131	-1	-1	-1	-1	-1	116
104	F	113	97	129	112	102	96	94	82	97	101	106	89
105	F	85	83	99	112	84	112	99	96	83	94	102	85
106	F	87	85	99	103	98	-1	95	104	94	94	121	109
107	F	108	113	140	112	-1	96	108	106	94	105	102	112
108	F	91	88	103	76	98	119	113	93	94	97	97	89
109	F	-1	-1	109	-1	-1	-1	-1	-1	-1	-1	-1	-1
110	F	106	97	131	112	-1	-1	113	101	-1	-1	-1	111
111	F	85	83	113	112	98	-1	113	96	97	94	118	97
112	F	102	92	130	112	102	104	106	85	103	87	77	112
113	F	130	108	127	108	120	-1	113	99	106	108	118	124
114	F	-1	-1	111	112	93	104	-1	-1	94	101	110	99
115	F	85	77	101	67	93	-1	89	75	89	133	114	83
116	F	102	76	99	112	84	123	73	95	77	123	109	91
117	F	91	101	128	112	120	-1	113	122	77	118	109	81
118	F	85	88	121	112	120	-1	103	140	92	118	109	91
119	F	82	93	109	112	-1	108	113	88	-1	-1	-1	-1
120	F	85	82	99	90	98	-1	87	79	92	118	113	81
121	F	102	85	109	59	93	-1	61	101	94	105	114	81
122	F	85	90	104	112	98	-1	63	98	77	112	117	85
123	F	-1	98	120	112	-1	84	106	95	94	94	87	-1
124	F	91	106	130	112	120	-1	107	116	103	108	97	101
125	F	113	94	130	112	120	119	108	105	109	94	101	112
126	F	-1	76	109	-1	-1	119	-1	-1	-1	-1	-1	-1
127	F	-1	96	109	112	120	100	87	-1	117	84	102	99
128	F	101	94	117	112	84	92	85	92	106	84	102	101
129	F	84	84	99	76	120	-1	104	125	49	133	117	81
130	F	91	90	110	99	93	-1	49	95	83	94	114	105
131	F	108	96	123	99	116	96	98	101	92	87	53	-1
132	F	85	84	108	76	98	116	97	96	103	94	109	87
133	F	96	84	96	112	89	-1	99	93	97	97	118	99
134	F	-1	92	112	112	93	112	113	91	66	123	109	87
135	F	82	92	108	-1	-1	135	-1	101	-1	-1	-1	-1
136	F	-1	83	94	-1	-1	119	-1	85	-1	-1	-1	-1
137	F	130	119	140	108	120	-1	113	-1	120	87	95	134
138	F	82	84	109	94	93	135	113	82	66	126	110	81
139	F	119	102	132	99	93	-1	101	92	117	84	87	112
140	F	113	106	140	94	120	-1	100	67	129	73	69	111
141	F	121	-1	118	-1	93	100	-1	80	77	108	81	79
142	F	85	78	108	112	-1	-1	113	82	-1	-1	-1	81
143	F	113	104	129	112	120	119	113	75	106	94	97	103
144	F	96	91	121	112	93	-1	94	115	117	90	91	105
145	F	85	85	97	94	93	135	113	85	74	108	110	85
146	F	85	83	105	-1	-1	100	-1	89	-1	-1	-1	-1
147	F	79	83	106	99	93	123	92	138.	83	97	105	85
148	F	96	87	126	112	-1	119	113	79	-1	-1	-1	91
149	F	113	101	125	112	120	112	81	86	109	90	105	120
150	F	92	88	110	112	-1	100	113	102	92	108	101	85
151	F	-1	-1	108	-1	-1	96	-1	-1	-1	-1	-1	-1
152	F	113	102	132	99	93	-1	105	122	112	76	79	105
153	F	-1	-1	99	112	-1	-1	113	-1	-1	-1	-1	-1
154	F	102	96	111	-1	-1	135	-1	88	-1	-1	-1	-1
155	F	-1	-1	123	112	116	112	107	86	117	84	77	109
156	F	-1	85	114	-1	-1	135	-1	102	-1	-1	-1	-1
157	F	-1	82	116	-1	-1	-1	-1	89	-1	-1	-1	-1
158	F	113	108	140	112	111	92	113	-1	92	108	109	107

Pupil	Sex	OF	DP	MA	RT23	RT22	RT27	NC	CW	SCM	ATM	TA	P-S
159	M	124	101	126	86	120	-1	113	109	109	84	113	122
160	M	113	88	115	112	116	92	113	89	103	90	89	105
161	M	113	88	140	103	120	-1	85	118	109	64	81	111
162	M	-1	-1	116	108	80	108	102	116	83	118	105	95
163	M	-1	92	124	112	120	-1	94	93	114	76	65	93
164	M	89	75	117	112	84	104	92	89	92	87	102	99
165	M	102	90	107	-1	57	135	-1	115	92	94	95	81
166	M	-1	83	107	-1	-1	-1	-1	-1	-1	-1	-1	-1
167	M	113	104	123	112	93	143	113	105	92	101	102	124
168	M	85	87	101	-1	89	-1	-1	-1	72	147	136	91
169	M	108	109	140	112	120	-1	103	115	103	105	99	122
170	M	96	90	119	112	93	119	97	89	69	129	101	83
171	M	85	81	113	112	84	112	87	109	97	97	85	107
172	M	96	99	140	99	120	-1	108	88	100	94	87	107
173	M	124	119	130	112	111	123	100	102	112	90	87	112
174	M	140	151	140	112	120	119	103	102	134	76	85	126
175	M	85	80	104	63	84	-1	94	99	94	115	95	89
176	M	96	87	105	112	-1	100	84	88	-1	-1	-1	79
177	M	113	90	117	112	93	119	113	95	97	105	95	87
178	M	113	104	130	86	120	-1	113	99	109	76	102	111
179	M	121	110	135	-1	102	88	-1	122	77	115	89	81
180	M	108	83	108	112	120	116	65	104	103	84	91	99
181	M	124	101	124	112	120	-1	101	95	100	90	85	101
182	M	91	83	103	81	84	-1	113	91	66	136	138	89
183	M	82	80	109	112	80	127	85	89	94	118	87	89
184	M	-1	-1	106	90	61	119	97	86	74	105	106	79
185	M	85	87	105	108	80	119	94	91	92	112	105	99
186	M	92	80	109	112	89	92	94	92	74	133	114	81
187	M	-1	-1	101	-1	-1	-1	-1	-1	-1	-1	-1	-1
188	M	113	98	131	-1	120	88	-1	109	117	101	113	109
189	M	108	81	117	112	89	108	113	106	86	101	91	95
190	M	-1	89	99	-1	-1	-1	-1	-1	-1	-1	-1	-1
191	M	106	107	124	112	120	92	101	79	103	76	85	103
192	M	89	81	106	112	84	108	99	99	94	94	71	128
193	M	85	82	120	86	93	-1	113	89	114	94	99	114
194	M	85	95	106	112	120	-1	55	108	112	97	91	103
195	M	-1	73	94	-1	-1	-1	-1	-1	-1	-1	-1	-1
196	M	96	100	126	99	120	-1	105	101	112	90	93	97
197	M	-1	-1	93	-1	-1	-1	-1	-1	-1	-1	-1	-1
198	M	163	165	140	112	120	84	113	134	137	69	63	134
199	M	146	172	140	112	120	84	113	132	126	76	85	134
200	M	85	86	104	-1	70	92	-1	119	109	105	102	83
201	M	96	89	112	72	116	-1	113	106	92	115	101	87
202	M	106	104	117	76	93	104	113	-1	109	80	75	89
203	M	102	95	122	-1	102	88	-1	101	94	101	118	87
204	M	130	117	139	112	120	100	113	112	117	90	81	124
205	M	101	74	86	112	75	96	113	104	64	129	121	75
206	M	108	110	131	99	120	-1	94	105	106	90	89	109
207	M	108	85	110	112	84	92	74	96	97	97	91	120

Pupil	Sex	OF	DP	MA	RT23	RT22	RT27	NC	CW	SCM	ATM	TA	F-S
208	F	102	98	115	112	102	88	108	92	89	118	106	95
209	F	85	101	124	94	107	-1	-1	137	103	90	102	105
210	F	-1	121	126	-1	-1	92	-1	-1	-1	-1	-1	-1
211	F	91	97	122	112	102	88	94	95	112	84	101	101
212	F	108	106	130	112	102	92	113	106	112	105	121	91
213	F	113	106	111	76	120	112	113	104	112	76	95	-1
214	F	-1	112	121	112	107	88	102	-1	106	94	91	103
215	F	91	93	115	103	98	88	103	102	103	97	91	101
216	F	85	108	116	67	75	104	73	102	89	90	99	93
217	F	85	100	101	99	70	108	65	79	92	140	109	81
218	F	101	127	140	99	111	88	-1	112	117	84	73	114
219	F	101	103	131	103	111	80	113	98	106	101	101	105
220	F	91	111	106	112	98	92	85	104	89	129	113	91
221	F	102	103	114	112	98	88	113	102	69	123	109	95
222	F	113	102	103	81	111	76	-1	105	86	105	110	107
223	F	108	105	117	76	102	88	92	141	81	123	114	87
224	F	91	114	134	99	107	76	97	111	92	118	102	124
225	F	85	96	113	99	107	92	76	112	92	90	93	89
226	F	80	91	107	86	66	96	84	79	86	115	99	91
227	F	102	106	119	108	102	88	103	86	77	108	106	93
228	F	96	91	111	81	75	100	76	105	92	101	77	91
229	F	111	100	118	112	84	84	95	106	109	87	91	105
230	F	124	95	128	108	111	-1	-1	-1	122	87	105	128
231	F	91	97	95	94	75	-1	56	86	83	123	109	93
232	F	130	139	140	94	107	88	-1	-1	114	94	91	132
233	F	121	112	130	99	111	76	113	98	106	97	89	122
234	F	85	98	123	94	89	88	113	73	103	94	105	103
235	F	85	-1	104	112	98	88	87	96	92	97	99	91
236	F	108	119	119	90	102	88	101	101	83	105	125	99
237	F	85	111	120	99	107	92	87	127	109	97	114	87
238	F	82	92	97	90	70	108	61	109	77	118	125	85
239	F	101	110	130	108	116	88	104	83	103	108	113	124

Pupil	Sex	OF	DP	MA	RT23	RT22	RT27	NC	CW	SCM	ATM	TA	P-S
240	M	109	98	114	94	93	104	97	124	72	118	101	91
241	M	130	118	134	-1	102	80	113	112	129	87	99	128
242	M	82	99	96	112	93	104	81	96	114	90	99	89
243	M	85	93	111	94	98	76	87	106	69	129	132	87
244	M	101	111	125	76	107	76	113	98	-1	-1	-1	91
245	M	85	84	100	86	66	119	85	121	92	133	132	77
246	M	96	96	112	99	102	104	113	112	97	101	101	99
247	M	85	101	107	112	102	108	-1	118	81	87	95	87
248	M	85	111	102	112	93	108	97	82	120	94	129	81
249	M	-1	81	85	-1	-1	116	97	124	-1	-1	-1	-1
250	M	80	110	102	99	107	104	-1	96	103	105	91	99
251	M	85	93	116	108	93	-1	70	129	92	87	87	83
252	M	85	113	115	112	107	104	-1	112	81	133	129	93
253	M	113	120	125	94	84	88	103	122	100	87	99	85
254	M	82	75	79	76	89	127	73	137	86	97	85	77
255	M	82	92	108	99	-1	119	81	104	-1	-1	-1	-1
256	M	123	106	128	72	98	80	-1	98	114	90	106	112
257	M	82	93	101	76	107	116	65	93	120	76	105	77
258	M	85	91	104	81	98	100	58	102	86	90	101	89
259	M	113	116	125	112	107	80	90	124	103	90	117	118
260	M	108	101	127	99	107	84	108	98	114	97	93	128
261	M	85	99	111	86	93	108	-1	-1	100	90	101	79
262	M	96	109	119	99	102	76	104	116	112	97	93	95
263	M	92	108	126	99	80	92	-1	119	86	105	105	116
264	M	121	97	114	90	93	76	90	104	117	87	91	101
265	M	113	119	140	99	98	88	108	112	92	108	95	107
266	M	-1	90	106	-1	-1	119	85	96	-1	-1	-1	-1
267	M	82	101	123	103	102	88	113	79	122	76	63	99
268	M	82	88	111	86	-1	84	89	141	-1	-1	-1	77
269	M	89	92	112	112	98	96	-1	115	83	112	117	85
270	M	91	93	108	103	102	-1	-1	91	106	97	106	83
271	M	99	94	118	112	120	88	100	115	89	84	79	112
272	M	-1	-1	123	112	120	84	100	109	100	84	93	114
273	M	102	102	125	112	102	-1	-1	-1	137	80	106	109
274	M	113	112	134	81	107	92	-1	114	103	101	109	97
275	M	116	99	118	103	107	104	61	111	126	94	113	95
276	M	85	101	113	90	102	92	101	115	100	101	91	97
277	M	85	99	113	108	102	96	-1	79	94	94	117	91
278	M	91	112	122	112	75	80	109	114	97	87	81	112
279	M	85	98	108	103	102	92	-1	96	97	112	99	81
280	M	96	-1	123	112	102	80	93	118	134	90	113	122
281	M	82	81	83	94	84	100	49	115	-1	-1	-1	85
282	M	96	109	128	108	75	76	101	108	97	97	91	99
283	M	91	103	97	112	107	-1	-1	102	114	87	85	79

APPENDIX 7

EXAMPLES OF INDIVIDUAL PUPILS' RESPONSES TO DIVERGENT PRODUCTION TESTS

For each of Tests 2, 3, 5a, 11, 12, 16, and 18 the following is provided:

1. Number and title of the test.
2. A brief description of the test task - see Appendix 1 for full details.
3. The index numbers of two pupils to be compared.
4. The Mathematics Attainment (MA) scores of these two pupils - these will be the same in each case.
5. The scores accredited to them on the divergent production test in question.
6. The complete set of responses provided by each pupil.
7. An indication of responses credited for originality (*).
8. An indication of responses judged inappropriate or incorrect (!).

Test 2 (Counters)

(In this test pupils are required to consider a child arranging 12 counters in a 3 x 4 rectangular array, and suggest mathematical questions the child might be trying to answer).

Pupil no. 199
MA score = 140
Score on this DP test = 35

$6 \times 2 = 12$
$2 \times 6 = 12$
$4 \times 3 = 12$
$3 \times 4 = 12$
$4 + 4 + 4 = 12$
$3 + 3 + 3 + 3 = 12$
$(3 \times 2) + (3 \times 2) = 12$
$(2 \times 3) + (2 \times 3) = 12$
$(2 \times 3) + (3 \times 2) = 12$
$(3 \times 2) + (2 \times 3) = 12$
$12 \div 6 = 2$
$12 \div 2 = 6$
$12 \div 4 = 3$
$12 \div 3 = 4$
$(3 \times 3) + (3 \times 1) = 12$
*$12 - 3 - 3 - 3 = 3$
$12 - 4 - 4 = 4$
$(3 \times 1) + (3 \times 3) = 12$
$(3 \times 4) + (3 \times 0) = 12$
$(3 \times 0) + (3 \times 4) = 12$
$(4 \times 3) + (0 \times 3) + 12$
$(0 \times 3) + (4 \times 3) = 12$
$(4 \times 3) + (3 \times 0) = 12$
*$(3 \times 2) \times 2 = 12$
*How many ways can 12 counters be arranged?
$(2 \times 3) \times 2 = 12$
$(3 \times 3) + 3 = 12$
*Find the factors of 12
$3 \times 2 \times 2 = 12$
*L = 3, M = 2, find LMM
Find MML

Pupil no. 59
MA score = 140
Score on this DP test = 8

$6 \times 2 = 12$
$12 \div 6 = 2$
$3 \times 4 = 12$
$12 \div 3 = 4$
!$6 \div 6 = 1$
!$6 + 5 = 11$
!$6 + 4 = 10$
!$6 + 3 = 9$
!$6 + 2 = 8$
!$6 + 1 = 7$
!$6 - 1 = 5$
!$3 \times 3 = 9$
!$2 \times 5 = 10$
! $9 = 3$
!$(6 \div 2) - (5 - 2) = 0$
!2^3
*$2/3$ of $12 = 8$

N.B. * indicates a response credited for originality
! indicates an inappropriate or incorrect response

Test 3 (Subsets)

(In this test pupils are required to make up subsets from the set of integers, 2 - 16, in each case stating what the rule is for membership of the subset).

Pupil no. 19
MA score = 129
Score on this DP test = 32

3, 5, 7, 9, 11, 13, 15 (odd)
*2, 3, 5, 7, 11, 13 (prime)
2, 3, 4, 5, 6, 7, 8, 9, (under 10)
*2, 3, 5, 7 (prime under 10)
*13, 14, 15, 16 (teen numbers)
*2, 6(have 3 letters)
4, 5, 9(have 4 letters)
4, 8, 12, 16(divisible by 4)
3, 6, 9, 12, 15(divisible by 3)
5, 10, 15 (divisible by 5)
6, 12 (divisible by 6)
7, 14 (divisible by 7)
8, 16 (divisible by 8)
*7, 11, 13, 14, 15, 16, (more than one syllable)
1,2,3,4,5,6,8,9,10,12, (one syllable)
6, 12 (divisible by both 2 and 3)
*2,4,5,7,8,10,11,13,14,16 (not divisible by 3)

Pupil no. 85
MA score = 129
Score on this DP test = 5

3,6,9,12,15 (divisible by 3)
4,8,12,16 (divisible by 4)
5,10,15 (divisible by 5)
6,12 (divisible by 6)
7,14 (divisible by 7)
!8,16,24 (divisible by 8)
!9,18,27,39 (divisible by 9)
!10,20,30,40 (divisible by 10)
!11,22,33,44 (divisible by 11)
!12,24,36,48 (divisible by.12)
!12,26,39 (divisible by 13)
!14,28,42 (divisible by 14)
!15,30,45 (divisible by 15)
!16,32,48 (divisible by 16)
!17,34,51 (divisible by 17)
!18,36,54 (divisible by 18)

N.B. * indicates a response credited for originality
! indicates an inappropriate or incorrect response

Test 5a (Similarities: numbers)

(In this test pupils are required to state what is the same about two given numbers, 16 and 36).

Pupil no. 53

MA score = 140

Score on this DP test = 18

Both even

Both divide by 2

Both divide by 4

Both have a 6 in them

*Both not prime

Both less than 40

Both above 15

*Both whole numbers

Both written in squares

*Both factors of 576

Pupil no. 107

MA score = 140

Score on this DP test = 4

Both have 6's in them

Both multiples of 2

Both in boxes

Both multiples of 4

N.B * indicates a response credited for originality

Test 11 (Crossnumber)

(In this test pupils are required to make up the clues for a cross-number puzzle in which the answers are already provided: 121, 32, 49, 100, 144, 13, 250, 91).

Pupil no. 24 MA score = 128 Score on this DP test = 20	Pupil no. 282 MA score = 128 Score on this DP test = 5
10 x 10 + 3 x 7	7 x 3 + 100
*If one dozen eggs cost £1.28 what will a quarter of a dozen cost?	8 x 4
147 - 3	5 x 8 + 9
*X x V + X + VIII + II + X x III	10^2
If a thousand cost £14.40 what will a hundred cost?	12^2
*Find a prime factor of 130	An unlucky number
5 x 100 - 2	50^2
*If I was 50 and you were 41 years older, how old are you?	!$8^2 + 8$

N.B. * indicates a response credited for originality

! indicates an inappropriate or incorrect response

Test 12 (Scattergram)

(In this test pupils are required to pose questions which can be answered from a given scattergram showing the distribution of boys and girls in 30 families).

Pupil no. 174
MA score = 140
Score on this DP test = 30

*How many families had twice as many boys as girls? (4)
*How many families had the same number of girls as boys? (7)
*How many girls altogether? (39)
How many boys altogether?(46)
*How many families have 1 child? (3)
*How many families with more than 6 children ? (1)
*What is the number of families with two kids? (12)
How many families altogether? (30)
*How many children altogether? (85)
*Are there any eight children families? (no)
*How many families have no children? (0)
How many families with the same number of boys as girls (7)

Pupil no. 137
MA score = 140
Score on this DP test = 3

How many families had 4 boys and no girls? (1)
How many families had 4 boys and 3 girls? (1)
How many families had 1 boy and no girls? (5)
How many families had 2 boys and 2 girls? (2)
How many families had 2 boys and 1 girl? (4)
How many families had 2 boys and 3 girls? (1)
How many families had 2 boys and 3 girls? (1)
How many families had 3 boys and 1 girl? (1)
How many families had 3 boys and 0 girls? (1)
How many families had 3 boys and 2 girls? (1)
How many families had 0 boys and 2 girls? (4)

N.B. * indicates a response credited for originality

Test 16 (Results)

(In this test pupils are required to deduce other results from the given result: 23 x 35 = 805).

Pupil no. 17 MA score = 129 Score on this DP test = 27	Pupil no. 7 MA score = 129 Score on this DP test = 7
230 x 35 = 8050	230 x 35 = 8050
35 x 23 = 805	23 x 3500 = 80500
*$17\frac{1}{2}$ x 23 = $402\frac{1}{2}$	!23 ÷ 805 = 35
35 x $11\frac{1}{2}$ = $402\frac{1}{2}$	!35 ÷ 805 = 23
*70 x 23 = 1610	230 x 350 = 80500
23 x 70 =1610	23 x 350000 = 8050000
46 x 35 = 1610	23000000 x 35000000 = 805000000000000
*(22 + 1) x 35 = 805	3500 x 230 = 805000
(21 + 2) x 35 = 805	230000 x 350000000 = 80500000000000
(20 + 3) x 35 = 805	2300 x 3500000000000000 = 8050000000000000000
*($17\frac{1}{2}$ x 2) x 23 = 805	
(30 + 5) x 23 = 805	23000000000000 x 3500000000000 = 80500000000000000000000000
(32 + 3) x 23 = 805	
*(70 ÷ 2) x 23 = 805	35000 x 23000 = 805000000
(46 ÷ 2) x 23 = 805	23 x 350000 = 8050000
35 x 46 = 1610	35 x 2300000 = 80500000
(19 + 2 + 2 x 1) x 35 = 805	23000 x 3500000000 = 80500000000000
$11\frac{1}{2}$ x 35 = $402\frac{1}{2}$	23 x 35000000000 = 805000000000
1610 ÷ 70 = 23	
1610 ÷ 23 = 70	
8050 ÷ 23 = 350	
8050 ÷ 350 = 23	
$402\frac{1}{2}$ ÷ 23 = $17\frac{1}{2}$	
$402\frac{1}{2}$ ÷ $17\frac{1}{2}$ = 23	
805 ÷ 23 = 35	
805 ÷ 35 = 23	

N.B. * indicates a response credited for originality

! indicates an inappropriate or incorrect response

Test 18 (Three Cards)

(In this test pupils are required to find possible values for A, B and C such that (A + B) x C = 9).

Pupil no. 43
MA score = 135
Score on this DP test = 31

A	B	C
2	1	3
1	2	3
!$1\frac{1}{2}$	$\frac{1}{2}$	3
3	0	3
0	3	3
*1.5	0	6
0	1.5	6
0.75	0.75	6
*3	3	1.5
6	0	1.5
0	6	1.5
*6	6	0.75
12	0	0.75
0	12	0.75
4.5	0	2
0	4.5	2
2.25	2.25	2
5	4	1
4	5	1
6	3	1
3	6	1
2	7	1
7	2	1
1	8	1
3	1.5	2
1.5	3	2
3.5	1	2
1	3.5	2
1.75	1.25	3
1.25	1.75	3
1.6	1.4	3
1.4	1.6	3

N.B.
* indicates a response credited for originality

! indicates an inappropriate or incorrect response

Pupil no. 179
MA score = 135
Score on this DP test = 4

A	B	C
1	2	3
2	1	3
!6	3	0
3	6	0
0	1	9
1	0	9
!1	8	0
!8	1	0
!8	0	1
!8	1	0
!1	1	8
!4	5	0
!5	2	2
!5	3	1
!1	3	5
!5	1	3
!6	2	1
!6	1	2
!4	3	2
!4	2	3
!3	2	4
!2	4	3
!3	4	2
!1	2	4
!3	2	6
!1	7	1
!2	7	0
!6	0	9
!0	5	9
!2	2	5
!5	1	4